
MATHEMATICS

9709/31

Paper 3

October/November 2016

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

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Cambridge is publishing the mark schemes for the October/November 2016 series for most Cambridge IGCSE[®], Cambridge International A and AS Level components and some Cambridge O Level components.

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- The symbol ∇ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously “correct” answers or results obtained from incorrect working.
- Note: B2 or A2 means that the candidate can earn 2 or 0.
B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.
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The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE Any Equivalent Form (of answer is equally acceptable) / Or Equivalent

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

CAO Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)

CWO Correct Working Only – often written by a ‘fortuitous’ answer

ISW Ignore Subsequent Working

SOI Seen or implied

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through $\frac{1}{2}$ ” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

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1	Solve for 3^x and obtain $3^x = \frac{18}{7}$ Use correct method for solving an equation of the form $3^x = a$, where $a > 0$ Obtain answer $x = 0.860$ 3 d.p. only	B1 M1 A1	[3]
2	State correct unsimplified first two terms of the expansion of $(1 + 2x)^{-\frac{3}{2}}$, e.g. $1 + (-\frac{3}{2})(2x)$ State correct unsimplified term in x^2, e.g. $(-\frac{3}{2})(-\frac{3}{2} - 1)(2x)^2 / 2!$ Obtain sufficient terms of the product of $(2 - x)$ and the expansion up to the term in x^2 Obtain final answer $2 - 7x + 18x^2$ Do not ISW	B1 B1 M1 A1	[4]
3	<i>EITHER:</i> Correctly restate the equation in terms of $\sin \theta$ and $\cos \theta$ Correct method to obtain a horizontal equation in $\sin \theta$ Reduce the equation to a correct quadratic in any form, e.g. $3\sin^2 \theta - \sin \theta - 2 = 0$ Solve a three-term quadratic for $\sin \theta$ Obtain final answer $\theta = -41.8^\circ$ only [Ignore answers outside the given interval.] <i>OR 1:</i> Square both sides of the equation and use $1 + \tan^2 \theta = \sec^2 \theta$ Correct method to obtain a horizontal equation in $\sin \theta$ Reduce the equation to a correct quadratic in any form, e.g. $9\sin^2 \theta - 6\sin \theta - 8 = 0$ Solve a three-term quadratic for $\sin \theta$ Obtain final answer $\theta = -41.8^\circ$ only <i>OR 2:</i> Multiply through by $(\sec \theta + \tan \theta)$ Use $\sec^2 \theta - \tan^2 \theta = 1$ Obtain $1 = 3 + 3\sin \theta$ Solve for $\sin \theta$ Obtain final answer $\theta = -41.8^\circ$ only	B1 M1 A1 M1 A1 B1 M1 A1 M1 B1 A1 M1 A1	[5]

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4	<i>EITHER:</i> <i>EITHER:</i> State $2xy + x^2 \frac{dy}{dx}$, or equivalent, as derivative of $x^2 y$ State $6y^2 + 12xy \frac{dy}{dx}$, or equivalent, as derivative of $6xy^2$ <i>OR:</i> Differentiating LHS using correct product rule, state term $xy(1 - 6 \frac{dy}{dx})$, or equivalent State term $(y + x \frac{dy}{dx})(x - 6y)$, or equivalent Equate attempted derivative of LHS to zero and set $\frac{dy}{dx}$ equal to zero Obtain a horizontal equation, e.g. $6y^2 - 2xy = 0$ (from correct work only) Explicitly reject $y = 0$ as a possibility $py^2 - qxy = 0$ Obtain an equation in x or y Obtain answer $(-3a, -a)$	B1 B1 B1 B1 M1* A1 A1 DM1 A1	[7]
	<i>OR:</i> Rearrange to $y = \frac{9a^3}{x(x - 6y)}$ and use correct quotient rule to obtain $-\frac{9a^3}{x^2(x - 6y)^2} \times \dots$ State term $(x - 6y) + x(1 - 6y')$, or equivalent Justify division by $x(x - 6y)$ Set $\frac{dy}{dx}$ equal to zero Obtain a horizontal equation, e.g. $6y^2 - 2xy = 0$ (from correct work only) Obtain an equation in x or y Obtain answer $(-3a, -a)$	B1 B1 B1 M1* A1 DM1 A1	
5 (i)	<i>EITHER:</i> Use $\tan 2A$ formula to express LHS in terms of $\tan \theta$ Express as a single fraction in any correct form Use Pythagoras or $\cos 2A$ formula Obtain the given result correctly <i>OR:</i> Express LHS in terms of $\sin 2\theta$, $\cos 2\theta$, $\sin \theta$ and $\cos \theta$ Express as a single fraction in any correct form Use Pythagoras or $\cos 2A$ formula or $\sin(A - B)$ formula Obtain the given result correctly	M1 A1 M1 A1 M1 A1 M1 A1	[4]
(ii)	Integrate and obtain a term of the form $a \ln(\cos 2\theta)$ or $b \ln(\cos \theta)$ (or secant equivalents) Obtain integral $-\frac{1}{2} \ln(\cos 2\theta) + \ln(\cos \theta)$, or equivalent Substitute limits correctly (expect to see use of <u>both</u> limits) Obtain the given answer following full and correct working	M1* A1 DM1 A1	[4]

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6	(i)	Make recognizable sketch of a relevant graph Sketch the other relevant graph and justify the given statement	B1 B1	[2]
	(ii)	Use calculations to consider the value of a relevant expression at $x = 1.4$ and $x = 1.6$, or the values of relevant expressions at $x = 1.4$ and $x = 1.6$ Complete the argument correctly with correct calculated values	M1 A1	[2]
	(iii)	State $x = 2 \sin^{-1} \left(\frac{3}{x+3} \right)$ Rearrange this in the form $\operatorname{cosec} \frac{1}{2}x = \frac{1}{3}x + 1$ If working in reverse, need $\sin \frac{x}{2} = \left(\frac{3}{x+3} \right)$ for first B1	B1 B1	[2]
	(iv)	Use the iterative formula correctly at least once Obtain final answer 1.471 Show sufficient iterations to 5 d.p. to justify 1.471 to 3 d.p., or show there is a sign change in the interval (1.4705, 1.4715)	M1 A1 A1	[3]
7	(i)	Use the correct product rule Obtain correct derivative in any form, e.g. $(2-2x)e^{\frac{1}{2}x} + \frac{1}{2}(2x-x^2)e^{\frac{1}{2}x}$ Equate derivative to zero and solve for x Obtain $x = \sqrt{5} - 1$ only	M1 A1 M1 A1	[4]
	(ii)	Integrate by parts and reach $a(2x-x^2)e^{\frac{1}{2}x} + b \int (2-2x)e^{\frac{1}{2}x} dx$ Obtain $2e^{\frac{1}{2}x}(2x-x^2) - 2 \int (2-2x)e^{\frac{1}{2}x} dx$, or equivalent Complete the integration correctly, obtaining $(12x-2x^2-24)e^{\frac{1}{2}x}$, or equivalent Use limits $x = 0, x = 2$ correctly having integrated by parts twice Obtain answer $24 - 8e$, or <u>exact</u> simplified equivalent	M1* A1 A1 DM1 A1	[5]

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8	(i)	State or imply a correct normal vector to either plane, e.g. $3\mathbf{i} + \mathbf{j} - \mathbf{k}$ or $\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ Use correct method to calculate their scalar product Show value is zero and planes are perpendicular	B1 M1 A1	[3]
	(ii)	<i>EITHER:</i> Carry out a complete strategy for finding a point on l the line of intersection Obtain such a point, e.g. $(0, 7, 5)$, $(1, 0, 1)$, $(5/4, -7/4, 0)$ <i>EITHER:</i> State two equations for a direction vector $a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ for l, e.g. $3a + b - c = 0$ and $a - b + 2c = 0$ Solve for one ratio, e.g. $a : b$ Obtain $a : b : c = 1 : -7 : -4$, or equivalent State a correct answer, e.g. $\mathbf{r} = 7\mathbf{j} + 5\mathbf{k} + \lambda(\mathbf{i} - 7\mathbf{j} - 4\mathbf{k})$ <i>OR1:</i> Obtain a second point on l, e.g. $(1, 0, 1)$ Subtract vectors and obtain a direction vector for l Obtain $-\mathbf{i} + 7\mathbf{j} + 4\mathbf{k}$, or equivalent State a correct answer, e.g. $\mathbf{r} = \mathbf{i} + \mathbf{k} + \lambda(-\mathbf{i} + 7\mathbf{j} + 4\mathbf{k})$ <i>OR2:</i> Attempt to find the vector product of the two normal vectors Obtain two correct components of the product Obtain $\mathbf{i} - 7\mathbf{j} - 4\mathbf{k}$, or equivalent State a correct answer, e.g. $\mathbf{r} = 7\mathbf{j} + 5\mathbf{k} + \lambda(\mathbf{i} - 7\mathbf{j} - 4\mathbf{k})$ <i>OR1:</i> Express one variable in terms of a second variable Obtain a correct simplified expression, e.g. $y = 7 - 7x$ Express the third variable in terms of the second Obtain a correct simplified expression, e.g. $z = 5 - 4x$ Form a vector equation for the line Obtain a correct equation, e.g. $\mathbf{r} = 7\mathbf{j} + 5\mathbf{k} + \lambda(\mathbf{i} - 7\mathbf{j} - 4\mathbf{k})$ <i>OR2:</i> Express one variable in terms of a second variable Obtain a correct simplified expression, e.g. $z = 5 - 4x$ Express the same variable in terms of the third Obtain a correct simplified expression e.g. $z = (7 + 4y) / 7$ Form a vector equation for the line Obtain a correct equation, e.g. $\mathbf{r} = \frac{5}{4}\mathbf{i} - \frac{7}{4}\mathbf{j} + \lambda(-\frac{1}{4}\mathbf{i} + \frac{7}{4}\mathbf{j} + \mathbf{k})$	M1 A1 B1 M1 A1 A1✓ B1 M1 A1 A1✓ M1 A1 A1 A1✓ M1 A1 M1 A1 M1 A1✓ M1 A1 M1 A1 M1 A1✓	[6]

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9	(a)	EITHER: Use quadratic formula to solve for w Use $i^2 = -1$ Obtain one of the answers $w = \frac{1}{2i+1}$ and $w = -\frac{5}{2i+1}$ Multiply numerator and denominator of an answer by $-2i + 1$, or equivalent Obtain final answers $\frac{1}{5} - \frac{2}{5}i$ and $-1 + 2i$ OR1: Multiply the equation by $1 - 2i$ Use $i^2 = -1$ Obtain $5w^2 + 4w(1 - 2i) - (1 - 2i)^2 = 0$, or equivalent Use quadratic formula or factorise to solve for w Obtain final answers $\frac{1}{5} - \frac{2}{5}i$ and $-1 + 2i$ OR2: Substitute $w = x + iy$ and form equations for real and imaginary parts Use $i^2 = -1$ Obtain $(x^2 - y^2) - 4xy + 4x - 1 = 0$ and $2(x^2 - y^2) + 2xy + 4y + 2 = 0$ o.e. Form equation in x only or y only and solve Obtain final answers $\frac{1}{5} - \frac{2}{5}i$ and $-1 + 2i$	M1 M1 A1 M1 A1 M1 M1 A1 M1 A1 M1 M1 A1 M1 A1	[5]
	(b)	Show a circle with centre $1 + i$ Show a circle with radius 2 Show half-line $\arg z = \frac{1}{4}\pi$ Show half-line $\arg z = -\frac{1}{4}\pi$ Shade the correct region	B1 B1 B1 B1 B1	[5]

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10	(i)	Separate variables correctly and integrate at least one side Integrate and obtain term kt, or equivalent Carry out a relevant method to obtain A and B such that $\frac{1}{x(4-x)} \equiv \frac{A}{x} + \frac{B}{4-x}$, or equivalent Obtain $A = B = \frac{1}{4}$, or equivalent Integrate and obtain terms $\frac{1}{4} \ln x - \frac{1}{4} \ln(4-x)$, or equivalent EITHER: Use a pair of limits in an expression containing $p \ln x$, $q \ln(4-x)$ and rt and evaluate a constant Obtain correct answer in any form, e.g. $\ln x - \ln(4-x) = 4kt - \ln 9$, or $\ln\left(\frac{x}{4-x}\right) = 4kt - 8k$ Use a second pair of limits and determine k Obtain the given exact answer correctly OR: Use both pairs of limits in a definite integral Obtain the given exact answer correctly Substitute k and either pair of limits in an expression containing $p \ln x$, $q \ln(4-x)$ and rt and evaluate a constant Obtain $\ln \frac{x}{4-x} = t \ln 3 - \ln 9$ or equivalent	M1 A1 M1* A1 A1✓ DM1 A1 DM1 A1 M1* A1 DM1 A1	[9]
	(ii)	Substitute $x = 3.6$ and solve for t Obtain answer $t = 4$	M1 A1	[2]

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5 (i)	<i>EITHER:</i> Use $\tan 2A$ formula to express LHS in terms of $\tan \theta$ Express as a single fraction in any correct form Use Pythagoras or $\cos 2A$ formula Obtain the given result correctly <i>OR:</i> Express LHS in terms of $\sin 2\theta$, $\cos 2\theta$, $\sin \theta$ and $\cos \theta$ Express as a single fraction in any correct form Use Pythagoras or $\cos 2A$ formula or $\sin(A - B)$ formula Obtain the given result correctly	M1 A1 M1 A1 M1 A1 M1 A1	[4]
(ii)	Integrate and obtain a term of the form $a \ln(\cos 2\theta)$ or $b \ln(\cos \theta)$ (or secant equivalents) Obtain integral $-\frac{1}{2} \ln(\cos 2\theta) + \ln(\cos \theta)$, or equivalent Substitute limits correctly (expect to see use of <u>both</u> limits) Obtain the given answer following full and correct working	M1* A1 DM1 A1	[4]

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	(ii)	Integrate by parts and reach $a(2x-x^2)e^{\frac{1}{2}x} + b \int (2-2x)e^{\frac{1}{2}x} dx$ Obtain $2e^{\frac{1}{2}x}(2x-x^2) - 2 \int (2-2x)e^{\frac{1}{2}x} dx$, or equivalent Complete the integration correctly, obtaining $(12x-2x^2-24)e^{\frac{1}{2}x}$, or equivalent Use limits $x = 0, x = 2$ correctly having integrated by parts twice Obtain answer $24 - 8e$, or <u>exact</u> simplified equivalent	M1* A1 A1 DM1 A1	[5]

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8	(i)	State or imply a correct normal vector to either plane, e.g. $3\mathbf{i} + \mathbf{j} - \mathbf{k}$ or $\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ Use correct method to calculate their scalar product Show value is zero and planes are perpendicular	B1 M1 A1	[3]
	(ii)	<i>EITHER:</i> Carry out a complete strategy for finding a point on l the line of intersection Obtain such a point, e.g. $(0, 7, 5)$, $(1, 0, 1)$, $(5/4, -7/4, 0)$ <i>EITHER:</i> State two equations for a direction vector $a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ for l, e.g. $3a + b - c = 0$ and $a - b + 2c = 0$ Solve for one ratio, e.g. $a : b$ Obtain $a : b : c = 1 : -7 : -4$, or equivalent State a correct answer, e.g. $\mathbf{r} = 7\mathbf{j} + 5\mathbf{k} + \lambda(\mathbf{i} - 7\mathbf{j} - 4\mathbf{k})$ <i>OR1:</i> Obtain a second point on l, e.g. $(1, 0, 1)$ Subtract vectors and obtain a direction vector for l Obtain $-\mathbf{i} + 7\mathbf{j} + 4\mathbf{k}$, or equivalent State a correct answer, e.g. $\mathbf{r} = \mathbf{i} + \mathbf{k} + \lambda(-\mathbf{i} + 7\mathbf{j} + 4\mathbf{k})$ <i>OR2:</i> Attempt to find the vector product of the two normal vectors Obtain two correct components of the product Obtain $\mathbf{i} - 7\mathbf{j} - 4\mathbf{k}$, or equivalent State a correct answer, e.g. $\mathbf{r} = 7\mathbf{j} + 5\mathbf{k} + \lambda(\mathbf{i} - 7\mathbf{j} - 4\mathbf{k})$ <i>OR1:</i> Express one variable in terms of a second variable Obtain a correct simplified expression, e.g. $y = 7 - 7x$ Express the third variable in terms of the second Obtain a correct simplified expression, e.g. $z = 5 - 4x$ Form a vector equation for the line Obtain a correct equation, e.g. $\mathbf{r} = 7\mathbf{j} + 5\mathbf{k} + \lambda(\mathbf{i} - 7\mathbf{j} - 4\mathbf{k})$ <i>OR2:</i> Express one variable in terms of a second variable Obtain a correct simplified expression, e.g. $z = 5 - 4x$ Express the same variable in terms of the third Obtain a correct simplified expression e.g. $z = (7 + 4y) / 7$ Form a vector equation for the line Obtain a correct equation, e.g. $\mathbf{r} = \frac{5}{4}\mathbf{i} - \frac{7}{4}\mathbf{j} + \lambda(-\frac{1}{4}\mathbf{i} + \frac{7}{4}\mathbf{j} + \mathbf{k})$	M1 A1 B1 M1 A1 A1✓ B1 M1 A1 A1✓ M1 A1 A1 A1✓ M1 A1 M1 A1 M1 A1✓ M1 A1 M1 A1 M1 A1✓	[6]

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9	(a)	EITHER: Use quadratic formula to solve for w Use $i^2 = -1$ Obtain one of the answers $w = \frac{1}{2i+1}$ and $w = -\frac{5}{2i+1}$ Multiply numerator and denominator of an answer by $-2i + 1$, or equivalent Obtain final answers $\frac{1}{5} - \frac{2}{5}i$ and $-1 + 2i$ OR1: Multiply the equation by $1 - 2i$ Use $i^2 = -1$ Obtain $5w^2 + 4w(1 - 2i) - (1 - 2i)^2 = 0$, or equivalent Use quadratic formula or factorise to solve for w Obtain final answers $\frac{1}{5} - \frac{2}{5}i$ and $-1 + 2i$ OR2: Substitute $w = x + iy$ and form equations for real and imaginary parts Use $i^2 = -1$ Obtain $(x^2 - y^2) - 4xy + 4x - 1 = 0$ and $2(x^2 - y^2) + 2xy + 4y + 2 = 0$ o.e. Form equation in x only or y only and solve Obtain final answers $\frac{1}{5} - \frac{2}{5}i$ and $-1 + 2i$	M1 M1 A1 M1 A1 M1 M1 A1 M1 A1 M1 M1 A1 M1 A1	[5]
	(b)	Show a circle with centre $1 + i$ Show a circle with radius 2 Show half-line $\arg z = \frac{1}{4}\pi$ Show half-line $\arg z = -\frac{1}{4}\pi$ Shade the correct region	B1 B1 B1 B1 B1	[5]

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10	(i)	Separate variables correctly and integrate at least one side Integrate and obtain term kt, or equivalent Carry out a relevant method to obtain A and B such that $\frac{1}{x(4-x)} \equiv \frac{A}{x} + \frac{B}{4-x}$, or equivalent Obtain $A = B = \frac{1}{4}$, or equivalent Integrate and obtain terms $\frac{1}{4} \ln x - \frac{1}{4} \ln(4-x)$, or equivalent EITHER: Use a pair of limits in an expression containing $p \ln x$, $q \ln(4-x)$ and rt and evaluate a constant Obtain correct answer in any form, e.g. $\ln x - \ln(4-x) = 4kt - \ln 9$, or $\ln\left(\frac{x}{4-x}\right) = 4kt - 8k$ Use a second pair of limits and determine k Obtain the given exact answer correctly OR: Use both pairs of limits in a definite integral Obtain the given exact answer correctly Substitute k and either pair of limits in an expression containing $p \ln x$, $q \ln(4-x)$ and rt and evaluate a constant Obtain $\ln \frac{x}{4-x} = t \ln 3 - \ln 9$ or equivalent	M1 A1 M1* A1 A1✓ DM1 A1 DM1 A1 M1* A1 DM1 A1	[9]
	(ii)	Substitute $x = 3.6$ and solve for t Obtain answer $t = 4$	M1 A1	[2]

MATHEMATICS

9709/33

Paper 3

October/November 2016

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the October/November 2016 series for most Cambridge IGCSE[®], Cambridge International A and AS Level components and some Cambridge O Level components.

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- The symbol ∇ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously “correct” answers or results obtained from incorrect working.
- Note: B2 or A2 means that the candidate can earn 2 or 0.
B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.
- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

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The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE Any Equivalent Form (of answer is equally acceptable) / Or Equivalent

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

CAO Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)

CWO Correct Working Only – often written by a ‘fortuitous’ answer

ISW Ignore Subsequent Working

SOI Seen or implied

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through $\frac{1}{2}$ ” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

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1	Use law of the logarithm of a quotient Remove logarithms and obtain a correct equation, e.g. $e^z = \frac{y+2}{y+1}$ Obtain answer $y = \frac{2-e^z}{e^z-1}$, or equivalent	M1 A1 A1	[3]
2	Use correct quotient or product rule Obtain correct derivative in any form Use Pythagoras to simplify the derivative to $\frac{1}{1+\cos x}$, or equivalent Justify the given statement, $-1 < \cos x < 1$ statement, or equivalent	M1 A1 A1 A1	[4]
3	Use the $\tan 2A$ formula to obtain an equation in $\tan \theta$ only Obtain a correct horizontal equation Rearrange equation as a quadratic in $\tan \theta$, e.g. $3\tan^2 \theta + 2\tan \theta - 1 = 0$ Solve for θ (usual requirements for solution of quadratic) Obtain answer, e.g. 18.4° Obtain second answer, e.g. 135°, and no others in the given interval	M1 A1 A1 M1 A1 A1	[6]
4 (i)	Commence division by $x^2 - x + 2$ and reach a partial quotient $4x^2 + kx$ Obtain quotient $4x^2 + 4x + a - 4$ or $4x^2 + 4x + b / 2$ Equate x or constant term to zero and solve for a or b Obtain $a = 1$ Obtain $b = -6$	M1 A1 M1 A1 A1	[5]
(ii)	Show that $x^2 - x + 2 = 0$ has no real roots Obtain roots $\frac{1}{2}$ and $-\frac{3}{2}$ from $4x^2 + 4x - 3 = 0$	B1 B1	[2]
5 (i)	State equation $\frac{dy}{dx} = \frac{1}{2}xy$	B1	[1]
(ii)	Separate variables correctly and attempts to integrate one side of equation Obtain terms of the form $a \ln y$ and bx^2 Use $x = 0$ and $y = 2$ to evaluate a constant, or as limits, in expression containing $a \ln y$ or bx^2 Obtain correct solution in any form, e.g. $\ln y = \frac{1}{4}x^2 + \ln 2$ Obtain correct expression for y, e.g. $y = 2e^{\frac{1}{4}x^2}$	M1 A1 M1 A1 A1	[5]
(iii)	Show correct sketch for $x \geq 0$. Needs through $(0, 2)$ and rapidly increasing positive gradient.	B1	[1]

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6	(i)	State or imply $du = \frac{1}{2\sqrt{x}} dx$ Substitute for x and dx throughout Justify the change in limits and obtain the given answer	B1 M1 A1	[3]
	(ii)	Convert integrand into the form $A + \frac{B}{u+1}$ Obtain integrand $A=1, B=-2$ Integrate and obtain $u - 2\ln(u+1)$ Substitute limits correctly in an integral containing terms au and $b\ln(u+1)$, where $ab \neq 0$ Obtain the given answer following full and correct working [The f.t. is on A and B .]	M1* A1 A1 [✓] + A1 [✓] DM1 A1	[6]
7	(i)	State modulus $2\sqrt{2}$, or equivalent State argument $-\frac{1}{3}\pi$ (or -60°)	B1 B1	[2]
	(ii) (a)	State answer $3\sqrt{2} + \sqrt{6}i$	B1	
	(b)	<i>EITHER:</i> Substitute for z and multiply numerator and denominator by conjugate of iz Simplify the numerator to $4\sqrt{3} + 4i$ or the denominator to 8 Obtain final answer $\frac{1}{2}\sqrt{3} + \frac{1}{2}i$ <i>OR:</i> Substitute for z , obtain two equations in x and y and solve for x or for y Obtain $x = \frac{1}{2}\sqrt{3}$ or $y = \frac{1}{2}$ Obtain final answer $\frac{1}{2}\sqrt{3} + \frac{1}{2}i$	M1 A1 A1 M1 A1 A1	[4]
	(iii)	Show points A and B in relatively correct positions Carry out a complete method for finding angle AOB , e.g. calculate the argument of $\frac{z^*}{iz}$ Obtain the given answer	B1 M1 A1	[3]
8	(i)	State or imply the form $\frac{A}{x+2} + \frac{Bx+C}{x^2+4}$ Use a correct method to determine a constant Obtain one of $A=2, B=1, C=-1$ Obtain a second value Obtain a third value	B1 M1 A1 A1 A1	[5]

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(ii)	Use correct method to find the first two terms of the expansion of $(x+2)^{-1}$, $(1+\frac{1}{2}x)^{-1}$, $(4+x^2)^{-1}$ or $(1+\frac{1}{4}x^2)^{-1}$ Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction Multiply out fully by $Bx + C$, where $BC \neq 0$ Obtain final answer $\frac{3}{4} - \frac{1}{4}x + \frac{5}{16}x^2$, or equivalent [Symbolic binomial coefficients, e.g. $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$ are not sufficient for the M1. The f.t. is on A, B, C.] [In the case of an attempt to expand $(3x^2 + x + 6)(x+2)^{-1}(x^2+4)^{-1}$, give M1A1A1 for the expansions, M1 for multiplying out fully, and A1 for the final answer.]	M1 A1✓ + A1✓ M1 A1	[5]
9 (i)	Differentiate both equations and equate derivatives Obtain equation $\cos a - a \sin a = -\frac{k}{a^2}$ State $a \cos a = \frac{k}{a}$ and eliminate k Obtain the given answer showing sufficient working	M1* A1 + A1 DM1 A1	[5]
(ii)	Show clearly correct use of the iterative formula at least once Obtain answer 1.077 Show sufficient iterations to 5 d.p. to justify 1.077 to 3 d.p., or show there is a sign change in the interval (1.0765, 1.0775)	M1 A1 A1	[3]
(iii)	Use a correct method to determine k Obtain answer $k = 0.55$	M1 A1	[2]
10 (i)	Express general point of l in component form e.g. $(1+2\lambda, 2-\lambda, 1+\lambda)$ Using the correct process for the modulus form an equation in λ Reduce the equation to a quadratic, e.g. $6\lambda^2 + 2\lambda - 4 = 0$ Solve for λ (usual requirements for solution of a quadratic) Obtain final answers $-\mathbf{i} + 3\mathbf{j}$ and $\frac{7}{3}\mathbf{i} + \frac{4}{3}\mathbf{j} + \frac{5}{3}\mathbf{k}$	B1 M1* A1 DM1 A1	[5]
(ii)	Using the correct process, find the scalar product of a direction vector for l and a normal for p Using the correct process for the moduli, divide the scalar product by the product of the moduli and equate the result to $\frac{2}{3}$ State a correct equation in any form, e.g. $\frac{2a-1+1}{\sqrt{(a^2+1+1)}\sqrt{(2^2+(-1)^2+1)}} = \pm \frac{2}{3}$ Solve for a^2 Obtain answer $a = \pm 2$	M1 M1 A1 M1 A1	[5]