

**MARK SCHEME for the October/November 2009 question paper
for the guidance of teachers**

9709 MATHEMATICS

9709/51

Paper 51, maximum raw mark 50

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- CIE will not enter into discussions or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the October/November 2009 question papers for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.



Page 2	Mark Scheme: Teachers' version	Syllabus	Paper
	GCE A/AS LEVEL – October/November 2009	9709	51

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- The symbol $\checkmark$ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously “correct” answers or results obtained from incorrect working.
- Note: B2 or A2 means that the candidate can earn 2 or 0.
B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.
- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

Page 3	Mark Scheme: Teachers' version	Syllabus	Paper
	GCE A/AS LEVEL – October/November 2009	9709	51

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)
CWO	Correct Working Only – often written by a ‘fortuitous’ answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through √” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Page 4	Mark Scheme: Teachers' version	Syllabus	Paper
	GCE A/AS LEVEL – October/November 2009	9709	51

1	[20 = 100x/0.25]	M1		For using $F = \lambda x/L$	
	Compressed by 0.05 m	A1			
	Distance is 0.2 m	A1√	3		[3]

2	Gain in KE = $\frac{1}{2} 0.2 \times 3^2$ (= 0.9 J)	B1			
	Gain in GPE = $0.2gx$ (= 2x J)	B1			
	Loss in EPE = $\frac{1}{2} 4x^2/0.6$ (= $10x^2/3$ J)	B1			
	[$10x^2/3 = 9/10 + 2x$]	M1		For using gain in KE + Gain in GPE = Loss in EPE	
	$x = 0.9$	A1	5		[5]

3	(i)	M1		For taking moments about the base	
	$8 \times 8 + 2 \times (16 + 3) = (8 + 2) \dot{y}$	A1			
	Distance of centre of mass is 10.2 cm	A1	3	AG	
	(ii) [$\tan \theta_{\max} = r/\dot{y}$]	M1		θ takes its max value when c.m. is vertically above point of contact	
	$\tan \theta_{\max} = 4/10.2$	A1√			
	Greatest possible value is 21.4	A1	3		[6]

4	(i) [$25 \times (24/25) = V \cos \theta$]	M1		For using $\dot{x} = V \cos \theta$	
	$V \cos \theta = 24$	A1	2		
	(ii) [$25 \times (7/25) = V \sin \theta - 0.3g$]	M1		For using $\dot{y} = V \sin \theta - gt$	
	$V \sin \theta = 10$	A1			
	[$\tan \theta = 10/24$, $V^2 = 24^2 + 10^2$]	M1		For using $\tan \theta = V \sin \theta / (V \cos \theta)$ or $V^2 = (V \cos \theta)^2 + (V \sin \theta)^2$	
	$\theta = 22.6^\circ$	A1√		ft dependent on both M marks	
	$V = 26$	A1√	5	ft dependent on both M marks	[7]

5	(i) $OG = 2 \times 0.5 \sin 30^\circ / (3 \times (\pi/6))$ (= $1/\pi$)	B1			
		M1		For taking moments about O	
	$3 \times (\sin 30^\circ / \pi) = F \times 0.5$	A1√			
	$F = 0.955$	A1	4		
	(ii)	M1		For resolving forces on the lamina horizontally and vertically	
	$X = F \cos 60^\circ$ (= 0.477)	A1			
	$Y = 3 - F \sin 60^\circ$ (= 2.17)	A1			
	Magnitude is 2.22 N	A1√	4	ft $(F^2 - 3\sqrt{3}F + 9)^{1/2}$	[8]

6	(i) $a = 1.5^2 / (0.7 \sin 40^\circ)$	B1			
	[$T \sin 40^\circ = 0.25a$]	M1		For using Newton's second law horizontally	
	Tension is 1.94 N	A1	3		
	(ii)	M1		For resolving forces vertically	
	$T \cos 40^\circ + R = 0.25g$	A1			
	Force exerted is 1.01 N	A1√	3	ft $2.5 - T \cos 40^\circ$	
	(iii)	M1		For using Newton's second law horizontally and $a = r\omega^2$	
	$T \sin 40^\circ = 0.25(0.7 \sin 40^\circ) \omega^2$	A1			
	$T \cos 40^\circ = 0.25g$ ($T = 3.2635\dots$)	B1			
	[$\tan 40^\circ = 0.7 \sin 40^\circ \omega^2 / g$ or $3.2635\dots \sin 40^\circ = 0.25(0.7 \sin 40^\circ) \omega^2$]	M1		For eliminating T or substituting for T	
	Maximum value of ω is 4.32	A1	5		[11]

Page 5	Mark Scheme: Teachers' version	Syllabus	Paper
	GCE A/AS LEVEL – October/November 2009	9709	51

7	(i)	M1		For using Newton's second law with $a = dv/dt$	
	$0.1dv/dt = -0.1g - 0.1v \Rightarrow \frac{1}{v+10} \frac{dv}{dt} = -1$	A1	2	AG	
	(ii) $[\int \frac{1}{v+10} dv = -\int dt]$	M1		For separating variables and attempting to integrate	
	$\ln(v+10) = -t (+A)$	A1			
	$A = \ln 30$	B1✓			
	$[\ln\{(v+10)/30\} = -t \Rightarrow (v+10)/30 = e^{-t}]$	M1		For transposing to eliminate $\ln$	
	$v = 30e^{-t} - 10$	A1			
	Until $0 = 30e^{-t} \Rightarrow$ valid for $0 \leq t \leq \ln 3$	B1	6		
	(iii)	M1		For substituting $v = 20$ into $a = -(v+10)$ or $t = 0$ into $a = -30e^{-t}$	
	Acceleration is -30 ms^{-2}	A1	2		[10]

**MARK SCHEME for the October/November 2009 question paper
for the guidance of teachers**

9709 MATHEMATICS

9709/52

Paper 52, maximum raw mark 50

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- CIE will not enter into discussions or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the October/November 2009 question papers for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.



Page 2	Mark Scheme: Teachers' version	Syllabus	Paper
	GCE A/AS LEVEL – October/November 2009	9709	52

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- The symbol $\checkmark$ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.
- Note: B2 or A2 means that the candidate can earn 2 or 0.
B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.
- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

Page 3	Mark Scheme: Teachers' version	Syllabus	Paper
	GCE A/AS LEVEL – October/November 2009	9709	52

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only – often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through $\sqrt{}$ " marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR–2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Page 4	Mark Scheme: Teachers' version	Syllabus	Paper
	GCE A/AS LEVEL – October/November 2009	9709	52

1	$[\tan \alpha = 2/3 \text{ or } \tan \alpha = \{\frac{1}{3} \times 4 \div \frac{2}{3} \times 3\}]$	M1		For finding the angle between CA and the median (or the angle between CA and CG where G is the centre of mass)	
	$\alpha = 33.7^\circ$	A1			
	$[60^\circ + \alpha > 90^\circ]$	M1		For comparing $60^\circ + \alpha$ with 90°	
	$\rightarrow$ prism falls on face containing BC	A1	4		4

2 (i)	$[\bar{y}_{\text{handle}} \text{ is } 20 \times 0.8/0.927.. \text{ from centre } (17.25)]$	M1		For using $\bar{y} = r \sin \alpha \div \alpha$	
	Centre of arc is 12 cm below top of cylinder	B1			
	Height = $2 + 46 - 12 + 17.25 = 53.25$ cm	A1	3	AG	
(ii)	$[(50 + 100 + 25) \bar{y} = 50 \times 1 + 100 \times 25 + 25 \times 53.25]$	M1		For taking moments about the base	
	Height is 22.2 cm	A1	2		5

3	$24 = 50 \times 0.96t$	M1		For using $x = (V \cos \theta)t$ to find t	
	$t = 0.5$	A1			
	$[y = 50 \times 0.5 \times (7/25) - \frac{1}{2} 10 \times 0.5^2 \text{ or } y = (7/24)24 - 10 \times 24^2/(2 \times 50^2 \times 0.96^2)]$	M1		For substituting $t = 0.5$ into $Y = Vt \sin \theta - \frac{1}{2} g t^2$ or $x = 24$ into the equation of the trajectory.	
	$y_Q = 5.75$	A1			
	$y_P = 7 - \frac{1}{2} g \times 0.5^2$	M1		For evaluating y_P when $t = 0.5$	
	$y_P = y_Q$ at time $t = 0.5 \rightarrow$ particles collide	A1	6		6

Page 5	Mark Scheme: Teachers' version	Syllabus	Paper
	GCE A/AS LEVEL – October/November 2009	9709	52

4 (i)	$EE = \frac{1}{2} (15\text{ m})x^2/3$	B1			
		M1		For using Loss of PE = Gain in KE + EE	
	$\frac{1}{2}mv^2 + \frac{1}{2} (15\text{ m})x^2/3 = mg(3 + x^2)$	A1ft		Ft error in EE	
	$v^2 = 5(12 + 4x - x)$	A1	4	AG	
(ii)	$[a = -2.5(2x - 4) \text{ or } a = g - 15x/3]$	M1		For using $a = v(dv/dx)$ or $a = (mg - \lambda x/L)/m$	
	$[v = 0 \rightarrow x = 6 \rightarrow a = -20]$	M1		For finding x at the lowest point and substituting	
	Magnitude is 20 ms^{-2} ; direction is upwards	A1	3		7

5 (i)	$[r = 0.8]$	M1		For using $v_P/v_A = r/0.5$	
		M1		For using $\sin\theta = (r - 0.5)/0.8$	
	$\sin\theta = \frac{3}{8}$	A1	3	AG	
(ii)	$[T\cos\theta = mg]$	M1		For resolving forces vertically	
	Tension is 4.31 N	A1	2		
(iii)	$[T\sin\theta = m\omega^2 r]$	M1		For using Newton's second law and $a = \omega^2 r$	
	$0.375T = 0.4 \times 0.8\omega^2$	A1			
	$\omega = 2.25$	A1	3		8

Page 6	Mark Scheme: Teachers' version	Syllabus	Paper
	GCE A/AS LEVEL – October/November 2009	9709	52

6 (i)	Moment of T_O about $P = T_O h \cos 20^\circ$	B1			
	Moment of W about $P = 5g \times 0.75h$	B1			
	$[T_O h \cos 20^\circ = 37.5h]$	M1		For taking moments about P	
	Tension in string at O is 39.9N	A1	4		
(ii)		M1		For resolving forces horizontally or vertically or for taking moments	
	$T_P \sin \theta = 39.9 \sin 20^\circ$	A1 ft		ft incorrect T_O	
	$T_P \cos \theta + 39.9 \cos 20^\circ = 5g$ or $(T_P \cos \theta)h = \frac{1}{4}h \times 50$ or $(T_P \cos \theta) \frac{3}{4}h = (T_O \cos 20^\circ) \frac{1}{4}h$	A1 ft		ft incorrect T_O	
		M1		For eliminating T_P	
	$\theta = 47.5$	A1			
	Tension in string at P is 18.5N	A1	6		10

7(i)	$[3 - 90/(v + 30)](dv/dx) + 1 = 0$ or $3 - 90/(v + 30) + (dx/dv) = 0$	B1			
		M1		For using $a = v(dv/dx)$	
	Acceleration is $-\frac{1}{3}(v + 30) \text{ ms}^{-2}$	A1	3	AG	
(ii)	$[0.3g + R = 0.3(v + 30)/3]$	M1		For using Newton's second law	
	Resisting force is $0.1v \text{ N}$	A1	2		
(iii)	$\int \frac{dv}{v + 30} = -\frac{1}{3} \int dt$	M1		For using $a = dv/dt$, separating variables and integrating	
	$\ln(v + 30) = -t/3 (+ A)$	A1			
	$\ln 50 = 0 + A$	B1			
	$[\ln 30 = -t/3 + \ln 50]$	M1		For finding t when $v = 0$	
	Time taken is 1.53 s	A1	5		10