



Cambridge International AS & A Level

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MATHEMATICS

9709/31

Paper 3 Pure Mathematics 3

May/June 2025

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
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- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
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This document has **20** pages.



1 (a) Sketch the graph of $y = |2x - 3|$.

[1]

(b) Solve the inequality $3x - 1 < |2x - 3|$.

[2]

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Find q in terms of p .

[3]

[illegible]

- [illegible]



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Handwriting practice area with 20 sets of dotted lines.



4

$$x = e^{\tan t}, \quad y = 3 \tan^2 t.$$

Find the equation of the tangent to the curve at the point $(e, 3)$. Give your answer in the form $y = mx + c$, where m and c are exact. [6]

[illegible]

This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- Find the values of p and q . [5]

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6 It is given that $z_1 = 3e^{\frac{1}{4}\pi i}$, $z_2 = \frac{3}{2}e^{\frac{1}{6}\pi i}$ and $\omega = 2e^{\frac{1}{2}\pi i}$.

- (a) State the values of ωz_1 and ωz_2 . Give your answers in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leq \pi$. [2]

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- (b) On a sketch of an Argand diagram with origin O , show the points A , B , C and D representing the complex numbers z_1 , z_2 , ωz_1 and ωz_2 respectively. [2]

- (c) State the geometric effects of multiplying z_1 and z_2 by ω . [2]

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[4]

[illegible]



- 8 With respect to the origin O , the points A and B have position vectors $2\mathbf{i}+4\mathbf{k}$ and $5\mathbf{i}+\mathbf{j}+6\mathbf{k}$ respectively. The line l_1 passes through the points A and B .

(a) Find a vector equation for the line l_1 . [2]

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The line l_2 has equation $\mathbf{r} = 2\mathbf{i} + \mathbf{j} + 5\mathbf{k} + \mu(\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$.

(b) Show that l_1 and l_2 do **not** intersect. [4]

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(c) Find the acute angle between the directions of l_1 and l_2 . [3]

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(a)

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(b) Verify by calculation that a lies between 2 and 2.1.

[2]

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(c) Use an iterative formula based on the equation in part (a) to determine a correct to 2 decimal places. Give the result of each iteration to 4 decimal places.

[3]

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- 10 (a) Find the quotient and remainder when $x^3 + 5x^2 - 2x - 15$ is divided by $x^2 - 3$. [3]

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- (b) The variables x and y satisfy the differential equation

$$\frac{dy}{dx} = \frac{x^3 + 5x^2 - 2x - 15}{6y(x^2 - 3)}.$$

It is given that $y = 2$ when $x = 2$.

Solve the differential equation to obtain an expression for y^2 in terms of x . [5]

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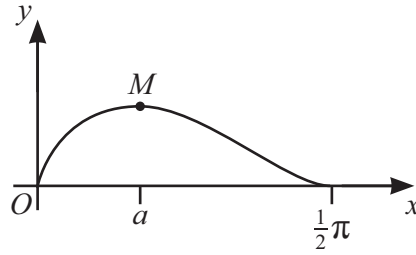
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(a) Find the exact value of a .

[6]

[illegible]



- Find the exact volume of the solid generated.

[5]

This image shows a full page of a worksheet designed for handwriting practice. It consists of approximately 20 horizontal rows. Each row is defined by two parallel dotted lines, creating a series of uniform gaps for writing. The entire page is otherwise blank, with no text or other markings.

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MATHEMATICS**9709/32**

Paper 3 Pure Mathematics 3

May/June 2025**1 hour 50 minutes**

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[5]

[illegible]

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[illegible]

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[illegible]



- 3 On an Argand diagram shade the region whose points represent complex numbers z which satisfy both the inequalities $|z - 3i| \leq 2$ and $\frac{1}{4}\pi \leq \arg(z - 1 - 2i) \leq \frac{3}{4}\pi$. [5]



[6]

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- By first forming a quartic equation in x or y , find the square roots of $-1 - 4\sqrt{5}i$ in exact Cartesian form. [5]

[illegible]

6

$$|x-2| = 2 \sin \frac{1}{2}x$$

has only one root in the interval $0 < x < \pi$.

[2]

(b) Show by calculation that this root lies between 1 and 1.5.

[2]

[illegible]



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[4]

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$$\sin 2\theta \frac{dx}{d\theta} = (4x + 3) \cos 2\theta,$$

and $x = 0$ when $\theta = \frac{1}{12}\pi$.

Solve the differential equation and obtain an expression for x in terms of θ .

[7]

[illegible]

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- 9 With respect to the origin O , the points A , B and C have position vectors given by

$$\overrightarrow{OA} = \begin{pmatrix} 1 \\ -4 \\ 2 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix}.$$

- (a) Find a vector equation for the line through A and B .

[2]

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- (b) Using a scalar product, find the exact value of $\cos BAC$.

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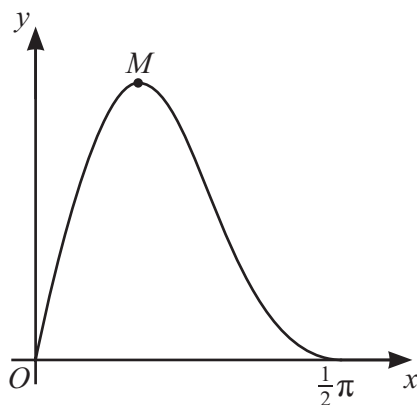
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[6]

[illegible]

11



The diagram shows the graph of $y = 5 \sin 2x \cos^2 x$ for $0 \leq x \leq \frac{1}{2}\pi$ and its maximum point M .

- (a) Find the exact x -coordinate of M .

[6]

[illegible]



[5]

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MATHEMATICS**9709/33**

Paper 3 Pure Mathematics 3

May/June 2025**1 hour 50 minutes**

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1 (a) Sketch the graph of $y = |3x - 2a|$, where a is a positive constant.

[1]

(b) Hence or otherwise solve the inequality $|3x - 2a| < x + 5a$.

[3]

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[4]

[illegible]

[4]

[illegible]



- 4 (a) It is given that $z_1 = r_1 e^{i\theta_1}$ and $z_2 = r_2 e^{i\theta_2}$.

Show that $(z_1 z_2)^* = z_1^* z_2^*$.

[3]

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- (b) $z = 3e^{\frac{1}{4}\pi i}$ is a root of the equation $z^2 + bz + c = 0$, where b and c are real.

State the other root and hence find the values of b and c .

[3]

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(a) Express $f(x)$ in partial fractions.

[3]

[illegible]



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[illegible]

[illegible]



[4]

[illegible]



- 9 With respect to the origin O , the points A , B and C have position vectors given by

$$\overrightarrow{OA} = \mathbf{i} + 2\mathbf{j}, \quad \overrightarrow{OB} = \mathbf{i} + 3\mathbf{j} - 2\mathbf{k} \quad \text{and} \quad \overrightarrow{OC} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k}.$$

The line l passes through B and C .

- (a) Find a vector equation for l .

[2]

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- (b) The point P is the foot of the perpendicular from A to l .

Find the position vector of P .

[4]

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(c) The point D is the reflection of A in l .

Find the position vector of D . [2]

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- $$\sin 4y \frac{dy}{dx} = x \sin 2y \sin 3x.$$

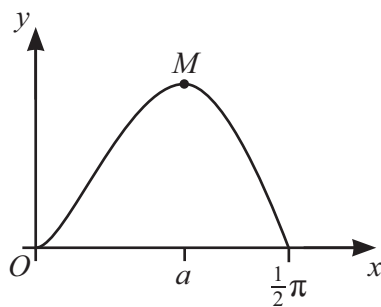
(a) Solve the differential equation, obtaining a relation between x and y . [8]

[illegible]



(b) Given that $0 < y < \frac{1}{2}\pi$, find the values of y when $x = 0$. [2]





The diagram shows the curve $y = \sqrt{x} \sin 2x$ for $0 \leq x \leq \frac{1}{2}\pi$. The curve has a maximum point at M , where $x = a$.

- (a) Show that $\tan 2a = -4a$ [4]

[illegible]

- (b)** Show by calculation that $0.9 < a < 0.95$. [2]





- converges, then it converges to a .

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[illegible]





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MATHEMATICS

9709/35

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May/June 2025

1 hour 50 minutes

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[5]

[illegible]



3 The complex numbers s and t are given by

$$s = 5(\cos 0.25 + i \sin 0.25) \quad \text{and} \quad t = 6e^{3i}.$$

(a) Express $\frac{s}{t}$ in the form $re^{i\theta}$, where $-\pi < \theta \leq \pi$ and $r > 0$. [2]

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(b) In an Argand diagram with origin O , the points A and B represent the complex numbers s and $\frac{s}{t}$ respectively.

By considering the line segments OA and OB , or otherwise, state the two geometric effects of dividing a complex number by $6e^{3i}$. [2]

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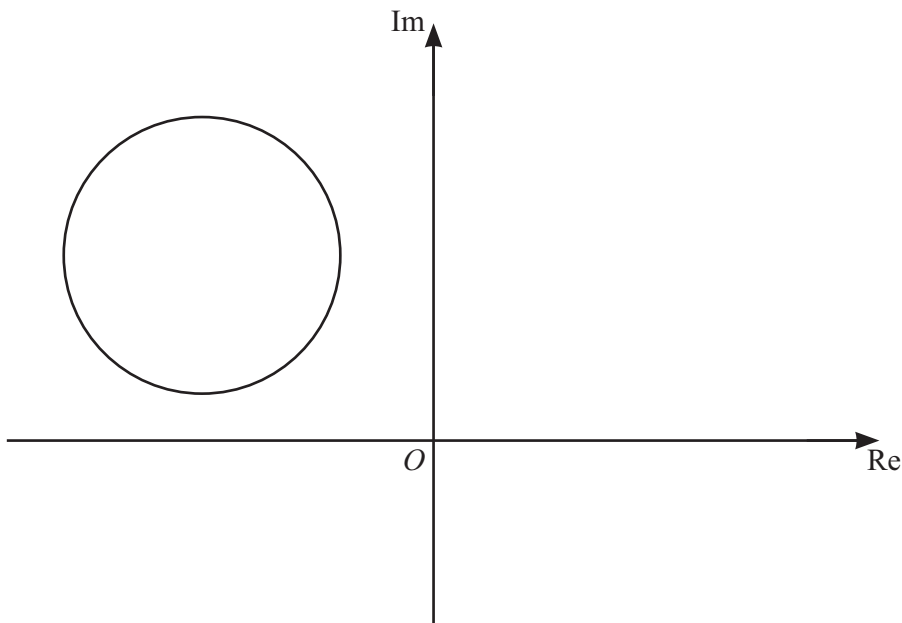
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The diagram shows the locus of points representing the complex numbers, z , satisfying $|z + 5 - 4i| = 3$.

- (a) For the points on this locus, determine the maximum and minimum possible values of $|z|$. [3]

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- (b) For the points on this locus, determine the minimum possible value of $\arg z$. [3]

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and

for $0 \leq t \leq 2\pi$.

(a) Show that $\frac{dy}{dx}$ can be written as $A \operatorname{cosec} 3t$, where A is a constant to be found.

[5]

[illegible]



- (b)** Find an equation of the normal to the curve at the point where $t = \frac{1}{12}\pi$. Give your answer in the form $y = mx + c$, where the constants m and c are exact. [4]

[illegible]

(a) Find the exact values of x when the gradient of the curve is $\frac{1}{4}$. [3]

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- (c) Show that, if a sequence of real values given by the iterative formula

$$x_{n+1} = \frac{1}{2} \cos^{-1} \left(\frac{-2}{4x_n + 1} \right)$$

converges, then it converges to the root of the equation in part (a).

[2]

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- (d) Use this iterative formula to calculate this root correct to 3 decimal places. Give the result of each iteration to 5 decimal places.

[3]

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[5]

[illegible]

[illegible]

(a) It is given that $|\overrightarrow{AB}| = |\overrightarrow{BC}|$.

(a) It is given that $|\overrightarrow{AB}| = |\overrightarrow{BC}|$.

(a) It is given that $|\overrightarrow{AB}| = |\overrightarrow{BC}|$.

[3]

[illegible]



(b) A, B, C and D are the vertices of a rhombus.

Find the position vector of D .

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- $$(x^2 + 3) \frac{dy}{dx} = e^{3y}(x - 2).$$

Solve the differential equation, and find the value of y when $x = 2$.

[8]

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[illegible]

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