

**MARK SCHEME for the October/November 2009 question paper
for the guidance of teachers**

9709 MATHEMATICS

9709/71

Paper 71, maximum raw mark 50

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1 $X \sim B(180, 0.02) \sim \text{Po}(3.6)$ $P(X < 4) = e^{-3.6} (1 + 3.6 + 3.6^2 / 2 + 3.6^3 / 6)$ $= 0.515$	B1 M1 A1 [3]	Poisson with mean 180×0.02 Poisson attempt with their λ allow end errors Correct answer SR ₁ Use of Bin scores B1 only for ans 0.514 SR ₂ Use of Normal scores B1 only for 0.479
2 $1.96 \times \frac{1.5}{\sqrt{n}} < \frac{1}{2}$ $n = 35$	B1 B1 M1 A1 [4]	$1.96 \times \frac{1.5}{\sqrt{n}}$ seen Confidence interval halved Solving an equation in their z , 1.5, n (2 and sq rt not needed) Correct answer (condone $n \geq 35$)
3 (i) $P(\bar{W} > 51) = P\left(z > \frac{51 - 48.5}{12.4 / \sqrt{5}}\right)$ $= 1 - \Phi(0.451)$ $= 1 - 0.674$ $= 0.326$	M1 M1 A1 [3]	Standardising with 51 and mean 48.5 Standardising using $\sqrt{5}$ Correct answer
(ii) $z = 1.5$ or 1.499 $\frac{51.6 - 48.5}{(12.4 / \sqrt{n})} = 1.5$ $\sqrt{n} = 6$ $n = 36$	B1 M1 M1 A1 [4]	1.5 or 1.499 seen Standardising must have $\sqrt{n}$ (no cc) Attempt to solve equation with $\sqrt{n}$, their z in correct answer
4 (i) $P(X > 4) = 1 - P(0, 1, 2, 3, 4)$ $= 1 - e^{-1.8} \left(1 + 1.8 + \frac{1.8^2}{2} + \frac{1.8^3}{3!} + \frac{1.8^4}{4!} \right)$ $= 1 - 0.9635$ $= 0.036(4)$ This is < 0.05 and so $X > 4$ is in the critical region $P(4) = e^{-1.8} \left(\frac{1.8^4}{4!} \right) = 0.0723$	M1 M1 A1 A1ft B1 [5]	Adding at least 3 relevant Poisson terms Poisson expression for $P(X > 4)$ (oe implied by later working) Correct prob 0.036 (or 0.96 subject to later working) Correct comparison and statement identifying CR (ft their prob < 0.05) Verification that $X = 4$ is not in the cr region
(ii) $P(\text{Type II error}) = P(X = 0, 1, 2, 3, 4)$ $= e^{-2.3} \left(1 + 2.3 + \frac{2.3^2}{2} + \frac{2.3^3}{3!} + \frac{2.3^4}{4!} \right)$ $= 0.916$	B1 M1 A1 [3]	Correct region Poisson expression $P(0, 1, 2, 3, 4)$ Correct answer

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5 (i) $\int_0^{\pi/4} k \cos x \, dx = 1$ $[k \sin x]_0^{\pi/4} = 1$ $k \sin(\pi/4) = 1 \Rightarrow k/\sqrt{2} = 1$ $k = \sqrt{2}$ AG	M1 A1 [2]	Equating to 1 and attempt to integrate with limits Correct answer legit obtained (no decimals seen)
(ii) $\int_{0.4}^{\pi/4} k \cos x \, dx = [k \sin x]_{0.4}^{\pi/4}$ $= 1 - k \sin(0.4)$ $= 0.449$	M1 A1 [2]	Attempt to integrate from 0.4 to $\pi/4$ o.e. Correct answer
(iii) $\int_0^{Q3} k \cos x \, dx = 0.75$ $[k \sin x]_0^{Q3} = 0.75$ $k \sin Q3 - 0 = 0.75$ $Q3 = 0.559$	M1 M1 A1 [3]	Equation with integral on one side and 0.75 on the other o.e. Attempt to solve their integral for Q3 Correct answer
(iv) ${}^5C_3 \times (0.25)^3 \times (0.75)^2$ $= 0.0879$ (45/512)	M1 A1 [2]	Binomial expression involving ${}^5C_3$, 0.25 and 0.75 Correct answer
6 (i) $\bar{x} = 14.8$ (890/60 oe) $s^2 = \frac{1}{59} \left(13780 - \frac{890^2}{60} \right)$ $= 9.80$	B1 M1 A1 [3]	Correct answer Substituting in formula from book, o.e. Correct answer
(ii) $H_0: \mu = 15.2$ $H_1: \mu < 15.2$ $P(\text{Type I error}) = 0.1$ (10%) Say the photographer has fewer discards when she doesn't	B1 B1 B1ft [3]	Correct H_1 and H_0 Correct answer o.e. must be related to question. No contradictions. ft their H_1
(iii) Test statistic $z = \frac{14.83 - 15.2}{\sqrt{\frac{9.802}{60}}}$ $= -0.915$ $CV \, z = \pm 1.282$ Not enough evidence to support photographer's claim.	M1 A1 M1 A1ft [4]	Standardising must have $\sqrt{60}$ Correct z (± 0.91 to 0.92) or correct area 0.18 Valid comparison with correct CV must be + with + or – with – and consistent with their H_1 oe comparison of areas Correct conclusion ft their z and their CV No contradictions

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7	(i) $3C \sim N(990, 5.2^2 \times 3)$ ($= N(990, 81.12)$) $2W \sim N(1000, 7.1^2 \times 2)$ ($= N(1000, 100.82)$) $3C - 2W \sim N(-10, 181.94)$ $P((3C - 2W) < 0) = \Phi\left(\frac{0 - (-10)}{\sqrt{181.94}}\right)$ $= \Phi(0.741)$ $= 0.771$	B1	Correct mean for both 3 cans cola and 2 bottles water
		B1	Correct variance for both 3 cans cola and 2 bottles water
		M1	Correct method for mean and variance for $3C - 2W$ or vice versa
		M1	Standardising and using tables, need the sq root and area > 0.5
		A1	[5] Correct answer
	(ii) new drink $\sim N(910, 2 \times 5.2^2 + 0.5^2 \times 7.1^2)$ $\sim N(910, 66.68)$ $P(ND > 900) = 1 - P\left(z < \frac{900 - 910}{\sqrt{66.68}}\right)$ $= 1 - P(z < -1.225)$ $= \Phi(1.225)$ $= 0.8897$ (0.890)	B1	Correct mean for new drink
		B1	Correct variance for new drink
		M1	Standardising with sq rt and using tables
		A1	[4] Correct answer

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1	(i) doubling only gives even numbers so odd numbers not included	B1 [1]	Needs to be aware that odd numbers aren't included
	(ii) $98\% \text{ CI} = 1.62 \pm 2.326 \times \frac{\sqrt{0.0052}}{\sqrt{5}}$ $= 1.62 \pm 0.0750$ $= (1.54, 1.70)$	M1 B1 A1 [3]	Correct shape with $\sqrt{5}$ seen in denom and their evaluated mean and sd (condone unbiased estimate of sample variance). 2.326 seen correct answer, cwo. Accept 1.55 and 1.70 .
2	(i) $\lambda = 6.35$ $P(> 1) = 1 - e^{-6.35} (1 + 6.35)$ $= 0.987$	M1 A1 [2]	Attempt at Poisson, $1 - (P(0) + P(1))$ with their λ . Accept $1 - P(0)$. Correct answer.
	(ii) $\lambda = 889$ $P(> 850) = 1 - \Phi\left(\frac{850.5 - 889}{\sqrt{889}}\right)$ $= \Phi(1.29)$ or $\Phi(1.31)$ with no cc $= 0.902$	B1 M1* M1*dep A1 [4]	Correct mean Standardising, with or without continuity correction using their $N(889, 889)$. Condone sd/var mixes. Correct area (> 0.5) with or without cc Correct answer
3	(i) Poisson mean 4.26 $P(\text{overbooked}) = P(0, 1, 2 \text{ not arrive})$ $= e^{-4.26} \left(1 + 4.26 + \frac{4.26^2}{2} \right)$ $= 0.202$	B1 M1 A1 ft A1 [4]	Correct mean Poisson attempt at $P(0,1,2)$, their 4.26, allow end errors. Correct unsimplified answer, ft their mean. Correct answer, as final answer.
	(ii) mean = $135/75 = 1.8$ Σ (Poissons) new mean = $4.26 + 1.8 = 6.06$ $P(5) = e^{-6.06} \left(\frac{6.06^5}{5!} \right)$ $= 0.159$	M1 M1 A1 [3]	Adding their two means. Attempt at Poisson $P(5)$ with their mean. Correct answer
(i) SR Normal B1 $N(4.26, 4.17(5))$ or $N(208.74 (209), 4.17(5))$ B1 Prob = 0.195 SR Binomial B(213 , 1/50) B1 correct unsimplified expression for $P(0,1,2)$ B1 Prob = 0.200 or 0.199 . (ii) SR Binomials M2 all cases of binomial products A1 prob = 0.160			

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4 (i) $P(0) = 0.5^{10} = 0.000976 = P(10)$ $P(1) = {}^{10}C_1 \times 0.5^1 \times 0.5^9 = 0.00976 = P(9)$ $\Sigma 4 \text{ probs} = 0.02147 < 10\%$ $P(2) = {}^{10}C_2 \times 0.5^2 \times 0.5^8 = 0.0439 = P(8)$ $\Sigma 6 \text{ probs} = 0.1093 > 10\%$ Values of h are 2, 3, 4, 5, 6, 7, 8	M1 M1 M1 A1 [4]	Evaluating two Binomial end probs of 0 and 1, (or 10 and 9). Summing either/both ends and comparing to 10% (doesn't have to be two-tailed here). Summing a third/last end probability and comparing to 10% (needn't be two-tailed here) Correct answer only. A1 dep on all 3 Ms. No errors seen.
(ii) $P(\text{Type II error}) =$ $P(2, 3, 4, 5, 6, 7, 8) (= 1 - P(0, 1, 9, 10))$ $= 1 - \left((0.3)^{10} + {}^{10}C_1(0.7)(0.3)^9 \right)$ $\quad \quad \quad + {}^{10}C_9(0.7)^9(0.3) + (0.7)^{10}$ $= 1 - 0.149$ $= 0.851$	B1ft M1 A1 [3]	Identifying correct probs ft their ans (i) Binomial expression for their P(Type II error) with powers summing to 10, ${}^{10}C_{\text{somethings}}$ and 0.3 and 0.7 seen. Correct final answer.
5 (i) $H_0: \mu = 750$ $H_1: \mu < 750$ Test statistic $z = \frac{746 - 750}{(11/\sqrt{20})} = +/-1.626$ CV $z = -1.751$ or -1.752 Not enough evidence to say mean is less.	B1 B1 M1 A1ft [4]	H_0 and H_1 correct Correct test statistic used in comparison. oe. Correct CV and valid comparison. oe. [0.0520 or 745.7] [2 tail 2.054/5] Correct conclusion ft their test statistic. No contradictions. Condone cc's.
(ii) $z = +/-1.881$ or $+/-1.882$ $-1.881 > \frac{745 - 750}{(11/\sqrt{n})}$ $n = 18$	B1 M1* M1*dep A1 [4]	Accept 1.881 or 1.882 Equation or inequality relating their z to standardised value. Condone cc's. Solving attempt for n Correct answer. cwo. 3 sf accuracy.

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6 (i) $\int_1^2 \frac{x}{3}(k-x) dx = 1$ $\left[\frac{kx^2}{6} - \frac{x^3}{9} \right]_1^2 = 1$ $\left[\frac{4k}{6} - \frac{8}{9} \right] - \left[\frac{k}{6} - \frac{1}{9} \right] = 1$ $k = 32/9 \text{ AG}$	M1 A1 A1 [3]	Equating to 1 and attempting to integrate, ignore limits here. Correct integration and correct limits. 32/9 correctly obtained.
(ii) $E(X) = \int_1^2 \frac{x^2}{3}(k-x) dx$ $= \left[\frac{kx^3}{9} - \frac{x^4}{12} \right]_1^2$ $= 1.52 \text{ (491/324)}$	M1 A1 [2]	attempting to evaluate $\int_1^2 \frac{x^2}{3}(k-x) dx$ correct answer.
(iii) $\int_1^{1.5} \frac{x}{3}(k-x) dx = \left[\frac{kx^2}{6} - \frac{x^3}{9} \right]_1^{1.5}$ $= 0.477 \text{ (or } 0.523)$ this is < 0.5 so 1.5 is < median ----- Alternative method	M1 A1 A1ft [3] M1 A1 A1	attempt to integrate using 1 and 1.5 or 1.5 and 2 correct reasoning and conclusion, ft their integration ----- $\int f(x) dx$ limits 1 to m (or m to 2) and equate to 0.5 attempted. Correct cubic in m in simplified form (no k's). $m = 1.52(2)$, so median > 1.5, or, 2 trials showing that median > 1.5.
7 (a) $\text{Var}(Y) = b^2 \text{Var}(X) = 4 \text{Var}(X)$ $b = 2 (> 0)$ $E(Y) = a + 2E(X) = E(X) + 0.8$ $a + 14.24 = 7.92$ $a = -6.32$	B1 M1 A1 [3]	Correct answer Using $E(Y) = a + 2E(X)$, their “2”. Correct equation in a, b subst in, using 0.8 correctly. Correct answer
(b) (i) $R + S \sim N(3\mu, 13)$ $z = -1.282 \text{ or } -1.281$ $1.282 = \frac{3\mu - 1}{\sqrt{13}}$ $\mu = 1.87$	B1 B1 M1 A1 [4]	Correct mean and variance. Correct z, must be -ve (or -ve implied). Standardising and solving, their mean, z, 13 (but combined), allow “13” or “$\sqrt{13}$”. Correct answer.
(ii) $S - R \sim N(\mu, 13)$ $P[(S - R) > 0] = P\left(z > \frac{0 - \mu}{\sqrt{13}}\right)$ $= \Phi(0.5197)$ $= 0.699$	B1 ft M1 A1 [3]	Correct mean and variance, ft their mean. Standardising and correct area (i.e. > 0.5 for correct), but consistent with their values. Accept 0.698