

**MARK SCHEME for the October/November 2011 question paper  
for the guidance of teachers**

**9709 MATHEMATICS**

**9709/71**

Paper 7, maximum raw mark 50

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

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|               |                                               |                 |              |
|---------------|-----------------------------------------------|-----------------|--------------|
| <b>Page 2</b> | <b>Mark Scheme: Teachers' version</b>         | <b>Syllabus</b> | <b>Paper</b> |
|               | <b>GCE AS/A LEVEL – October/November 2011</b> | <b>9709</b>     | <b>71</b>    |

## **Mark Scheme Notes**

Marks are of the following three types:

**M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

**A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

**B** Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep\*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- The symbol  $\surd$  implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously “correct” answers or results obtained from incorrect working.
- Note: B2 or A2 means that the candidate can earn 2 or 0.  
B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.
- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking  $g$  equal to 9.8 or 9.81 instead of 10.

|               |                                               |                 |              |
|---------------|-----------------------------------------------|-----------------|--------------|
| <b>Page 3</b> | <b>Mark Scheme: Teachers' version</b>         | <b>Syllabus</b> | <b>Paper</b> |
|               | <b>GCE AS/A LEVEL – October/November 2011</b> | <b>9709</b>     | <b>71</b>    |

The following abbreviations may be used in a mark scheme or used on the scripts:

|     |                                                                                                                                                                                         |
|-----|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| AEF | Any Equivalent Form (of answer is equally acceptable)                                                                                                                                   |
| AG  | Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)                                                     |
| BOD | Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)                                                                                                  |
| CAO | Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)                                                                                             |
| CWO | Correct Working Only – often written by a ‘fortuitous’ answer                                                                                                                           |
| ISW | Ignore Subsequent Working                                                                                                                                                               |
| MR  | Misread                                                                                                                                                                                 |
| PA  | Premature Approximation (resulting in basically correct work that is insufficiently accurate)                                                                                           |
| SOS | See Other Solution (the candidate makes a better attempt at the same question)                                                                                                          |
| SR  | Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance) |

### **Penalties**

|       |                                                                                                                                                                                                                                                                                                                                                                                                                                                               |
|-------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| MR –1 | A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through ✓” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting. |
| PA –1 | This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.                                                                                                                                                                                                                                                                                                                             |

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|---------------|-----------------------------------------------|-----------------|--------------|
| <b>Page 4</b> | <b>Mark Scheme: Teachers' version</b>         | <b>Syllabus</b> | <b>Paper</b> |
|               | <b>GCE AS/A LEVEL – October/November 2011</b> | <b>9709</b>     | <b>71</b>    |

|                                                                                                                                                                                                                                                     |           |                                                                 |
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| <b>1 (i)</b> Mean = 2.6<br><br>Var = $4 \times 1.3$<br><br>= 5.2                                                                                                                                                                                    | B1        |                                                                 |
|                                                                                                                                                                                                                                                     | M1        | M1 for either $4 \times$ , or for $\text{Var}(X) = 1.3$ implied |
|                                                                                                                                                                                                                                                     | A1 [3]    |                                                                 |
| <b>(ii)</b> Var $\neq$ mean<br>or $2X$ does not take all integer values                                                                                                                                                                             | B1<br>[1] | $X$ and $X$ are not independent oe                              |
| <b>2</b> $H_0$ : $P(\text{correct}) = \frac{1}{5}$<br>$H_1$ : $P(\text{correct}) > \frac{1}{5}$<br>$B(100, \frac{1}{5}) \approx N(20, 16)$<br><br>$\frac{26.5 - 20}{4} = 1.625$<br><br>comp $z = 1.645$<br><br>Claim not justified                  | B1        | Accept p<br>Accept $H_0$ : $\mu = 20$ $H_1$ : $\mu > 20$        |
|                                                                                                                                                                                                                                                     | M1<br>A1  | Allow wrong or no cc or denom = 16<br>For $\pm 1.625$           |
|                                                                                                                                                                                                                                                     | A1        |                                                                 |
|                                                                                                                                                                                                                                                     | M1        | Valid comparison of $z$ or areas<br>( $0.0521 > 0.05$ )         |
|                                                                                                                                                                                                                                                     | A1ft [5]  | In context. No contradictions. Ft their $z$ .                   |
| <b>3</b> $\text{Var}(\text{Tot}) = 0.02^2 + 0.03^2 + 0.01^2 = 0.0014$<br><br>Mean(Tot) = 0.37<br>Tot $\sim N(0.37, 0.0014)$<br><br>$\frac{0.30 - 0.37}{\sqrt{0.0014}} (= -1.871)$<br><br>$\Phi(-1.871) = 1 - \Phi(1.871)$<br><br>= 0.0306 or 0.0307 | B1        |                                                                 |
|                                                                                                                                                                                                                                                     | B1        |                                                                 |
|                                                                                                                                                                                                                                                     | M1        | Allow without $\sqrt{\phantom{x}}$ . No cc                      |
|                                                                                                                                                                                                                                                     | M1        |                                                                 |
|                                                                                                                                                                                                                                                     | A1 [5]    | Correct area                                                    |
| <b>4 (i)</b> Est( $\mu$ ) = 331(.125)<br><br>Est( $\sigma^2$ ) = $\frac{8}{7} \left( \frac{877179}{8} - 331.125^2 \right)$<br><br>= 4.125 or 4.13                                                                                                   | B1        |                                                                 |
|                                                                                                                                                                                                                                                     | M1        | Allow their $\Sigma x^2$                                        |
|                                                                                                                                                                                                                                                     | A1 [3]    |                                                                 |
| <b>(ii)</b> $z = 2.326$<br><br>$331 \pm z \times \sqrt{\frac{4.2}{50}}$<br><br>= 330 to 332 (3 sfs)                                                                                                                                                 | B1        |                                                                 |
|                                                                                                                                                                                                                                                     | M1        | Allow incorrect $z$ ( $\neq 1, 0$ ), not a prob                 |
|                                                                                                                                                                                                                                                     | A1 [3]    | Ignore brackets, if given. CWO                                  |
| <b>(iii)</b> No, because 333 is not within CI                                                                                                                                                                                                       | B1ft [1]  |                                                                 |

|               |                                               |                 |              |
|---------------|-----------------------------------------------|-----------------|--------------|
| <b>Page 5</b> | <b>Mark Scheme: Teachers' version</b>         | <b>Syllabus</b> | <b>Paper</b> |
|               | <b>GCE AS/A LEVEL – October/November 2011</b> | <b>9709</b>     | <b>71</b>    |

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| <b>5 (i)</b> $\pm 1.645$ used<br><br>$\frac{\bar{x} - 22}{\frac{3.5}{\sqrt{12}}} > 1.645$<br><br>$\bar{x} > 23.66(20)$<br>$\bar{x} > 23.7$ <b>AG</b>                                                | B1<br><br>M1<br><br><br>A1<br><br>[3] | Accept '='<br>(standardising using 23.7 scores M1A0)<br>or $\bar{x} = 23.66(20)$                                                                                                      |
| <b>(ii)</b> $P(\bar{x} < 23.7 \mid \mu = 25.8)$<br><br>$\frac{23.662 - 25.8}{\frac{3.5}{\sqrt{12}}} = -2.116$<br><br>$\Phi(-2.116) = 1 - \Phi(2.116)$<br>$(= 1 - 0.9828)$<br><br>$= 0.0172$ (3 sfs) | M1<br><br>A1<br><br>M1<br><br>A1 [4]  | For attempt type II error and standardising<br><br>$\frac{23.7 - 25.8}{\frac{3.5}{\sqrt{12}}} = -2.078$<br><br>$\Phi(-2.078) = 1 - \Phi(2.078)$<br>$(= 1 - 0.9812)$<br><br>$= 0.0188$ |
| <b>6 (i)</b> Customers arrive independently or randomly                                                                                                                                             | B1 [1]                                | In context. Allow "singly"                                                                                                                                                            |
| <b>(ii)</b> $e^{-6} \times \frac{6^5}{5!}$<br><br>$= 0.161$ (3 sfs)                                                                                                                                 | M1<br><br>A1 [2]                      | Poisson P(5), allow any mean                                                                                                                                                          |
| <b>(iii)</b> $\lambda = 2.4$<br><br>$e^{-2} \left( 1 + 2.4 + \frac{2.4^2}{2!} \right)$<br><br>$= 0.570$ (3 sfs)                                                                                     | B1<br><br>M1<br><br>A1 [3]            | Poisson P(0, 1, 2), allow their mean<br>allow one end error                                                                                                                           |
| <b>(iv)</b> N(24, 24)<br><br>$\frac{295 - 24}{\sqrt{24}} (= 1.123)$<br><br>$\Phi(1.123)$<br><br>$= 0.869$ (3 sfs)                                                                                   | B1<br><br>M1<br><br>M1<br><br>A1 [4]  | Stated or implied<br><br>Allow with wrong or no cc and/or no $\sqrt{\phantom{x}}$<br>Correct area                                                                                     |

|               |                                               |                 |              |
|---------------|-----------------------------------------------|-----------------|--------------|
| <b>Page 6</b> | <b>Mark Scheme: Teachers' version</b>         | <b>Syllabus</b> | <b>Paper</b> |
|               | <b>GCE AS/A LEVEL – October/November 2011</b> | <b>9709</b>     | <b>71</b>    |

|                                                                                                                                                                                                       |                                          |                                                                                                                                                                                             |
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| <b>7 (i) (a)</b> $X$ or 5                                                                                                                                                                             | B1 [1]                                   |                                                                                                                                                                                             |
| <b>(b)</b> $V$ or 3<br><br>Higher and lower values more likely or there are more higher and lower values or more prob at both extremes                                                                | B1<br><br>B1dep [2]                      | Should mention values or prob<br>Not just graph or spread eg not “More spread”                                                                                                              |
| <b>(ii)</b> $\frac{2+1}{2} \times 0.5$ or $\int_0^{0.5} (2-2x)dx$<br><br>= 0.75                                                                                                                       | M1<br><br>A1 [2]                         | (‘or’ method requires linear function and correct limits)<br><br>CWO                                                                                                                        |
| <b>(iii) (a)</b> $\int_0^1 ax^n dx = 1$<br><br>$\left[ \frac{ax^{n+1}}{n+1} \right]_0^1 = 1$<br><br>$\frac{a}{n+1} = 1$<br><br>( $a = n + 1$ <b>AG</b> )                                              | M1<br><br>A1<br><br>A1 [3]               | Attempt integ of correct form = 1 (ignore limits)<br><br>Correct integrand & limits<br><br>No errors seen                                                                                   |
| <b>(b)</b> $\int_0^1 ax^{n+1} dx = \frac{5}{6}$ oe<br><br>$\left[ \frac{ax^{n+2}}{n+2} \right]_0^1 = \frac{5}{6}$ oe<br><br>$\frac{a}{n+2} = \frac{5}{6}$<br>( $6a = 5n + 10$ )<br><br>$a = 5, n = 4$ | M1*<br><br>A1<br><br>M1dep<br><br>A1 [4] | Integral of form $\int xf(x)dx = \frac{5}{6}$ , ignore limits<br><br>Correct integrand & limits<br><br>Attempt to use $a = n + 1$ within 2 <sup>nd</sup> equ to get an equ in $n$ (or $a$ ) |

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|---------------|-----------------------------------------------|-----------------|--------------|
| <b>Page 2</b> | <b>Mark Scheme: Teachers' version</b>         | <b>Syllabus</b> | <b>Paper</b> |
|               | <b>GCE AS/A LEVEL – October/November 2011</b> | <b>9709</b>     | <b>72</b>    |

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|---------------|-----------------------------------------------|-----------------|--------------|
| <b>Page 3</b> | <b>Mark Scheme: Teachers' version</b>         | <b>Syllabus</b> | <b>Paper</b> |
|               | <b>GCE AS/A LEVEL – October/November 2011</b> | <b>9709</b>     | <b>72</b>    |

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| BOD | Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)                                                                                                  |
| CAO | Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)                                                                                             |
| CWO | Correct Working Only – often written by a ‘fortuitous’ answer                                                                                                                           |
| ISW | Ignore Subsequent Working                                                                                                                                                               |
| MR  | Misread                                                                                                                                                                                 |
| PA  | Premature Approximation (resulting in basically correct work that is insufficiently accurate)                                                                                           |
| SOS | See Other Solution (the candidate makes a better attempt at the same question)                                                                                                          |
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### **Penalties**

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|---------------|-----------------------------------------------|-----------------|--------------|
| <b>Page 4</b> | <b>Mark Scheme: Teachers' version</b>         | <b>Syllabus</b> | <b>Paper</b> |
|               | <b>GCE AS/A LEVEL – October/November 2011</b> | <b>9709</b>     | <b>72</b>    |

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|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------|-----------------------------------------------------------------|
| <b>1</b> (i) Mean = 2.6<br><br>Var = $4 \times 1.3$<br><br>= 5.2                                                                                                                                                                                    | B1        |                                                                 |
|                                                                                                                                                                                                                                                     | M1        | M1 for either $4 \times$ , or for $\text{Var}(X) = 1.3$ implied |
|                                                                                                                                                                                                                                                     | A1 [3]    |                                                                 |
| (ii) Var $\neq$ mean<br>or $2X$ does not take all integer values                                                                                                                                                                                    | B1<br>[1] | $X$ and $X$ are not independent oe                              |
| <b>2</b> $H_0$ : $P(\text{correct}) = \frac{1}{5}$<br>$H_1$ : $P(\text{correct}) > \frac{1}{5}$<br>$B(100, \frac{1}{5}) \approx N(20, 16)$<br><br>$\frac{26.5 - 20}{4} = 1.625$<br><br>comp $z = 1.645$<br><br>Claim not justified                  | B1        | Accept p<br>Accept $H_0$ : $\mu = 20$ $H_1$ : $\mu > 20$        |
|                                                                                                                                                                                                                                                     | M1<br>A1  | Allow wrong or no cc or denom = 16<br>For $\pm 1.625$           |
|                                                                                                                                                                                                                                                     | A1        |                                                                 |
|                                                                                                                                                                                                                                                     | M1        | Valid comparison of $z$ or areas<br>( $0.0521 > 0.05$ )         |
|                                                                                                                                                                                                                                                     | A1ft [5]  | In context. No contradictions. Ft their $z$ .                   |
| <b>3</b> $\text{Var}(\text{Tot}) = 0.02^2 + 0.03^2 + 0.01^2 = 0.0014$<br><br>Mean(Tot) = 0.37<br>Tot $\sim N(0.37, 0.0014)$<br><br>$\frac{0.30 - 0.37}{\sqrt{0.0014}} (= -1.871)$<br><br>$\Phi(-1.871) = 1 - \Phi(1.871)$<br><br>= 0.0306 or 0.0307 | B1        |                                                                 |
|                                                                                                                                                                                                                                                     | B1        |                                                                 |
|                                                                                                                                                                                                                                                     | M1        | Allow without $\sqrt{\cdot}$ . No cc                            |
|                                                                                                                                                                                                                                                     | M1        |                                                                 |
|                                                                                                                                                                                                                                                     | A1 [5]    | Correct area                                                    |
| <b>4</b> (i) $\text{Est}(\mu) = 331(.125)$<br><br>$\text{Est}(\sigma^2) = \frac{8}{7} \left( \frac{877179}{8} - 331.125^2 \right)$<br><br>= 4.125 or 4.13                                                                                           | B1        |                                                                 |
|                                                                                                                                                                                                                                                     | M1        | Allow their $\Sigma x^2$                                        |
|                                                                                                                                                                                                                                                     | A1 [3]    |                                                                 |
| (ii) $z = 2.326$<br><br>$331 \pm z \times \sqrt{\frac{4.2}{50}}$<br><br>= 330 to 332 (3 sfs)                                                                                                                                                        | B1        |                                                                 |
|                                                                                                                                                                                                                                                     | M1        | Allow incorrect $z$ ( $\neq 1, 0$ ), not a prob                 |
|                                                                                                                                                                                                                                                     | A1 [3]    | Ignore brackets, if given. CWO                                  |
| (iii) No, because 333 is not within CI                                                                                                                                                                                                              | B1ft [1]  |                                                                 |

|               |                                               |                 |              |
|---------------|-----------------------------------------------|-----------------|--------------|
| <b>Page 5</b> | <b>Mark Scheme: Teachers' version</b>         | <b>Syllabus</b> | <b>Paper</b> |
|               | <b>GCE AS/A LEVEL – October/November 2011</b> | <b>9709</b>     | <b>72</b>    |

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|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----|----------------------------------------------------------------------------------|
| <b>5 (i)</b> $\pm 1.645$ used<br><br>$\frac{\bar{x} - 22}{\frac{3.5}{\sqrt{12}}} > 1.645$<br><br>$\bar{x} > 23.66(20)$<br>$\bar{x} > 23.7$ <b>AG</b>                                                | B1  |                                                                                  |
|                                                                                                                                                                                                     | M1  |                                                                                  |
|                                                                                                                                                                                                     | A1  | Accept '='<br>(standardising using 23.7 scores M1A0)<br>or $\bar{x} = 23.66(20)$ |
|                                                                                                                                                                                                     | [3] |                                                                                  |
| <b>(ii)</b> $P(\bar{x} < 23.7 \mid \mu = 25.8)$<br><br>$\frac{23.662 - 25.8}{\frac{3.5}{\sqrt{12}}} = -2.116$<br><br>$\Phi(-2.116) = 1 - \Phi(2.116)$<br>$(= 1 - 0.9828)$<br><br>$= 0.0172$ (3 sfs) | M1  | For attempt type II error and<br>standardising                                   |
|                                                                                                                                                                                                     | A1  | $\frac{23.7 - 25.8}{\frac{3.5}{\sqrt{12}}} = -2.078$                             |
|                                                                                                                                                                                                     | M1  | $\Phi(-2.078) = 1 - \Phi(2.078)$<br>$(= 1 - 0.9812)$                             |
|                                                                                                                                                                                                     | A1  | $= 0.0188$                                                                       |
|                                                                                                                                                                                                     | [4] |                                                                                  |
| <b>6 (i)</b> Customers arrive independently or randomly                                                                                                                                             | B1  | [1] In context. Allow "singly"                                                   |
| <b>(ii)</b> $e^{-6} \times \frac{6^5}{5!}$<br><br>$= 0.161$ (3 sfs)                                                                                                                                 | M1  | Poisson P(5), allow any mean                                                     |
|                                                                                                                                                                                                     | A1  | [2]                                                                              |
| <b>(iii)</b> $\lambda = 2.4$<br><br>$e^{-2} \left( 1 + 2.4 + \frac{2.4^2}{2!} \right)$<br><br>$= 0.570$ (3 sfs)                                                                                     | B1  |                                                                                  |
|                                                                                                                                                                                                     | M1  | Poisson P(0, 1, 2), allow their mean<br>allow one end error                      |
|                                                                                                                                                                                                     | A1  | [3]                                                                              |
| <b>(iv)</b> N(24, 24)<br><br>$\frac{295 - 24}{\sqrt{24}} (= 1.123)$<br><br>$\Phi(1.123)$<br><br>$= 0.869$ (3 sfs)                                                                                   | B1  | Stated or implied                                                                |
|                                                                                                                                                                                                     | M1  | Allow with wrong or no cc and/or no $\sqrt{\phantom{x}}$<br>Correct area         |
|                                                                                                                                                                                                     | M1  |                                                                                  |
|                                                                                                                                                                                                     | A1  | [4]                                                                              |

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|---------------|-----------------------------------------------|-----------------|--------------|
| <b>Page 6</b> | <b>Mark Scheme: Teachers' version</b>         | <b>Syllabus</b> | <b>Paper</b> |
|               | <b>GCE AS/A LEVEL – October/November 2011</b> | <b>9709</b>     | <b>72</b>    |

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| <b>7 (i) (a)</b> $X$ or 5                                                                                                                                                                             | B1 [1]                                   |                                                                                                                                                                                             |
| <b>(b)</b> $V$ or 3<br><br>Higher and lower values more likely or there are more higher and lower values or more prob at both extremes                                                                | B1<br><br>B1dep [2]                      | Should mention values or prob<br>Not just graph or spread eg not “More spread”                                                                                                              |
| <b>(ii)</b> $\frac{2+1}{2} \times 0.5$ or $\int_0^{0.5} (2-2x)dx$<br><br>= 0.75                                                                                                                       | M1<br><br>A1 [2]                         | (‘or’ method requires linear function and correct limits)<br><br>CWO                                                                                                                        |
| <b>(iii) (a)</b> $\int_0^1 ax^n dx = 1$<br><br>$\left[ \frac{ax^{n+1}}{n+1} \right]_0^1 = 1$<br><br>$\frac{a}{n+1} = 1$<br><br>( $a = n + 1$ <b>AG</b> )                                              | M1<br><br>A1<br><br>A1 [3]               | Attempt integ of correct form = 1 (ignore limits)<br><br>Correct integrand & limits<br><br>No errors seen                                                                                   |
| <b>(b)</b> $\int_0^1 ax^{n+1} dx = \frac{5}{6}$ oe<br><br>$\left[ \frac{ax^{n+2}}{n+2} \right]_0^1 = \frac{5}{6}$ oe<br><br>$\frac{a}{n+2} = \frac{5}{6}$<br>( $6a = 5n + 10$ )<br><br>$a = 5, n = 4$ | M1*<br><br>A1<br><br>M1dep<br><br>A1 [4] | Integral of form $\int xf(x)dx = \frac{5}{6}$ , ignore limits<br><br>Correct integrand & limits<br><br>Attempt to use $a = n + 1$ within 2 <sup>nd</sup> equ to get an equ in $n$ (or $a$ ) |

**MARK SCHEME for the October/November 2011 question paper  
for the guidance of teachers**

**9709 MATHEMATICS**

**9709/73**

Paper 7, maximum raw mark 50

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- Cambridge will not enter into discussions or correspondence in connection with these mark schemes.

Cambridge is publishing the mark schemes for the October/November 2011 question papers for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.



|               |                                               |                 |              |
|---------------|-----------------------------------------------|-----------------|--------------|
| <b>Page 2</b> | <b>Mark Scheme: Teachers' version</b>         | <b>Syllabus</b> | <b>Paper</b> |
|               | <b>GCE AS/A LEVEL – October/November 2011</b> | <b>9709</b>     | <b>73</b>    |

## **Mark Scheme Notes**

Marks are of the following three types:

**M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

**A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

**B** Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep\*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- The symbol  $\checkmark$  implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously “correct” answers or results obtained from incorrect working.
- Note: B2 or A2 means that the candidate can earn 2 or 0.  
B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.
- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking  $g$  equal to 9.8 or 9.81 instead of 10.

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|---------------|-----------------------------------------------|-----------------|--------------|
| <b>Page 3</b> | <b>Mark Scheme: Teachers' version</b>         | <b>Syllabus</b> | <b>Paper</b> |
|               | <b>GCE AS/A LEVEL – October/November 2011</b> | <b>9709</b>     | <b>73</b>    |

The following abbreviations may be used in a mark scheme or used on the scripts:

|     |                                                                                                                                                                                         |
|-----|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| AEF | Any Equivalent Form (of answer is equally acceptable)                                                                                                                                   |
| AG  | Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)                                                     |
| BOD | Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)                                                                                                  |
| CAO | Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)                                                                                             |
| CWO | Correct Working Only – often written by a ‘fortuitous’ answer                                                                                                                           |
| ISW | Ignore Subsequent Working                                                                                                                                                               |
| MR  | Misread                                                                                                                                                                                 |
| PA  | Premature Approximation (resulting in basically correct work that is insufficiently accurate)                                                                                           |
| SOS | See Other Solution (the candidate makes a better attempt at the same question)                                                                                                          |
| SR  | Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance) |

### **Penalties**

|       |                                                                                                                                                                                                                                                                                                                                                                                                                                                               |
|-------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| MR –1 | A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through ✓” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting. |
| PA –1 | This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.                                                                                                                                                                                                                                                                                                                             |

| Page 4 | Mark Scheme: Teachers' version         | Syllabus | Paper |
|--------|----------------------------------------|----------|-------|
|        | GCE AS/A LEVEL – October/November 2011 | 9709     | 73    |

|                                                                                                                                                                                            |                                |                                                                                                                                                                                                                        |
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| <b>1</b> $50 = a + b \times 54$<br>$100 = b^2 \times 144$ or $10 = b \times 12$<br>$b = \frac{5}{6}$ oe<br>$a = 5$                                                                         | B1<br>B1<br>M1<br>A1 [4]       | Solving two simultaneous equations<br>Both correct                                                                                                                                                                     |
| <b>2</b> $2 \times z \times \sqrt{\frac{0.35 \times 0.65}{n}} = 0.157$<br>$z = 2.326$<br>$n = 4 \times 2.326^2 \times 0.35 \times 0.65 \div 0.157^2$<br>(=199.738...)<br>$n = 200$         | M1<br>M1<br>B1<br>M1<br>A1 [5] | For $\sqrt{(pq/n)}$ in equation<br>For equation of the form<br>$2 \times z \times f(n) = 0.157$<br>Rearrange to form $n = \dots$ from a correct equation in $n$ , but allow any $z$ and/or factor of “2” errors<br>cao |
| <b>3 (i)</b> Number all members<br>Explain the selection of 3-digit random numbers<br>Omit repeats OR omit nos. over 750 (until have 8 nos.)                                               | B1<br>B1<br>B1 [3]             |                                                                                                                                                                                                                        |
| <b>(ii)</b> Est $(\mu) = 20$<br>$\text{Est } (\sigma^2) = \frac{8}{7} \left( \frac{3636}{8} - 20^2 \right)$<br>$= \frac{436}{7}$ or 62.3 (3 sfs)                                           | B1<br>M1<br>A1 [3]             | $1/7 \times (3636 - 160^2/8)$<br>$(7.89\dots)^2$ M1A1, but 7.89... only M1A0                                                                                                                                           |
| <b>(iii)</b> Amounts spent last week in café by all club members                                                                                                                           | B1 [1]                         |                                                                                                                                                                                                                        |
| <b>4 (i)</b> $\int_0^1 ke^{-x} dx = 1$<br>$\left[ -ke^{-x} \right]_0^1 = 1$<br>$(= -ke^{-1} - (-ke^0))$<br>$= k \times \frac{e-1}{e} = 1$ or $k(e-1) = e$<br>$k = \frac{e}{e-1}$ <b>AG</b> | M1<br>A1<br>A1 [3]             | Int = 1, ignore limits<br>Correct integral & limits, & = 1<br>Correctly obtained, no errors seen                                                                                                                       |

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|---------------|-----------------------------------------------|-----------------|--------------|
| <b>Page 5</b> | <b>Mark Scheme: Teachers' version</b>         | <b>Syllabus</b> | <b>Paper</b> |
|               | <b>GCE AS/A LEVEL – October/November 2011</b> | <b>9709</b>     | <b>73</b>    |

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| <p><b>(ii)</b> <math>\frac{e}{e-1} \int_0^1 x e^{-x} dx</math></p> <p><math>= \frac{e}{e-1} ( [x(-e^{-x})]_0^1 - \int_0^1 (-e^{-x}) dx )</math></p> <p><math>= \frac{e}{e-1} ( [-x e^{-x}]_0^1 - [e^{-x}]_0^1 )</math></p> <p><math>(= \frac{e}{e-1} (-e^{-1} - 0 - (e^{-1} - 1)) )</math></p> <p><math>= \frac{e}{e-1} (1 - \frac{2}{e}) \text{ or } \frac{e-2}{e-1} \text{ oe}</math></p>                               | M1                | Attempt $\int x f(x) dx$ , ignore limits                                                                                                               |
|                                                                                                                                                                                                                                                                                                                                                                                                                           | M1*               | Attempt integration by parts the correct way round, ignore limits                                                                                      |
|                                                                                                                                                                                                                                                                                                                                                                                                                           | M1dep*            | Attempt second integral of the form $\pm \int e^{-x} dx$ , ignore limits                                                                               |
|                                                                                                                                                                                                                                                                                                                                                                                                                           | A1<br>[4]         | Accept $k$ instead of $\frac{e}{e-1}$ throughout except ans                                                                                            |
| <p><b>5 (i)</b> Assume pop sd same (105)</p> <p><math>H_0</math>: Pop mean = 1150<br/> <math>H_1</math>: Pop mean &lt; 1150</p> <p><math>\frac{\frac{21800}{20} - 1150}{\frac{105}{\sqrt{20}}}</math></p> <p><math>= \pm 2.556 \text{ or } 2.56</math></p> <p>Compare with <math>z = \pm 2.326</math><br/> (for a clear 2 tail test compare with <math>\pm 2.576</math>)</p> <p>Evidence that mean distance decreased</p> | B1                |                                                                                                                                                        |
|                                                                                                                                                                                                                                                                                                                                                                                                                           | B1                | Allow “ $\mu$ ” but not just “mean”                                                                                                                    |
|                                                                                                                                                                                                                                                                                                                                                                                                                           | M1                | Allow $\div \frac{105}{20}$ . (Accept “totals” method)                                                                                                 |
|                                                                                                                                                                                                                                                                                                                                                                                                                           | A1                | Or 0.0053 if prob/area comparison used                                                                                                                 |
|                                                                                                                                                                                                                                                                                                                                                                                                                           | M1<br>A1ft<br>[6] | Correct comparison of $z$ or prob/area consistent with their test<br>In context.<br>Allow mean dist decreased<br>ft their $z$ and/or clear 2 tail test |
| <p><b>(ii)</b> 0.01</p> <p>Concluding there has been a decrease when there has not.</p>                                                                                                                                                                                                                                                                                                                                   | B1                |                                                                                                                                                        |
|                                                                                                                                                                                                                                                                                                                                                                                                                           | B1<br>[2]         | In context                                                                                                                                             |

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|---------------|-----------------------------------------------|-----------------|--------------|
| <b>Page 6</b> | <b>Mark Scheme: Teachers' version</b>         | <b>Syllabus</b> | <b>Paper</b> |
|               | <b>GCE AS/A LEVEL – October/November 2011</b> | <b>9709</b>     | <b>73</b>    |

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|----------|----------------------------------------------------------------------------------------------|--------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>6</b> | <b>(i)</b> $E(\text{Tot}) = 2 \times 36 + 55$ ( $= 127$ )                                    | B1           | (Or $\pm 13$ )                                                                                                                                                                                   |
|          | $\text{Var}(\text{Tot}) = 2 \times 3.5^2 + 5.2^2$ ( $= 51.54$ )                              | B1           |                                                                                                                                                                                                  |
|          | $\frac{140 - 127}{\sqrt{51.54}}$ ( $= 1.811$ )                                               | M1           | For standardising with their mean and var. Allow without $\sqrt{\quad}$ or with false cc, but their mean and variance must involve parameters from both given distributions                      |
|          | $\Phi(1.811)$<br>$= 0.965$ (3 sfs)                                                           | A1 [4]       |                                                                                                                                                                                                  |
|          | <b>(ii)</b> $E(RM) = 36 + 1.5 \times 55$ ( $= 118.5$ )                                       | B1           | (Or $\pm 18.5$ )                                                                                                                                                                                 |
|          | $\text{Var}(RM) = 3.5^2 + 1.5^2 \times 5.2^2$ ( $= 73.09$ )                                  | B1           |                                                                                                                                                                                                  |
|          | $\frac{100 - 118.5}{\sqrt{73.09}}$ ( $= -2.164$ )                                            | M1           | For standardising with their mean and var. Allow without $\sqrt{\quad}$ or with false cc, but their mean and variance must involve parameters from both given distributions                      |
|          | $1 - \Phi(-2.164) = \Phi(2.164)$<br>$0.985$ (3 sfs)                                          | A1 [4]       |                                                                                                                                                                                                  |
| <b>7</b> | <b>(i) (a)</b> $\frac{1 - e^{-1.2}(1 + 1.2)}{1 - e^{-1.4}}$ ( $= 0.3374$ )<br>( $= 0.7534$ ) | M1<br>A1     | M1 for Poisson either $P(0 \text{ or } 1)$ or $P(0)$ with<br>$\lambda = 1.2$ or $2.4$ or $1.4$ or $2.8$ , accept one end error<br>Both expressions fully correct                                 |
|          | $(1 - e^{-1.2}(1 + 1.2)) \times (1 - e^{-1.4})$<br>$= 0.254$ (3 sfs)                         | M1<br>A1 [4] | Their Poisson $P(0 \text{ or } 1) \times P(0)$                                                                                                                                                   |
|          | <b>(i) (b)</b> $\lambda = 2.6$ seen<br>$1 - e^{-2.6}(1 + 2.6 + 2.6^2 \div 2)$                | B1<br>M1     | Poisson $1 - P(0, 1, 2)$ , allow $1 - P(0, 1, 2, 3)$ , with attempt at combined $\lambda$ for M and W.<br>Accept combination method: at least 4 correct terms and "1 –" M1; all terms correct B1 |
|          | $= 0.482$ (3 sfs)                                                                            | A1 [3]       |                                                                                                                                                                                                  |

|               |                                               |                 |              |
|---------------|-----------------------------------------------|-----------------|--------------|
| <b>Page 7</b> | <b>Mark Scheme: Teachers' version</b>         | <b>Syllabus</b> | <b>Paper</b> |
|               | <b>GCE AS/A LEVEL – October/November 2011</b> | <b>9709</b>     | <b>73</b>    |

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| <b>(ii)</b> $N(52, 52)$<br><br>$\frac{60.5 - 52}{\sqrt{52}} \quad (= 1.179)$<br><br>$1 - \Phi("1.179")$<br>$(= 1 - 0.8808)$<br>$= 0.119 \text{ (3 sfs)}$ | B1     | Seen or implied                                                                                                                                       |
|                                                                                                                                                          | M1     | Standardising with $N(\lambda, \lambda)$ with<br>$\lambda = 10 \times 5.2$ or $10 \times 2.6$<br>Allow with wrong or no cc or no $\sqrt{\phantom{x}}$ |
|                                                                                                                                                          | M1     | Their correct area                                                                                                                                    |
|                                                                                                                                                          | A1 [4] |                                                                                                                                                       |