

NOVEMBER 2002

**GCE Advanced Level
GCE Advanced Subsidiary Level**

MARK SCHEME

MAXIMUM MARK : 50

SYLLABUS/COMPONENT : 9709 /5, 8719 /5

**MATHEMATICS
(Mechanics 2)**



Page 1	Mark Scheme	Syllabus	Paper
	A & AS Level Examinations – November 2002	9709, 8719	5

1		$r = 4\text{cm}$	B1	
		Uses $v = \omega r$	M1	
		Speed is 20cm s^{-1} (FT if $r = \frac{1}{3} \times$ candidates perp. distance from B)	A1	3

2	(i)	Takes moments about B [$T \cos 60^\circ \times 2 = 10\text{g} \times 1$]	M1	
		Obtains tension as 100 N	A1	2
	(ii)	Uses Hooke's Law (for expression in x or L only)	M1	
		Obtains $100 = 200(3 - L)/L$ or $100 = 200x/(3 - x)$	A1 ft	
		Obtains natural length as 2 m	A1	3

3	(i)	$x = 10t, y = -5t^2$	B1	
		Eliminates t to find an equation in x and y (allow if candidate derives the general trajectory equation)	M1	
		Obtains $y = -x^2/20$ (Allow B1/3 for putting $\theta=0$ and $v=10$ in traji. equation)	A1	3
	(ii)	Uses $\tan \theta = dy/dx$ or $\tan \theta = \dot{y}/\dot{x}$	M1	
		Obtains $x = 30$ when $y = -45$, or $t = 3$ when $y = -45$, or $\dot{x} = 10$ and $\dot{y} = (\pm)30$	A1	
		Obtains angle as 108.4° (108.435) or 71.6° (71.565)	A1	3

4		$a = 4^2/0.8$ [= 20]	B1	
		Uses Newton's 2 nd law horizontally to obtain a 3 term equation	M1	
		Obtains $(T_P + T_Q) \cos 30^\circ = 0.5 \times 20$ [$T_P + T_Q = \frac{20}{\sqrt{3}}$]	A1 ft	
		Resolves forces vertically to obtain a 3 term equation	M1	
		Obtains $T_P \cos 60^\circ = T_Q \cos 60^\circ + 5$ [$T_P - T_Q = 10$]	A1	
Alternatively for the above 4 marks				
		Uses Newton's 2 nd law perpendicular to BQ to obtain a 3 term equation	M2	
		Obtains $T_P \cos 30^\circ - 0.5g \cos 30^\circ = 0.5 \times 20 \cos 60^\circ$ [$T_P = 5 + \frac{10}{\sqrt{3}}$]	A2 ft	
[SR Allow A1 with 1 sign or trigonometric error]				
		Obtains tension in PB as 10.8 N (10.7735)	A1	6

NB Use of equal tensions can score B1, M1, A0, M1, A0 at most.

Page 2	Mark Scheme	Syllabus	Paper
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5	(i)	GPE = $0.075g(d \sin 30^\circ)$ or $0.075g(d + x)\sin 30^\circ$	B1	
		EPE = $1.5(d - 2)^2/2 \times 2$ or $1.5x^2/2 \times 2$	B1	
		Uses the principle of conservation of energy to form an equation with GPE and EPE terms $\left[\frac{3}{8}d = \frac{3}{8}(d - 2)^2 \text{ or } \frac{3}{8}(2 + x) = \frac{3}{8}x^2 \right]$	M1*	
		Attempts to solve a quadratic equation in d $[(d - 1)(d - 4) = 0]$ or attempts to solve a quadratic equation in x and uses $d = x + 2$ $[(x + 1)(x - 2) = 0 \text{ and } d = 2 + 2]$	M1 dep	
		Obtains distance as 4m	A1	5
	(ii)	Obtains the tension at the lowest point as 1.5 N ft for $1.5(d - 2)/2$	B1 ft	
		Uses Newton's 2 nd law to obtain a 3 term equation	M1	
		Obtains $1.5 - 0.075g \sin 30^\circ = 0.075a$	A1 ft	
		Obtains acceleration as 15ms^{-2}	A1	4

Page 3	Mark Scheme	Syllabus	Paper
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6	(i)	Uses Newton's 2 nd law and $a = v \, dv/dx$, and attempts to integrate $[(1/10) v \, dv/dx = -v/200]$	M1*	
		$v = -x/20 \quad (+C)$	A1	
		Uses $v(0) = 5$ to find C	M1 dep	
		Obtains $v = -x/20 + 5 \quad (\text{a.e.f.})$	A1	4
	(ii)	Uses $v = dx/dt$, separates the variables and integrates $[\int \frac{1}{100-x} dx = \int \frac{1}{20} dt]$	M1*	
		Obtains $\ln(100-x) = -t/20 (+C)$	A1	
		Uses $x = 0$ when $t = 0$ to obtain $t = 20[\ln 100 - \ln(100-x)]$ ft only if the term in x is logarithmic	A1 ft	
		For taking anti-logarithms throughout the equation $[100-x = 100e^{-t/20}]$ <i>N.B. $\ln(100-x) = -t/20 + C \rightarrow 100-x = e^{-t/20 + C} = e^{-t/20} + e^C$ is Mo</i>	M1 dep	
		Obtains $x = 100(1 - e^{-t/20}) \quad (\text{a.e.f.})$	A1	5
Alternatively for the above 9 marks Uses Newton's 2nd law with $a = dv/dt$, separates the variables and integrates $[\int \frac{1}{v} dv = -\int \frac{1}{20} dt]$ Obtains $\ln v = -t/20 \quad (+C)$ Uses $v = 5$ when $t = 0$ to obtain $t = 20[\ln 5 - \ln v]$ ft only if the term in v is logarithmic For taking anti-logarithms throughout the equation $[v = 5e^{-t/20}]$ Uses $v = dx/dt$ and integrates $[x = \int 5e^{-t/20} dt]$ Obtains $x = -100 e^{-t/20} (+C)$ Uses $x = 0$ when $t = 0$ to obtain $x = 100(1 - e^{-t/20})$ Eliminates the exponential term from $x = 100(1 - e^{-t/20})$ and $v = 5e^{-t/20}$ to obtain an equation in x and v $[x = 100(1 - v/5)]$ Obtains $v = -x/20 + 5$				
	(iii)	$x = 100(1 - e^{-t/20})$ and $e^{-t/20}$ is +ve for all $t \rightarrow x < 100$	B1	1

N.B. If (i) is solved as in scheme and then (ii) is solved using the alternative method, the 5 marks awarded for (ii) from the alternative method are M1* (Ae), A1 (not FT), M1 (dep), M1* (uses $v = \frac{dx}{dt}$ and integrate or substit. for v from (i)), (Ae) A1 (Mo dep) (Ae).

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7	(i)	Uses $(A_1 \pm A_2)x = A_1x_1 \pm A_2x_2$ to find x $[(25 \times 5 + 15 \times 5)x = 25 \times 5 \times 12.5 + 15 \times 5 \times 2.5]$	M1	
		Obtains $x = 8.75$	A1	
		Uses $(A_1 \pm A_2)y = A_1y_1 \pm A_2y_2$ to find y $[(25 \times 5 + 15 \times 5)y = 25 \times 5 \times 2.5 + 15 \times 5 \times 12.5]$	M1	
		Obtains $y = 6.25$	A1	4
	(ii)	States or obtains $\mu = \tan \alpha$ for prism on point of sliding	B1	
		States or obtains $\tan \alpha \leq x/y$ for prism not toppled	M1	
		Eliminates $\tan \alpha$ from $\mu = \tan \alpha$ and $\tan \alpha \leq x/y$, and substitutes for x and y $[\mu \leq 8.75/6.25]$ obtains $\tan \alpha \leq 8.75/6.25$	A1	
		Coefficient of friction is less than $7/5$ (convincing explanation for inequality)	A1	4
	(iii)	States or obtains $\tan \beta = y/x$ for prism on point of toppling	M1	
		States or obtains $\mu > \tan \beta$ for prism not sliding (or on the point of sliding)	B1	
		Eliminates $\tan \beta$ from $\tan \beta = y/x$ and $\mu > \tan \beta$, and substitutes for x and y $[\mu > 6.25/8.75]$ to obtain the least value of the coefficient of friction as $5/7$ (convincing explanation for inequality)	A1	3