

CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge International Advanced Level

MARK SCHEME for the October/November 2015 series

9709 MATHEMATICS

9709/31

Paper 3, maximum raw mark 75

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Mark Scheme Notes

Marks are of the following three types:

- M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- The symbol ∇ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously “correct” answers or results obtained from incorrect working.
- Note: B2 or A2 means that the candidate can earn 2 or 0.
B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.
- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

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The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
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SOS	See Other Solution (the candidate makes a better attempt at the same question)
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Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through ✓” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

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- 1 *EITHER*: State or imply non-modular inequality $(2x-5)^2 > (3(2x+1))^2$, or corresponding quadratic equation, or pair of linear equations $(2x-5) = \pm 3(2x+1)$ **B1**
 Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations for x **M1**
 Obtain critical values -2 and $\frac{1}{4}$ **A1**
 State final answer $-2 < x < \frac{1}{4}$ **A1**
OR: Obtain critical value $x = -2$ from a graphical method, or by inspection, or by solving a linear equation or inequality **B1**
 Obtain critical value $x = \frac{1}{4}$ similarly **B2**
 State final answer $-2 < x < \frac{1}{4}$ **B1** [4]
 [Do not condone $\leq$ for $<$]
- 2 State or imply $1+u=u^2$ **B1**
 Solve for u **M1**
 Obtain root $\frac{1}{2}(1+\sqrt{5})$, or decimal in $[1.61, 1.62]$ **A1**
 Use correct method for finding x from a positive root **M1**
 Obtain $x = 0.438$ and no other answer **A1** [5]
- 3 Use $\tan(A \pm B)$ and obtain an equation in $\tan \theta$ and $\tan \phi$ **M1***
 Substitute throughout for $\tan \theta$ or for $\tan \phi$ **dep M1***
 Obtain $3 \tan^2 \theta - \tan \theta - 4 = 0$ or $3 \tan^2 \phi - 5 \tan \phi - 2 = 0$, or 3-term equivalent **A1**
 Solve a 3-term quadratic and find an angle **M1**
 Obtain answer $\theta = 135^\circ$, $\phi = 63.4^\circ$ **A1**
 Obtain answer $\theta = 53.1^\circ$, $\phi = 161.6^\circ$ **A1** [6]
 [Treat answers in radians as a misread. Ignore answers outside the given interval.]
 [SR: Two correct values of θ (or ϕ) score A1; then A1 for both correct θ, ϕ pairs.]
- 4 (i) Evaluate, or consider the sign of, $x^3 - x^2 - 6$ for two integer values of x , or equivalent **M1**
 Obtain the pair $x = 2$ and $x = 3$, with no errors seen **A1** [2]
- (ii) State a suitable equation, e.g. $x = \sqrt{(x + (6/x))}$ **B1**
 Rearrange this as $x^3 - x^2 - 6 = 0$, or work *vice versa* **B1** [2]
- (iii) Use the iterative formula correctly at least once **M1**
 Obtain final answer 2.219 **A1**
 Show sufficient iterates to 5 d.p. to justify 2.219 to 3 d.p., or show there is a sign change in the interval (2.2185, 2.2195) **A1** [3]

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- 5 (i) State or imply that the derivative of e^{-2x} is $-2e^{-2x}$ **B1**
Use product or quotient rule **M1**
Obtain correct derivative in any form **A1**
Use Pythagoras **M1**
Justify the given form **A1** [5]
- (ii) Fully justify the given statement **B1** [1]
- (iii) State answer $x = \frac{1}{4}\pi$ **B1** [1]
- 6 (i) Substitute $x = -1$, equate to zero and simplify at least as far as $-8 + a - b - 1 = 0$ **B1**
Substitute $x = -\frac{1}{2}$ and equate the result to 1 **M1**
Obtain a correct equation in any form, e.g. $-1 + \frac{1}{4}a - \frac{1}{2}b - 1 = 1$ **A1**
Solve for a or for b **M1**
Obtain $a = 6$ and $b = -3$ **A1** [5]
- (ii) Commence division by $(x + 1)$ reaching a partial quotient $8x^2 + kx$ **M1**
Obtain quadratic factor $8x^2 - 2x - 1$ **A1**
Obtain factorisation $(x + 1)(4x + 1)(2x - 1)$ **A1** [3]
[The M1 is earned if inspection reaches an unknown factor $8x^2 + Bx + C$ and an equation in B and/or C , or an unknown factor $Ax^2 + Bx - 1$ and an equation in A and/or B .]
[If linear factors are found by the factor theorem, give B1B1 for $(2x - 1)$ and $(4x + 1)$, and B1 for the complete factorisation.]
- 7 (i) Use correct method to form a vector equation for AB **M1**
Obtain a correct equation, e.g. $\mathbf{r} = \mathbf{i} + 2\mathbf{j} + \lambda(2\mathbf{i} - 2\mathbf{j} + \mathbf{k})$ or $\mathbf{r} = 3\mathbf{i} + \mathbf{k} + \mu(2\mathbf{i} - 2\mathbf{j} + \mathbf{k})$ **A1** [2]
- (ii) Using a direction vector for AB and a relevant point, obtain an equation for m in any form **M1**
Obtain answer $2x - 2y + z = 4$, or equivalent **A1** [2]
- (iii) Express general point of AB in component form, e.g. $(1 + 2\lambda, 2 - 2\lambda, \lambda)$ or $(3 + 2\mu, -2\mu, 1 + \mu)$ **B1**
Substitute in equation of m and solve for λ or for μ **M1**
Obtain final answer $\frac{7}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}$ for the position vector of N , from $\lambda = \frac{2}{3}$ or $\mu = -\frac{1}{3}$ **A1**
Carry out a correct method for finding CN **M1**
Obtain the given answer $\sqrt{13}$ **A1** [5]
[The f.t. is on the direction vector for AB .]

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8	Separate variables and integrate one side	B1	
	Obtain term $\ln(x+2)$	B1	
	Use $\cos 2A$ formula to express $\sin^2 2\theta$ in the form $a + b \cos 4\theta$	M1	
	Obtain correct form $(1 - \cos 4\theta)/2$, or equivalent	A1	
	Integrate and obtain term $\frac{1}{2}\theta - \frac{1}{8}\sin 4\theta$, or equivalent	A1	
	Evaluate a constant, or use $\theta = 0, x = 0$ as limits in a solution containing terms $c \ln(x+2), d \sin(4\theta), e\theta$	M1	
	Obtain correct solution in any form, e.g. $\ln(x+2) = \frac{1}{2}\theta - \frac{1}{8}\sin 4\theta + \ln 2$	A1	
	Use correct method for solving an equation of the form $\ln(x+2) = f$	M1	
	Obtain answer $x = 0.962$	A1	[9]
9	(i) Show u in a relatively correct position	B1	
	Show u^* in a relatively correct position	B1	
	Show $u^* - u$ in a relatively correct position	B1	
	State or imply that $OABC$ is a parallelogram	B1	[4]
	(ii) EITHER: Substitute for u and multiply numerator and denominator by $3 + i$, or equivalent	M1	
	Simplify the numerator to $8 + 6i$ or the denominator to 10	A1	
	Obtain final answer $\frac{4}{5} + \frac{3}{5}i$, or equivalent	A1	
	OR: Substitute for u , obtain two equations in x and y and solve for x or for y	M1	
	Obtain $x = \frac{4}{5}$ or $y = \frac{3}{5}$, or equivalent	A1	
	Obtain final answer $\frac{4}{5} + \frac{3}{5}i$, or equivalent	A1	[3]
	(iii) State or imply $\arg(u^*/u) = \tan^{-1}(\frac{3}{4})$	B1	
	Substitute exact arguments in $\arg(u^*/u) = \arg u^* - \arg u$	M1	
	Fully justify the given statement using exact values	A1	[3]
10	(i) Use the quotient rule	M1	
	Obtain correct derivative in any form	A1	
	Equate derivative to zero and solve for x	M1	
	Obtain answer $x = \sqrt[3]{2}$, or exact equivalent	A1	[4]
	(ii) State or imply indefinite integral is of the form $k \ln(1+x^3)$	M1	
	State indefinite integral $\frac{1}{3} \ln(1+x^3)$	A1	
	Substitute limits correctly in an integral of the form $k \ln(1+x^3)$	M1	
	State or imply that the area of R is equal to $\frac{1}{3} \ln(1+p^3) - \frac{1}{3} \ln 2$, or equivalent	A1	
	Use a correct method for finding p from an equation of the form $\ln(1+p^3) = a$		
	or $\ln((1+p^3)/2) = b$	M1	
	Obtain answer $p = 3.40$	A1	[2]

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	Use a correct method for finding p from an equation of the form $\ln(1+p^3) = a$		
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- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.

When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

The symbol ∇ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously “correct” answers or results obtained from incorrect working.

Note: B2 or A2 means that the candidate can earn 2 or 0.

B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

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The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)
CWO	Correct Working Only – often written by a ‘fortuitous’ answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through $\frac{1}{2}$ ” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

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- 1 Draw curve with increasing gradient existing for negative and positive values of x M1
 Draw correct curve passing through the origin A1 [2]
- 2 Either State correct unsimplified x^2 or x^3 term M1
 Obtain $a = -9$ A1
 Obtain $b = 45$ A1
- Or Use chain rule to differentiate twice to obtain form $k(1 + 9x)^{-\frac{5}{3}}$ M1
 Obtain $f''(x) = -18(1 + 9x)^{-\frac{5}{3}}$ and hence $a = -9$ A1
 Obtain $f'''(x) = 270(1 + 9x)^{-\frac{8}{3}}$ and hence $b = 45$ A1 [3]
- 3 Use correct quotient rule or equivalent to find first derivative M1*
 Obtain $\frac{-(1 + \tan x) \sec^2 x - \sec^2 x(2 - \tan x)}{(1 + \tan x)^2}$ or equivalent A1
 Substitute $x = \frac{1}{4}\pi$ to find gradient dep M1*
 Obtain $-\frac{3}{2}$ A1
 Form equation of tangent at $x = \frac{1}{4}\pi$ M1
 Obtain $y = -\frac{3}{2}x + 1.68$ or equivalent A1 [6]
- 4 (i) Use $\frac{dy}{dx} = \frac{\dot{y}}{\dot{x}}$ and equate $\frac{dy}{dx}$ to 4 M1
 Obtain $\frac{4p^3}{2p + 3} = 4$ or equivalent A1
 Confirm given result $p = \sqrt[3]{2p + 3}$ correctly A1 [3]
- (ii) Evaluate $p - \sqrt[3]{2p + 3}$ or $p^3 - 2p - 3$ or equivalent at 1.8 and 2.0 M1
 Justify result with correct calculations and argument
 (−0.076 and 0.087 or −0.77 and 1 respectively) A1 [2]
- (iii) Use the iterative process correctly at least once with $1.8 \leq p_n \leq 2.0$ M1
 Obtain final answer 1.89 A1
 Show sufficient iterations to at least 4 d.p. to justify 1.89 or show sign change in interval (1.885, 1.895) A1 [3]

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- 5 State $du = 3 \sin x \, dx$ or equivalent **B1**
 Use identity $\sin 2x = 2 \sin x \cos x$ **B1**
 Carry out complete substitution, for x and dx **M1**
 Obtain $\int \frac{8-2u}{\sqrt{u}} du$, or equivalent **A1**

 Integrate to obtain expression of form $au^{\frac{1}{2}} + bu^{\frac{3}{2}}$, $ab \neq 0$ **M1***
 Obtain correct $16u^{\frac{1}{2}} - \frac{4}{3}u^{\frac{3}{2}}$ **A1**

 Apply correct limits correctly **dep M1***
 Obtain $\frac{20}{3}$ or exact equivalent **A1** [8]
- 6 State or imply $\sin A \times \cos 45 + \cos A \times \sin 45 = 2\sqrt{2} \cos A$ **B1**
 Divide by $\cos A$ to find value of $\tan A$ **M1**
 Obtain $\tan A = 3$ **A1**
 Use identity $\sec^2 B = 1 + \tan^2 B$ **B1**
 Solve three-term quadratic equation and find $\tan B$ **M1**
 Obtain $\tan B = \frac{3}{2}$ only **A1**

 Substitute **numerical values** in $\frac{\tan A - \tan B}{1 + \tan A \tan B}$ **M1**
 Obtain $\frac{3}{11}$ **A1** [8]
- 7 (i) Either Substitute $x = -1$ and evaluate **M1**
 Obtain 0 and conclude $x + 1$ is a factor **A1**

Or Divide by $x + 1$ and obtain a constant remainder **M1**
 Obtain remainder = 0 and conclude $x + 1$ is a factor **A1** [2]
- (ii) Attempt division, or equivalent, at least as far as quotient $4x^2 + kx$ **M1**
 Obtain complete quotient $4x^2 - 5x - 6$ **A1**
 State form $\frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{4x+3}$ **A1**
 Use relevant method for finding at least one constant **M1**
 Obtain one of $A = -2, B = 1, C = 8$ **A1**
 Obtain all three values **A1**
 Integrate to obtain three terms each involving natural logarithm of linear form **M1**
 Obtain $-2 \ln(x+1) + \ln(x-2) + 2 \ln(4x+3)$, condoning no use of modulus signs
 and absence of $\dots + c$ **A1** [8]

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- 8 (i) Express a general point on the line in single component form, e.g. $(\lambda, 2 - 3\lambda, -8 + 4\lambda)$,
substitute in equation of plane and solve for λ M1
Obtain $\lambda = 3$ A1
Obtain $(3, -7, 4)$ A1 [3]
- (ii) State or imply normal vector to plane is $4\mathbf{i} - \mathbf{j} + 5\mathbf{k}$ B1
Carry out process for evaluating scalar product of two relevant vectors M1
Using the correct process for the moduli, divide the scalar product by the product
of the moduli and evaluate $\sin^{-1}$ or $\cos^{-1}$ of the result. M1
Obtain 54.8° or 0.956 radians A1 [4]
- (iii) Either Find at least one position of C by translating by appropriate multiple
of direction vector $\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$ from A or B M1
Obtain $(-3, 11, -20)$ A1
Obtain $(9, -25, 28)$ A1
- Or Form quadratic equation in λ by considering $BC^2 = 4AB^2$ M1
Obtain $26\lambda^2 - 156\lambda - 702 = 0$ or equivalent and hence $\lambda = -3, \lambda = 9$ A1
Obtain $(-3, 11, -20)$ and $(9, -25, 28)$ A1 [3]
- 9 (a) Either Find w using conjugate of $1 + 3i$ M1
Obtain $\frac{7-i}{5}$ or equivalent A1
Square $x + iy$ form to find w^2 M1
Obtain $w^2 = \frac{48-14i}{25}$ and confirm modulus is 2 A1
Use correct process for finding argument of w^2 M1
Obtain -0.284 radians or -16.3° A1
- Or 1 Find w using conjugate of $1 + 3i$ M1
Obtain $\frac{7-i}{5}$ or equivalent A1
Find modulus of w and hence of w^2 M1
Confirm modulus is 2 A1
Find argument of w and hence of w^2 M1
Obtain -0.284 radians or -16.3° A1
- Or 2 Square both sides to obtain $(-8 + 6i)w^2 = -12 + 16i$ B1
Find w^2 using relevant conjugate M1
Use correct process for finding modulus of w^2 M1
Confirm modulus is 2 A1
Use correct process for finding argument of w^2 M1
Obtain -0.284 radians or -16.3° A1

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<u>Or 3</u>	Find modulus of LHS and RHS	M1	
	Find argument of LHS and RHS	M1	
	Obtain $\sqrt{10} e^{1.249i}$ $w = \sqrt{20} e^{1.107i}$ or equivalent	A1	
	Obtain $w = \sqrt{2} e^{-0.1419i}$ or equivalent	A1	
	Use correct process for finding w^2	M1	
	Obtain 2 and -0.284 radians or -16.3°	A1	
<u>Or 4</u>	Find moduli of $2 + 4i$ and $1 + 3i$	M1	
	Obtain $\sqrt{20}$ and $\sqrt{10}$	A1	
	Obtain $ w^2 = 2$ correctly	A1	
	Find $\arg(2 + 4i)$ and $\arg(1 + 3i)$	M1	
	Use correct process for $\arg(w^2)$	A1	
	Obtain -0.284 radians or -16.3°	A1	
<u>Or 5</u>	Let $w = a + ib$, form and solve simultaneous equations in a and b	M1	
	$a = \frac{7}{5}$ and $b = -\frac{1}{5}$	A1	
	Find modulus of w and hence of w^2	M1	
	Confirm modulus is 2	A1	
	Find argument of w and hence of w^2	M1	
<u>Or 6</u>	Obtain -0.284 radians or -16.3°	A1	
	Find w using conjugate of $1 + 3i$	M1	
	Obtain $\frac{7-i}{5}$ or equivalent	A1	
	Use $ w^2 = w\bar{w}$	M1	
	Confirm modulus is 2	A1	
<u>Or 6</u>	Find argument of w and hence of w^2	M1	
	Obtain -0.284 radians or -16.3°	A1	[6]
(b)	Draw circle with centre the origin and radius 5	B1	
	Draw straight line parallel to imaginary axis in correct position	B1	
	Use relevant trigonometry on a correct diagram to find argument(s)	M1	
	Obtain $5e^{\pm \frac{1}{3}\pi i}$ or equivalents in required form	A1	[4]

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- 10 (i) State $\frac{dN}{dt} = k(N - 150)$ B1 [1]
- (ii) Substitute $\frac{dN}{dt} = 60$ and $N = 900$ to find value of k M1
 Obtain $k = 0.08$ A1
 Separate variables and obtain general solution involving $\ln(N - 150)$ M1*
 Obtain $\ln(N - 150) = 0.08t + c$ (following their k) or $\ln(N - 150) = kt + c$ A1[✓]
 Substitute $t = 0$ and $N = 650$ to find c dep M1*
 Obtain $\ln(N - 150) = 0.08t + \ln 500$ or equivalent A1
 Obtain $N = 500e^{0.08t} + 150$ A1 [7]
- (iii) Either Substitute $t = 15$ to find N or solve for t with $N = 2000$ M1
 Obtain Either $N = 1810$ or $t = 16.4$ and conclude target not met A1 [2]