

# Cambridge International A Level

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**MATHEMATICS****9709/31**

Paper 3 Pure Mathematics 3

**May/June 2025****MARK SCHEME**Maximum Mark: 75

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**Published**

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

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This document consists of **21** printed pages.

**PUBLISHED****Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

**GENERIC MARKING PRINCIPLE 1:**

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

**GENERIC MARKING PRINCIPLE 2:**

Marks awarded are always **whole marks** (not half marks, or other fractions).

**GENERIC MARKING PRINCIPLE 3:**

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

**GENERIC MARKING PRINCIPLE 4:**

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

**GENERIC MARKING PRINCIPLE 5:**

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

**GENERIC MARKING PRINCIPLE 6:**

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

**Mathematics-Specific Marking Principles**

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

**Annotations guidance for centres**

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

**Annotations**

| <b>Annotation</b>                                                                   | <b>Meaning</b>                         |
|-------------------------------------------------------------------------------------|----------------------------------------|
|    | More information required              |
|    | Accuracy mark awarded zero             |
|    | Accuracy mark awarded one              |
|    | Independent accuracy mark awarded zero |
|  | Independent accuracy mark awarded one  |
|  | Independent accuracy mark awarded two  |
|  | Benefit of the doubt                   |
|  | Blank Page                             |
|  | Incorrect                              |
| Dep                                                                                 | Used to indicate DM0 or DM1            |

| <b>Annotation</b>                                                                   | <b>Meaning</b>                                                                                                                             |
|-------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------|
| DM1                                                                                 | Dependent on the previous M1 mark(s)                                                                                                       |
|    | Follow through                                                                                                                             |
|    | Indicate working that is right or wrong                                                                                                    |
| Highlighter                                                                         | Highlight a key point in the working                                                                                                       |
|    | Ignore subsequent work                                                                                                                     |
|    | Judgement                                                                                                                                  |
|    | Judgement                                                                                                                                  |
|    | Method mark awarded zero                                                                                                                   |
|    | Method mark awarded one                                                                                                                    |
|    | Method mark awarded two                                                                                                                    |
|    | Misread                                                                                                                                    |
|    | Omission or Other solution                                                                                                                 |
| Off-page comment                                                                    | Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to. |
| On-page comment                                                                     | Allows comments to be entered in speech bubbles on the candidate response.                                                                 |
|  | Judgment made by the PE                                                                                                                    |
|  | Premature approximation                                                                                                                    |
|  | Special case                                                                                                                               |
|  | Indicates that work/page has been seen                                                                                                     |

| Annotation                                                                        | Meaning                                |
|-----------------------------------------------------------------------------------|----------------------------------------|
|  | Error in number of significant figures |
|  | Correct                                |
|  | Transcription error                    |
|  | Correct answer from incorrect working  |

**PUBLISHED****Mark Scheme Notes**

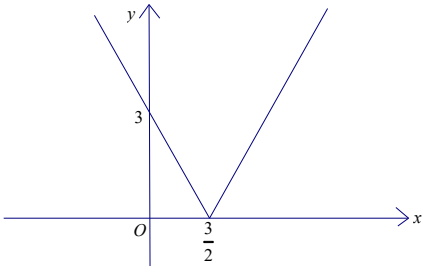
The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

**Types of mark**

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
  - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
  - The total number of marks available for each question is shown at the bottom of the Marks column.
  - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
  - Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

**Abbreviations**

|        |                                                                                                                                                                                       |
|--------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| AEF/OE | Any Equivalent Form (of answer is equally acceptable) / Or Equivalent                                                                                                                 |
| AG     | Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)                                                   |
| CAO    | Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)                                                                                           |
| CWO    | Correct Working Only                                                                                                                                                                  |
| ISW    | Ignore Subsequent Working                                                                                                                                                             |
| SOI    | Seen Or Implied                                                                                                                                                                       |
| SC     | Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance) |
| WWW    | Without Wrong Working                                                                                                                                                                 |
| AWRT   | Answer Which Rounds To                                                                                                                                                                |

| Question | Answer                                                                            | Marks     | Guidance                                                                                                                                                                                                                    |
|----------|-----------------------------------------------------------------------------------|-----------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 1(a)     |  | <b>B1</b> | Symmetrical. In correct position.<br>Condone if no complete scale shown, but must see 3 and $\frac{3}{2}$ marked.<br>Needs to exist for negative $x$ .<br>Must be intending straight lines.<br>Ignore $y = 3x - 1$ if seen. |
|          |                                                                                   | <b>1</b>  |                                                                                                                                                                                                                             |
| 1(b)     | Obtain critical value $\frac{4}{5}$ from $3x - 1 = 3 - 2x$                        | <b>B1</b> |                                                                                                                                                                                                                             |
|          | State final answer $x < \frac{4}{5}$                                              | <b>B1</b> |                                                                                                                                                                                                                             |
|          | <b>Alternative Method for Question 1(b)</b>                                       |           |                                                                                                                                                                                                                             |
|          | Obtain critical value $\frac{4}{5}$ from $(3x - 1)^2 = (3 - 2x)^2$                | <b>B1</b> | Ignore $x = -2$ if seen.                                                                                                                                                                                                    |
|          | State final answer $x < \frac{4}{5}$                                              | <b>B1</b> |                                                                                                                                                                                                                             |
|          |                                                                                   | <b>2</b>  |                                                                                                                                                                                                                             |

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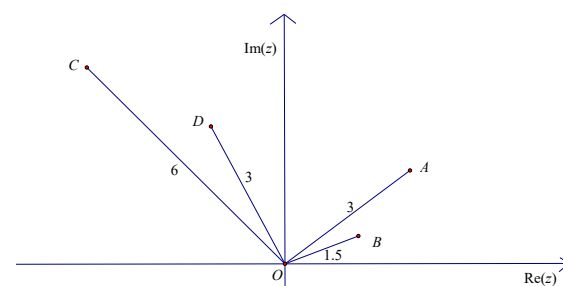
| Question | Answer                                                 | Marks     | Guidance                                                                                 |
|----------|--------------------------------------------------------|-----------|------------------------------------------------------------------------------------------|
| 2        | Use logarithm of a root or a power                     | <b>M1</b> | E.g.<br>$2 \ln p = \ln p^2$ , $\frac{1}{2} \ln(q+1) = \ln \sqrt{q+1}$ or $3 = \ln e^3$ . |
|          | Obtain $p^2(p-1) = e^3 \sqrt{q+1}$                     | <b>A1</b> | OE without logs.                                                                         |
|          | Obtain $q = \left( \frac{p^2(p-1)}{e^3} \right)^2 - 1$ | <b>A1</b> | OE, ISW                                                                                  |
|          |                                                        | <b>3</b>  |                                                                                          |

| Question | Answer                                                          | Marks      | Guidance                                                                            |
|----------|-----------------------------------------------------------------|------------|-------------------------------------------------------------------------------------|
| 3        | $\frac{x+i(y+5)}{(x-5)+iy} \times \frac{(x-5)-iy}{(x-5)-iy}$    | <b>*M1</b> | Multiply numerator (and denominator) by the conjugate of the denominator.           |
|          | Equate imaginary part of numerator to zero                      | <b>DM1</b> | Numerator<br>$= x(x-5) + y(y+5) + i((y+5)(x-5) - xy)$ .                             |
|          | Obtain $x - y = 5$                                              | <b>A1</b>  | OE                                                                                  |
|          | Correct use of modulus and solve for $x$ and $y$                | <b>M1</b>  | $x^2 + (x-5)^2 = 17 \Rightarrow x^2 - 5x + 4 = 0$<br>Or $x - y = 5$ and $xy = -4$ . |
|          | $\Rightarrow z = 4 - i$                                         | <b>A1</b>  |                                                                                     |
|          | $\Rightarrow z = 1 - 4i$                                        | <b>A1</b>  |                                                                                     |
|          | <b>Alternative Method for the first two marks of Question 3</b> |            |                                                                                     |
|          | Quotient real $\Rightarrow \frac{x}{x+5} = \frac{y+5}{y}$       | <b>M1</b>  |                                                                                     |
|          | Rearrange to linear form and simplify                           | <b>M1</b>  |                                                                                     |
|          |                                                                 | <b>6</b>   |                                                                                     |

| Question | Answer                                                                                                                         | Marks      | Guidance                                                                  |
|----------|--------------------------------------------------------------------------------------------------------------------------------|------------|---------------------------------------------------------------------------|
| 4        | At $(e, 3)$ , $\tan t = 1$ or $t = \frac{\pi}{4}$                                                                              | <b>B1</b>  | SOI                                                                       |
|          | $\frac{dx}{dt} = \sec^2 t \times e^{\tan t}$                                                                                   | <b>B1</b>  |                                                                           |
|          | $\frac{dy}{dt} = 6 \tan t \times \sec^2 t$                                                                                     | <b>B1</b>  |                                                                           |
|          | $\frac{dy}{dx} = \frac{6 \tan t \times \sec^2 t}{\sec^2 t \times e^{\tan t}} (= 6 \tan t \times e^{-\tan t})$                  | <b>*M1</b> | Correct use of <i>their</i> derivatives.                                  |
|          | $y - 3 = \frac{6}{e}(x - e)$                                                                                                   | <b>DM1</b> | Substitute for $t$ , and use correct method for the equation of the line. |
|          | $y = \frac{6}{e}x - 3$                                                                                                         | <b>A1</b>  | Or exact equivalent.                                                      |
|          | <b>Alternative Method for Question 4</b>                                                                                       |            |                                                                           |
|          | Correct cartesian form, e.g. $x = e^{\sqrt{\frac{y}{3}}}$ or $\ln x = \sqrt{\frac{y}{3}}$ or $y = 3(\ln x)^2$                  | <b>B1</b>  | SOI                                                                       |
|          | Differentiate function of a function                                                                                           | <b>*M1</b> | Complete method.                                                          |
|          | Obtain $\frac{dx}{dy} = k \sqrt{\frac{e}{y}} e^{\sqrt{\frac{y}{3}}}$ or $\frac{dy}{dx} = k \times \frac{1}{x} \ln x$           | <b>A1</b>  |                                                                           |
|          | Obtain $\frac{dx}{dy} = \frac{1}{6} \sqrt{\frac{3}{y}} e^{\sqrt{\frac{y}{3}}}$ or $\frac{dy}{dx} = 6 \times \frac{1}{x} \ln x$ | <b>A1</b>  |                                                                           |
|          | $y - 3 = \frac{6}{e}(x - e)$                                                                                                   | <b>DM1</b> | Use correct method for the equation of the line.                          |

| Question | Answer                 | Marks | Guidance             |
|----------|------------------------|-------|----------------------|
| 4        | $y = \frac{6}{e}x - 3$ | A1    | Or exact equivalent. |
|          |                        | 6     |                      |

| Question | Answer                                                                   | Marks | Guidance                                                           |
|----------|--------------------------------------------------------------------------|-------|--------------------------------------------------------------------|
| 5        | Use $f(-2a) = -22a^3$                                                    | M1    | Or use long division and equate a constant remainder to $-22a^3$ . |
|          | Obtain $-24a^3 + 4pa^3 - 14a^3 + qa^3 = -22a^3$                          | A1    | Must evaluate the terms.<br>OE, e.g. $4p + q = 16$ .               |
|          | Use $f\left(\frac{a}{3}\right) = -a^3$                                   | M1    | Or use long division and equate a constant remainder to $-a^3$ .   |
|          | Obtain $\frac{1}{9}a^3 + \frac{1}{9}pa^3 + \frac{7}{3}a^3 + qa^3 = -a^3$ | A1    | Must evaluate the terms.<br>OE, e.g. $p + 9q = -31$                |
|          | Obtain $p = 5, q = -4$                                                   | A1    |                                                                    |
|          |                                                                          | 5     |                                                                    |

| Question | Answer                                                                                              | Marks       | Guidance                                                                                                                           |
|----------|-----------------------------------------------------------------------------------------------------|-------------|------------------------------------------------------------------------------------------------------------------------------------|
| 6(a)     | Obtain $\omega z_1 = 6e^{i\frac{3}{4}\pi}$                                                          | <b>B1</b>   |                                                                                                                                    |
|          | Obtain $\omega z_2 = 3e^{i\frac{2}{3}\pi}$                                                          | <b>B1</b>   |                                                                                                                                    |
|          |                                                                                                     |             | <b>SC B1</b> for both moduli correct or both arguments correct.                                                                    |
|          |                                                                                                     | <b>2</b>    |                                                                                                                                    |
| 6(b)     | $A$ and $B$ plotted correctly                                                                       | <b>B1</b>   |  <p>Follow <i>their</i> answers to part (a)</p> |
|          | $C$ and $D$ plotted with angles relatively correct and the same stretch implied.                    | <b>B1FT</b> | $C$ and $D$ correct, or FT <i>their</i> $A$ and $B$ .                                                                              |
|          |                                                                                                     | <b>2</b>    |                                                                                                                                    |
| 6(c)     | Rotation $\frac{\pi}{2}$ radians (anticlockwise),<br>or rotation $\frac{3\pi}{2}$ radians clockwise | <b>B1</b>   | Accept $90^\circ$ .<br>Not required to state the centre of the rotation, but B0 if an incorrect statement seen.                    |
|          | Enlargement (scale) factor 2                                                                        | <b>B1</b>   | Not required to state the centre of the enlargement, but B0 if an incorrect statement seen.<br>Allow ‘expansion’ or ‘stretch’.     |
|          |                                                                                                     | <b>2</b>    |                                                                                                                                    |

| Question | Answer                                                                                                           | Marks       | Guidance                                                                                                                                                                                                                                                                |
|----------|------------------------------------------------------------------------------------------------------------------|-------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 7(a)     | Expand $5\sin\left(x + \frac{\pi}{6}\right) - 4\cos x$ to obtain $\frac{5}{2}\sqrt{3}\sin x - \frac{3}{2}\cos x$ | <b>B1</b>   | Or exact 2 term equivalent.                                                                                                                                                                                                                                             |
|          | State $R = \sqrt{21}$                                                                                            | <b>B1ft</b> | Follow <i>their</i> $\frac{5}{2}\sqrt{3}$ and $\frac{3}{2}$ .                                                                                                                                                                                                           |
|          | Use correct trig formulae to obtain $\tan \alpha$                                                                | <b>M1</b>   | OE, WWW.<br>E.g. $\tan \alpha = \frac{3}{5\sqrt{3}} \left( = \frac{\sqrt{3}}{5} \right)$ ,<br>$\sin \alpha = \frac{3}{2\sqrt{21}} \left( = \frac{\sqrt{3}}{2\sqrt{7}} \right)$ ,<br>$\cos \alpha = \frac{5\sqrt{3}}{2\sqrt{21}} \left( = \frac{5}{2\sqrt{7}} \right)$ . |
|          | Obtain $\alpha = 0.333$                                                                                          | <b>A1</b>   |                                                                                                                                                                                                                                                                         |
|          |                                                                                                                  | <b>4</b>    |                                                                                                                                                                                                                                                                         |
| 7(b)     | $\sin^{-1}\left(\frac{\sqrt{7}}{\sqrt{21}}\right)$                                                               | <b>B1FT</b> | SOI<br>0.615... can be implied by one correct answer.<br>Follow <i>their</i> $\sqrt{21}$ .                                                                                                                                                                              |
|          | Use a correct method to obtain a value of $\theta$ in the interval                                               | <b>M1</b>   |                                                                                                                                                                                                                                                                         |
|          | Obtain one correct answer, e.g. 0.47                                                                             | <b>A1</b>   |                                                                                                                                                                                                                                                                         |
|          | Obtain second correct answer, e.g. 1.43, and no others in the interval                                           | <b>A1</b>   |                                                                                                                                                                                                                                                                         |
|          |                                                                                                                  | <b>4</b>    |                                                                                                                                                                                                                                                                         |

| Question | Answer                                                                                                                                                            | Marks       | Guidance                                                                                                                                                                                                                         |
|----------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 8(a)     | Use a correct method to form an equation for $l_1$                                                                                                                | <b>M1</b>   | Accept column vectors.                                                                                                                                                                                                           |
|          | Obtain $\mathbf{r} = (2\mathbf{i} + 4\mathbf{k}) + \lambda(3\mathbf{i} + \mathbf{j} + 2\mathbf{k})$                                                               | <b>A1</b>   | OE, e.g. $\mathbf{r} = (5\mathbf{i} + \mathbf{j} + 6\mathbf{k}) + \lambda(3\mathbf{i} + \mathbf{j} + 2\mathbf{k})$ .<br>Must have $\mathbf{r} = \dots$ , or in $x, y, z$ or $\mathbf{R} = \dots$                                 |
|          |                                                                                                                                                                   | <b>2</b>    |                                                                                                                                                                                                                                  |
| 8(b)     | Express general point of a line in component form                                                                                                                 | <b>B1ft</b> | E.g. $\begin{pmatrix} 2 + \mu \\ 1 + 2\mu \\ 5 + 3\mu \end{pmatrix}$ or $\begin{pmatrix} 2 + 3\lambda \\ \lambda \\ 4 + 2\lambda \end{pmatrix}$ or $\begin{pmatrix} 5 + 3\lambda \\ 1 + \lambda \\ 6 + 2\lambda \end{pmatrix}$ . |
|          | Equate two pairs of components of $l_2$ and <i>their</i> $l_1$ , and solve for $\lambda$ or $\mu$                                                                 | <b>M1</b>   | $\begin{pmatrix} 2 + \mu \\ 1 + 2\mu \\ 5 + 3\mu \end{pmatrix} = \begin{pmatrix} 2 + 3\lambda \\ \lambda \\ 4 + 2\lambda \end{pmatrix}$                                                                                          |
|          | Obtain e.g. $\lambda = -\frac{1}{5}, \mu = -\frac{3}{5}$                                                                                                          | <b>A1</b>   | Or $\lambda = -\frac{1}{7}, \mu = -\frac{3}{7}$ or $\lambda = -1, \mu = -1$ .                                                                                                                                                    |
|          | Show that this does not fit the third component and hence the lines do not intersect.                                                                             | <b>A1</b>   | $\frac{16}{5} \neq \frac{18}{5}$ or $\frac{1}{7} \neq -\frac{1}{7}$ or $1 \neq -1$ .                                                                                                                                             |
|          |                                                                                                                                                                   | <b>4</b>    |                                                                                                                                                                                                                                  |
| 8(c)     | Carry out the correct process for evaluating the scalar product of the direction vector of $l_1$ and $l_2$                                                        | <b>M1</b>   | E.g. $(\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}) \cdot (3\mathbf{i} + \mathbf{j} + 2\mathbf{k}) = 3 + 2 + 6$ .                                                                                                                     |
|          | Using the correct process for the moduli, divide their scalar product by the product of the moduli of their vectors and evaluate the inverse cosine of the result | <b>M1</b>   | $\theta = \cos^{-1}\left(\frac{3 + 2 + 6}{\sqrt{14} \times \sqrt{14}}\right)$                                                                                                                                                    |
|          | Obtain AWRT $38.2^\circ$ or 0.667 radians                                                                                                                         | <b>A1</b>   |                                                                                                                                                                                                                                  |
|          |                                                                                                                                                                   | <b>3</b>    |                                                                                                                                                                                                                                  |

| Question | Answer                                                                                                         | Marks      | Guidance                                                                                                                                                                                       |
|----------|----------------------------------------------------------------------------------------------------------------|------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 9(a)     | Commence integration and reach $px^2 \ln x + q \int x \, dx$                                                   | <b>M1*</b> | Allow M1 with $q \int \frac{x^2}{x} \, dx$ .                                                                                                                                                   |
|          | Obtain $3x^2 \ln x - 3 \int x \, dx$                                                                           | <b>A1</b>  | $\frac{x^2}{x}$ must be simplified.                                                                                                                                                            |
|          | Complete integration and obtain $3x^2 \ln x - \frac{3}{2}x^2$                                                  | <b>A1</b>  |                                                                                                                                                                                                |
|          | Use limits correctly in an expression of the form $ax^2 \ln x - bx^2$ and equate to 4, having integrated twice | <b>DM1</b> | $3a^2 \ln a - \frac{3}{2}a^2 - 0 + \frac{3}{2} = 4$                                                                                                                                            |
|          | Obtain $a = \exp\left(\frac{1}{6}\left(\frac{5}{a^2} + 3\right)\right)$ correctly                              | <b>A1</b>  | AG                                                                                                                                                                                             |
|          |                                                                                                                | <b>5</b>   |                                                                                                                                                                                                |
| 9(b)     | Calculate the values of a relevant expression or pair of expressions at $a = 2$ and $a = 2.1$                  | <b>M1</b>  | Not from using calculator to evaluate the original integral.                                                                                                                                   |
|          | Justify the given statement with correct calculated values                                                     | <b>A1</b>  | E.g., using $f(x) = 6x^2 \ln x - 3x^2$<br>$f(2) = 4.636 < 5$ and $f(2.1) = 6.402 > 5$ ,<br>or using the exponential,<br>$2 < 2.0306$ and $2.1 > 1.9917$<br>or $0.0306 > 0$ and $-0.1083 < 0$ . |
|          |                                                                                                                | <b>2</b>   |                                                                                                                                                                                                |

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| Question | Answer                                                                                                                           | Marks     | Guidance                                                                                                                                                                                                                                                       |
|----------|----------------------------------------------------------------------------------------------------------------------------------|-----------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 9(c)     | Use the iterative process $a_{n+1} = \exp\left(\frac{1}{6}\left(\frac{5}{a_n^2} + 3\right)\right)$ correctly at least once.      | <b>M1</b> |                                                                                                                                                                                                                                                                |
|          | Obtain final answer 2.02                                                                                                         | <b>A1</b> |                                                                                                                                                                                                                                                                |
|          | Show sufficient iterations to at least 4 d.p. to justify 2.02 to 2 d.p.<br>or show that there is a sign change in (2.015, 2.025) | <b>A1</b> | E.g. $2 \rightarrow 2.0306 \rightarrow 2.0180 \rightarrow 2.0231 \dots$<br>$2.05 \rightarrow 2.0103 \rightarrow 2.0263 \rightarrow 2.0197 \rightarrow 2.0224 \dots$<br>$2.1 \rightarrow 1.9917 \rightarrow 2.0342 \rightarrow 2.0166 \rightarrow 2.0237 \dots$ |
|          |                                                                                                                                  | <b>3</b>  |                                                                                                                                                                                                                                                                |



| Question | Answer                                                                                                   | Marks       | Guidance                                                                 |
|----------|----------------------------------------------------------------------------------------------------------|-------------|--------------------------------------------------------------------------|
| 10(b)    | Separate variables correctly and obtain $\int 6y \, dy = 3y^2$                                           | <b>B1</b>   | OE from $\int 6y \, dy = \int \frac{x^3 + 5x^2 - 2x - 15}{x^2 - 3} dx$ . |
|          | Obtain $\frac{1}{2}x^2 + 5x$                                                                             | <b>B1ft</b> | Follow <i>their</i> linear quotient.                                     |
|          | Obtain $\frac{1}{2} \ln(x^2 - 3)$                                                                        | <b>B1ft</b> | From the $x$ term in <i>their</i> remainder $ax + b$ .                   |
|          | Use $y = 2$ , $x = 2$ to evaluate the constant of integration in an integral containing $k \ln(x^2 - 3)$ | <b>M1</b>   | $12 = 2 + 10 + \frac{1}{2} \ln 1 + C \Rightarrow C = 0$                  |
|          | Obtain $y^2 = \frac{1}{6}x^2 + \frac{5}{3}x + \frac{1}{6} \ln(x^2 - 3)$                                  | <b>A1</b>   | OE                                                                       |
|          |                                                                                                          | <b>5</b>    |                                                                          |

| Question | Answer                                                                                                                                    | Marks      | Guidance                                                                                                                                                                                                                                                                   |
|----------|-------------------------------------------------------------------------------------------------------------------------------------------|------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 11(a)    | $\frac{d}{dx} \sqrt{\sin 2x} = \frac{2 \cos 2x}{2\sqrt{\sin 2x}}$ Accept $k \frac{\cos 2x}{\sqrt{\sin 2x}}$                               | <b>B1</b>  | SOI                                                                                                                                                                                                                                                                        |
|          | Use correct product rule                                                                                                                  | <b>M1*</b> |                                                                                                                                                                                                                                                                            |
|          | Obtain correct derivative in any form                                                                                                     | <b>A1</b>  | E.g. $\frac{dy}{dx} = -\sin x \sqrt{\sin 2x} + \cos x \times \frac{2 \cos 2x}{2\sqrt{\sin 2x}}$ .                                                                                                                                                                          |
|          | Equate the derivative to zero and obtain a horizontal equation                                                                            | <b>DM1</b> | E.g. $-\sin a \sin 2a + \cos a \cos 2a = 0$                                                                                                                                                                                                                                |
|          | Use correct trig formulae to obtain an equation in one trig function                                                                      | <b>DM1</b> | E.g. $\cos 3a = 0$ or $\sin^2 a = \frac{1}{4}$ .                                                                                                                                                                                                                           |
|          | Obtain $a = \frac{1}{6}\pi$                                                                                                               | <b>A1</b>  | Exact answer in radians only.                                                                                                                                                                                                                                              |
|          |                                                                                                                                           | <b>6</b>   |                                                                                                                                                                                                                                                                            |
| 11(b)    | Use $[\pi] \int y^2 dx$                                                                                                                   | <b>M1*</b> |                                                                                                                                                                                                                                                                            |
|          | Use double angle formula to obtain a form that can be integrated directly,<br>e.g. $\int \cos^2 x \sin 2x dx = \int 2 \cos^3 x \sin x dx$ | <b>DM1</b> | Or $\int \frac{1}{4} \sin 4x + \frac{1}{2} \sin 2x dx$ ,<br>or $\int 2(u - u^3) du$ by substituting $u = \sin x$ .                                                                                                                                                         |
|          | Obtain $-(\pi) \times \frac{1}{2} \cos^4 x$                                                                                               | <b>A1</b>  | Or $-[\pi] \left( \frac{1}{16} \cos 4x + \frac{1}{4} \cos 2x \right)$ , OE.                                                                                                                                                                                                |
|          | Use correct limits correctly                                                                                                              | <b>DM1</b> | $\left[ -[\pi] \times \frac{1}{2} \cos^4 x \right]_0^{\frac{\pi}{2}} = -[\pi] \left( \frac{1}{2} \cos \left( \frac{1}{2} \pi \right) - \frac{1}{2} \right)$ ,<br>or $-[\pi] \left( \frac{1}{16} \cos 2\pi + \frac{1}{4} \cos \pi - \frac{1}{16} - \frac{1}{4} \right)$ OE. |
|          | Obtain final answer $\frac{1}{2}\pi$                                                                                                      | <b>A1</b>  | If the factor of $\pi$ is missing throughout, allow the first 4 marks and A0 here.                                                                                                                                                                                         |
|          |                                                                                                                                           | <b>5</b>   |                                                                                                                                                                                                                                                                            |

# Cambridge International A Level

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**MATHEMATICS****9709/32**

Paper 3 Pure Mathematics 3

**May/June 2025****MARK SCHEME**Maximum Mark: 75

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**Published**

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

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This document consists of **25** printed pages.

**PUBLISHED****Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

**GENERIC MARKING PRINCIPLE 1:**

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

**GENERIC MARKING PRINCIPLE 2:**

Marks awarded are always **whole marks** (not half marks, or other fractions).

**GENERIC MARKING PRINCIPLE 3:**

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

**GENERIC MARKING PRINCIPLE 4:**

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

**GENERIC MARKING PRINCIPLE 5:**

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

**GENERIC MARKING PRINCIPLE 6:**

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

**Mathematics-Specific Marking Principles**

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

**Annotations guidance for centres**

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

**Annotations**

| <b>Annotation</b>                                                                   | <b>Meaning</b>                         |
|-------------------------------------------------------------------------------------|----------------------------------------|
|    | More information required              |
|    | Accuracy mark awarded zero             |
|    | Accuracy mark awarded one              |
|    | Independent accuracy mark awarded zero |
|  | Independent accuracy mark awarded one  |
|  | Independent accuracy mark awarded two  |
|  | Benefit of the doubt                   |
|  | Blank Page                             |
|  | Incorrect                              |
| Dep                                                                                 | Used to indicate DM0 or DM1            |

| <b>Annotation</b>                                                                   | <b>Meaning</b>                                                                                                                             |
|-------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------|
| DM1                                                                                 | Dependent on the previous M1 mark(s)                                                                                                       |
|    | Follow through                                                                                                                             |
|    | Indicate working that is right or wrong                                                                                                    |
| Highlighter                                                                         | Highlight a key point in the working                                                                                                       |
|    | Ignore subsequent work                                                                                                                     |
|    | Judgement                                                                                                                                  |
|    | Judgement                                                                                                                                  |
|    | Method mark awarded zero                                                                                                                   |
|    | Method mark awarded one                                                                                                                    |
|    | Method mark awarded two                                                                                                                    |
|    | Misread                                                                                                                                    |
|    | Omission or Other solution                                                                                                                 |
| Off-page comment                                                                    | Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to. |
| On-page comment                                                                     | Allows comments to be entered in speech bubbles on the candidate response.                                                                 |
|  | Judgment made by the PE                                                                                                                    |
|  | Premature approximation                                                                                                                    |
|  | Special case                                                                                                                               |
|  | Indicates that work/page has been seen                                                                                                     |

| Annotation                                                                        | Meaning                                |
|-----------------------------------------------------------------------------------|----------------------------------------|
|  | Error in number of significant figures |
|  | Correct                                |
|  | Transcription error                    |
|  | Correct answer from incorrect working  |

**PUBLISHED****Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

**Types of mark**

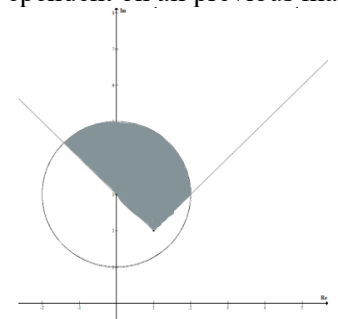
- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
  - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
  - The total number of marks available for each question is shown at the bottom of the Marks column.
  - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
  - Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

**Abbreviations**

|        |                                                                                                                                                                                       |
|--------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| AEF/OE | Any Equivalent Form (of answer is equally acceptable) / Or Equivalent                                                                                                                 |
| AG     | Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)                                                   |
| CAO    | Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)                                                                                           |
| CWO    | Correct Working Only                                                                                                                                                                  |
| ISW    | Ignore Subsequent Working                                                                                                                                                             |
| SOI    | Seen Or Implied                                                                                                                                                                       |
| SC     | Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance) |
| WWW    | Without Wrong Working                                                                                                                                                                 |
| AWRT   | Answer Which Rounds To                                                                                                                                                                |

| Question | Answer                                                      | Marks      | Guidance                                                                                        |
|----------|-------------------------------------------------------------|------------|-------------------------------------------------------------------------------------------------|
| 1        | Obtain a 3-term quadratic in $e^x$                          | <b>*M1</b> |                                                                                                 |
|          | Obtain e.g. $3e^{2x} - 12e^x - 2 = 0$ or 3-term equivalent  | <b>A1</b>  | E.g. $3m^2 - 12m - 2 = 0$ .<br>'= 0' could be implied by subsequent working.                    |
|          | Solve a 3-term quadratic to obtain a value for $x$ or $e^x$ | <b>DM1</b> | Need to get as far as a value for $x$ or $e^x$ .                                                |
|          | Obtain root $\frac{6 + \sqrt{42}}{3}$ or 4.16...            | <b>A1</b>  | OE<br>Ignore second root if seen.                                                               |
|          | Obtain answer 1.426 only                                    | <b>A1</b>  | CAO, must be 3 d.p.<br>0/5 for answer with no working.<br>Second root must be rejected if seen. |
|          |                                                             | <b>5</b>   |                                                                                                 |

| Question | Answer                                                                                                                                              | Marks     | Guidance                                                           |
|----------|-----------------------------------------------------------------------------------------------------------------------------------------------------|-----------|--------------------------------------------------------------------|
| 2(a)     | Find the first two terms of the expansion of $(1-2x)^{-\frac{3}{2}}$                                                                                | <b>B1</b> |                                                                    |
|          | Obtain correct third term $\frac{-\frac{3}{2}\left(-\frac{3}{2}-1\right)}{2!}(-2x)^2$ or $\frac{-\frac{3}{2}\left(-\frac{3}{2}-1\right)}{2!}(2x)^2$ | <b>B1</b> | $\frac{15}{2}x^2$<br>Ignore extra terms.                           |
|          | Multiply <i>their</i> 3 term expansion $a+bx+cx^2$ by $(6-x)$ obtaining all necessary terms                                                         | <b>M1</b> | $6+18x+45x^2-x-3x^2$ ....<br>Ignore extra terms.                   |
|          | $6+17x+42x^2$                                                                                                                                       | <b>A1</b> | Ignore extra terms.<br>Allow with the terms in any order.          |
|          |                                                                                                                                                     | <b>4</b>  |                                                                    |
| 2(b)     | $ x  < \frac{1}{2}$ or $-\frac{1}{2} < x < \frac{1}{2}$ or $(-0.5, 0.5)$ or $] -0.5, 0.5[$                                                          | <b>B1</b> | OE<br>B0 for an ambiguous statement.<br>Must be strict inequality. |
|          |                                                                                                                                                     | <b>1</b>  |                                                                    |

| Question | Answer                                                                                                                 | Marks       | Guidance                                                                                                                                                                                                                                             |
|----------|------------------------------------------------------------------------------------------------------------------------|-------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 3        | Show a circle centre (0, 3)                                                                                            | <b>B1</b>   | Allow for a circle in the correct place even if their scales require an ellipse.<br>'3' marked on the axis, centre implied by vertical intercepts or dashes to indicate a scale.                                                                     |
|          | Show a circle with radius 2                                                                                            | <b>B1FT</b> | FT centre not at the origin.<br>Consistent with <i>their</i> scale if they have unequal scales on the axes.                                                                                                                                          |
|          | Show the point representing (1, 2)                                                                                     | <b>B1</b>   | Position can be implied provided there is some indication of scale, e.g. numbers or marks on the axes.                                                                                                                                               |
|          | Show correct half-lines from (1, 2), one at an angle of $\frac{1}{4}\pi$ and the other at an angle of $\frac{3}{4}\pi$ | <b>B1FT</b> | FT from <i>their</i> (1, 2) not at the origin.<br>Correct lines can imply previous B1.<br>Half-lines must be intended to be 'symmetrical'.<br>Angles consistent with <i>their</i> scale: correct lines need to pass very close to (0, 3) and (2, 3). |
|          | Shade the correct region                                                                                               | <b>B1</b>   | Dependent on all previous marks.<br>                                                                                                                             |
|          |                                                                                                                        | <b>5</b>    |                                                                                                                                                                                                                                                      |

| Question | Answer                                                                                                                                                                                                                                                                                                                                                                                                                                           | Marks      | Guidance                                                                                                                                                                                                                     |
|----------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 4        | Use correct trigonometric formulae to form an equation in $\tan x$ only, or an equation in terms of $\sin x$ and $\cos x$ only<br><br>e.g. $\frac{3}{\tan x} - 4\left(\frac{1 - \tan^2 x}{2 \tan x}\right) = 3$<br><br>$\frac{3}{\tan x} - \frac{4}{\frac{2 \tan x}{1 - \tan^2 x}} = 3$<br><br>$3 \frac{\cos x}{\sin x} - 4 \frac{\cos^2 x - \sin^2 x}{2 \sin x \cos x} = 3$<br><br>$3 \cot x - 4\left(\frac{\cot^2 x - 1}{2 \cot x}\right) = 3$ | <b>*M1</b> | Condone one slip in manipulating the original equation provided correct trig formulae used.                                                                                                                                  |
|          | Obtain a correct horizontal equation, in $\tan x$ or in $\cos x$ and $\sin x$ , in any form                                                                                                                                                                                                                                                                                                                                                      | <b>A1</b>  | E.g. $3 - (2 - 2 \tan^2 x) = 3 \tan x$<br>or $2 \sin^2 x - 3 \sin x \cos x + \cos^2 x = 0$ .                                                                                                                                 |
|          | Reduce equation to a 3-term quadratic<br><br>Allow if they square both sides and obtain a quartic in $\sin x$ or $\cos x$                                                                                                                                                                                                                                                                                                                        | <b>A1</b>  | E.g. $2 \tan^2 x - 3 \tan x + 1 = 0$ ,<br>$2 \sin^2 x - 3 \sin x \cos x + \cos^2 x = 0$ .<br>Allow if they leave the equation as a cubic with a factor of $\tan x$ and go on to give the correct solutions to the quadratic. |
|          | Solve a 3-term quadratic to obtain a value for $x$<br>The quadratic could be arrived at from a correct cubic<br>Not available if they have an incorrect cubic with an incorrect quadratic factor.                                                                                                                                                                                                                                                | <b>DM1</b> | If the initial equation is correct, then M1 is implied by a correct solution.                                                                                                                                                |
|          | Obtain one answer, e.g. $(x =) 45^\circ$                                                                                                                                                                                                                                                                                                                                                                                                         | <b>A1</b>  |                                                                                                                                                                                                                              |
|          | Obtain a second answer, e.g. AWRT $(x =) 26.6^\circ$ and no other in the given interval, e.g. $x = 0$                                                                                                                                                                                                                                                                                                                                            | <b>A1</b>  | Ignore answers outside the given interval.                                                                                                                                                                                   |
|          |                                                                                                                                                                                                                                                                                                                                                                                                                                                  | <b>6</b>   |                                                                                                                                                                                                                              |

| Question | Answer                                                                                               | Marks | Guidance                                                                                                                                                                                                  |
|----------|------------------------------------------------------------------------------------------------------|-------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 5        | Square $x + iy$ and equate real and imaginary parts to $-1$ and $-4\sqrt{5}$ respectively            | *M1   |                                                                                                                                                                                                           |
|          | Obtain equations $x^2 - y^2 = -1$ and $2xy = -4\sqrt{5}$ from <i>their</i> expansion                 | A1    |                                                                                                                                                                                                           |
|          | Working from two equations in two unknowns, eliminate one variable and find an equation in the other | DM1   | Do not condone incorrect algebra, e.g. $x = -2\sqrt{5}y$ .                                                                                                                                                |
|          | Obtain $x^4 + x^2 - 20 = 0$ or $y^4 - y^2 - 20 = 0$                                                  | A1    | Accept 3-term equivalents.<br>Condone missing “= 0” if implied by subsequent working.                                                                                                                     |
|          | Obtain answers $\pm(2 - \sqrt{5}i)$ and no others                                                    | A1    | Must state the square roots, not $x$ and $y$ separately, and not coordinates.<br>Do not allow $\sqrt{4}$ in place of 2.<br>A0 if there are additional incorrect solutions.<br>No working seen scores 0/5. |
|          |                                                                                                      | 5     |                                                                                                                                                                                                           |

| Question | Answer                                                                                                                                                                                                                                                                                                                                        | Marks     | Guidance                                                                                                                                                                                                       |
|----------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 6(a)     | <p>Sketch a relevant graph for <math>0 &lt; x &lt; \pi</math></p> <p>For <math>y =  x - 2 </math> graph should be symmetrical and have correct intercepts on the axes</p> <p>For <math>y = 2 \sin \frac{1}{2}x</math>, graph should pass through the origin, have correct curvature and <math>\max y = 2</math> when <math>x = \pi</math></p> | <b>B1</b> | <p>Ignore anything outside <math>0 &lt; x &lt; \pi</math>.<br/>Ignore what happens in <math>y &lt; 0</math>.</p>                                                                                               |
|          | <p>Sketch second relevant graph and confirm root.</p> <p>The vertex of the modulus graph in roughly correct position relative to <math>\pi</math> and/or <math>\frac{1}{2}\pi</math></p> <p>If no mark on the graph, check to see if they have written something below the graph.</p>                                                         | <b>B1</b> | <p>Needs to mark intersection with a dot, a cross, or say roots at points of intersection, OE.<br/>If the intersection is highlighted in some way, then they do not need to make a comment.</p>                |
|          |                                                                                                                                                                                                                                                                                                                                               |           | <p><b>SC</b> A sketch <math>y = x - 2</math> <b>and</b> <math>y = \pm 2 \sin \frac{1}{2}x</math> (above and below the <math>x</math>-axis) scores B1.<br/>A clear indication of the root scores second B1.</p> |
|          |                                                                                                                                                                                                                                                                                                                                               | <b>2</b>  |                                                                                                                                                                                                                |

| Question | Answer                                                                                                                                                                                                                                                                                                                                                                                                | Marks     | Guidance                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |
|----------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 6(b)     | <p>Calculate the values of a relevant expression or pair of expressions at <math>x = 1</math> and <math>x = 1.5</math></p> <p>e.g. <math>f(x) =  x - 2  - 2\sin \frac{1}{2}x</math><br/> <math>f(1) = 0.0411... &gt; 0</math> <math>f(1.5) = -0.863... &lt; 0</math></p> <p>e.g. <math>f(x) = (x - 2)^2 - 4\sin^2\left(\frac{1}{2}x\right) \Rightarrow f(1) = 0.0806..., f(1.5) = -1.60...</math></p> | <b>M1</b> | <p>Or comparing <math> x - 2 </math> and <math>2\sin \frac{1}{2}x</math>.</p> <p>Using 1: <math> x - 2  = 1</math>, <math>2\sin \frac{1}{2}x = 0.958...</math><br/> <math>\Rightarrow 1 &gt; 0.958...</math></p> <p>Using 1.5: <math> x - 2  = 0.5</math> <math>2\sin \frac{1}{2}x = 1.36...</math><br/> <math>\Rightarrow 0.5 &lt; 1.36...</math></p> <p>Need all values but condone one error.</p> <p>If the solution involves 4 values, the pairing must be clear. Embedded values are not sufficient, e.g. <math>f_1(1) = ...</math> and <math>f_2(1) = ...</math> etc.</p> <p>M0 if working in degrees (gives 0.98... and 0.47... if using <math>f(x) = 0</math>).</p> <p>Allow if working on a smaller interval.</p> |
|          | Complete the argument correctly with correct calculated values                                                                                                                                                                                                                                                                                                                                        | <b>A1</b> | <p>Values correct.</p> <p>They must have a conclusion in words or symbols, but they do not need to say that the function is continuous.</p> <p>A correct statement with correct inequalities is sufficient.</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |
|          |                                                                                                                                                                                                                                                                                                                                                                                                       | <b>2</b>  |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |

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| Question | Answer                                                                                                                                                                      | Marks     | Guidance                                                                                                                                               |
|----------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------|--------------------------------------------------------------------------------------------------------------------------------------------------------|
| 6(c)     | Use the iterative process correctly at least once starting at 1.03 (get as far as 1.0281)                                                                                   | <b>M1</b> | M0 if working in degrees.                                                                                                                              |
|          | Obtain final answer 1.02                                                                                                                                                    | <b>A1</b> | No working seen at all scores 0/3.                                                                                                                     |
|          | Show sufficient iterations to 4 d.p. to justify 1.02 to 2 d.p.<br>or show there is a sign change in the interval (1.015, 1.025) (or a smaller interval containing the root) | <b>A1</b> | 1.03, 1.0149, 1.0281, 1.0166, 1.0266, 1.0179,<br>1.0255, 1.0189, 1.0246<br>Incorrect starting point is M0.<br>Once the convergence is established ISW. |
|          |                                                                                                                                                                             | <b>3</b>  |                                                                                                                                                        |

| Question | Answer                                                                                                                                                                            | Marks        | Guidance                                                                                                                                                                                                                                                                   |
|----------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 7(a)     | State $R = 25$                                                                                                                                                                    | <b>B1</b>    | From correct work.<br>BOD if correct value follows use of <i>their</i> $\alpha$ , but not if a decimal approximation to $R$ is seen first.                                                                                                                                 |
|          | Use correct trig formula to find $\alpha$ , e.g. $\alpha = \tan^{-1}\left(\frac{7}{24}\right)$                                                                                    | <b>M1</b>    | If $\cos \alpha = 24$ and $\sin \alpha = 7$ seen, then M0 A0.                                                                                                                                                                                                              |
|          | Obtain $\alpha = 0.2838$                                                                                                                                                          | <b>A1</b>    | CAO                                                                                                                                                                                                                                                                        |
|          |                                                                                                                                                                                   | <b>3</b>     |                                                                                                                                                                                                                                                                            |
| 7(b)     | $\cos^{-1}\left(\frac{24.5}{25}\right)$                                                                                                                                           | <b>B1 FT</b> | Can be implied by $\pm 0.2(0033\dots)$ , or by $\frac{1}{3}x = 0.08\dots$ or $0.48\dots$ or a correct value of $x$ following M1.<br>FT <i>their</i> $R$ from <b>(a)</b> .<br>Allow B1 if $\cos^{-1}\left(\frac{24.5}{R}\right)$ is not evaluated, provided $R \geq 24.5$ . |
|          | Carry out a complete correct method to find a value of $x$<br><br>Sight of a correct equation using <b>(a)</b> is needed, e.g. $\cos\left(\frac{1}{3}x - 0.2838\right) = 0.98$ OE | <b>M1</b>    | Need some method shown, but might not show interim values if working on a calculator. Incorrect answers and insufficient evidence scores M0.                                                                                                                               |
|          | Obtain answer ( $x =$ ) 1.45                                                                                                                                                      | <b>A1</b>    | AWRT<br>Need 3 sf or better.                                                                                                                                                                                                                                               |
|          | Obtain ( $x =$ ) 0.250 or ( $x =$ ) 0.251, and no other in the interval                                                                                                           | <b>A1</b>    | AWRT<br>Ignore answers outside the given interval.                                                                                                                                                                                                                         |
|          |                                                                                                                                                                                   | <b>4</b>     |                                                                                                                                                                                                                                                                            |

| Question | Answer                                                                                                                                                                                        | Marks     | Guidance                                                                                                                                                                                                                                                |
|----------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 8        | Separate variables correctly                                                                                                                                                                  | <b>B1</b> | $\int \frac{1}{4x+3} dx = \int \frac{\cos 2\theta}{\sin 2\theta} d\theta$<br>Can be implied by obtaining both correct integrals.                                                                                                                        |
|          | Obtain term $\frac{1}{4} \ln(4x+3)$                                                                                                                                                           | <b>B1</b> | OE<br>or $\frac{1}{4} \ln\left(x + \frac{3}{4}\right)$ .                                                                                                                                                                                                |
|          | Obtain term of the form $A \ln(\sin 2\theta)$                                                                                                                                                 | <b>M1</b> | Or $A \ln(k \sin 2\theta)$ OE, e.g. $P \ln a \sin \theta + Q \ln b \cos \theta$ from using the $\tan 2\theta$ formula.<br>Or expanding $\cos 2\theta$ as $\cos^2 \theta - \sin^2 \theta$ .                                                              |
|          | Obtain term $\frac{1}{2} \ln(\sin 2\theta)$                                                                                                                                                   | <b>A1</b> | OE<br>Correct in any form, e.g. $\frac{1}{2} \ln a \sin \theta + \frac{1}{2} \ln b \cos \theta$ .                                                                                                                                                       |
|          | Use $x = 0$ when $\theta = \frac{1}{12}\pi$ to evaluate a constant or as limits in a solution containing terms of the form $\ln(\sin 2\theta)$ and $\ln(4x+3)$<br>$c = \dots$ seen or implied | <b>M1</b> | E.g. $c = \frac{1}{4} \ln 3 - \frac{1}{2} \ln \sin\left(\frac{1}{6}\pi\right)$<br>$\frac{1}{4} \ln(4x+3) - \frac{1}{4} \ln 3 = \frac{1}{2} \ln(\sin 2\theta) - \frac{1}{2} \ln \frac{1}{2}$ .<br>Note that the constant may be expressed as a logarithm |
|          | Obtain correct answer in any form with the trigonometry evaluated                                                                                                                             | <b>A1</b> | E.g. $\frac{1}{4} \ln(4x+3) = \frac{1}{2} \ln(\sin 2\theta) + \frac{1}{4} \ln 12$<br>$\frac{1}{4} \ln(4x+3) = \frac{1}{2} \ln(\sin 2\theta) + 0.621$<br>$\frac{1}{4} \ln \frac{4x+3}{3} = \frac{1}{2} \ln(2 \sin 2\theta)$                              |
|          | Obtain final answer $x = \frac{12 \sin^2 2\theta - 3}{4}$                                                                                                                                     | <b>A1</b> | OE<br>Allow $\frac{e^{2.48 \dots} \sin^2 2\theta - 3}{4}$ .<br>Allow 2.48.<br>Allow $x = \frac{1}{4} \left( e^{2 \ln(\sin 2\theta) + 2.48 \dots} - 3 \right)$ .                                                                                         |
|          |                                                                                                                                                                                               | <b>7</b>  |                                                                                                                                                                                                                                                         |

| Question | Answer                                                                                                                                                            | Marks     | Guidance                                                                                                                                                                                                                                                                                                 |
|----------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 9(a)     | Carry out a correct method for finding a vector equation for the line through $A$ and $B$                                                                         | <b>M1</b> | A complete method.<br>Can be working from any point on $AB$ .                                                                                                                                                                                                                                            |
|          | Obtain $\mathbf{r} = \begin{pmatrix} 1 \\ -4 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} -3 \\ 5 \\ 1 \end{pmatrix}$                                             | <b>A1</b> | OE<br>Must have $\mathbf{r} = \dots$ not $l = \dots$<br>Accept $\mathbf{r}_{AB} = \dots$<br>Accept $\mathbf{R} = \dots$                                                                                                                                                                                  |
|          |                                                                                                                                                                   | <b>2</b>  |                                                                                                                                                                                                                                                                                                          |
| 9(b)     | Obtain a direction vector for $\overrightarrow{AC} = \begin{pmatrix} 1 \\ 7 \\ 3 \end{pmatrix}$                                                                   | <b>B1</b> | OE<br>Or $\overrightarrow{CA}$ .                                                                                                                                                                                                                                                                         |
|          | Carry out correct process for evaluating the scalar product of $\overrightarrow{AB}$ and $\overrightarrow{AC}$ or $\overrightarrow{BA}$ and $\overrightarrow{CA}$ | <b>M1</b> | $-3 \times 1 + 5 \times 7 + 1 \times 3 = 35$<br>Allow for correct answer and no working seen.                                                                                                                                                                                                            |
|          | Using the correct process for the moduli, divide their scalar product by the product of the moduli                                                                | <b>M1</b> | Independent M0M1 is possible.<br>ISW finding the angle.                                                                                                                                                                                                                                                  |
|          | Obtain $(\cos BAC) = \frac{\sqrt{35}}{\sqrt{59}}$ or exact simplified equivalent                                                                                  | <b>A1</b> | From correct working. The answer needs to come from using a scalar product.<br>Accept $\frac{35}{\sqrt{2065}}$ or $\frac{35}{\sqrt{35}\sqrt{59}}$ or $\frac{\sqrt{2065}}{59}$<br>$\theta = \cos^{-1} \frac{\sqrt{35}}{\sqrt{59}}$ without a statement of $\cos BAC$ scores A0.<br>ISW finding the angle. |
|          |                                                                                                                                                                   | <b>4</b>  |                                                                                                                                                                                                                                                                                                          |

| Question | Answer                                                                                                                                                                                                                                                                                  | Marks     | Guidance                                                                                                                                                                                                                                                                                                                                        |
|----------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 9(c)     | Use area = $\frac{1}{2} \overrightarrow{AB}  \overrightarrow{AC} \sin BAC$ with <i>their</i> $ \overrightarrow{AB}  \overrightarrow{AC} $                                                                                                                                               | <b>M1</b> | For “hence”, must be using the angle at <i>A</i> .<br>Need not substitute for the trigonometry.<br>Accept any equivalent form for <i>their</i> $\sin BAC$ .                                                                                                                                                                                     |
|          | Use $\sin x = \sqrt{1 - \cos^2 x}$ or an equivalent exact method with <i>their</i> $\cos x (< 1)$ to obtain an exact value for $\sin x$<br><br>$\frac{1}{2}\sqrt{35}\sqrt{59}\sqrt{1 - \frac{35^2}{35 \times 59}} = \frac{1}{2}\sqrt{35}\sqrt{59}\frac{\sqrt{840}}{\sqrt{35}\sqrt{59}}$ | <b>M1</b> | NB: These two M marks are independent.<br>Might not quote the formula. Could draw a triangle and use Pythagoras, which is equivalent.<br>$\sin x = \sqrt{\frac{24}{59}}$<br>$\sin\left(\cos^{-1}\sqrt{\frac{35}{59}}\right)$ is allowed for the first M1, but not for the second.                                                               |
|          | Obtain answer $\sqrt{210}$ from correct working                                                                                                                                                                                                                                         | <b>A1</b> | Accept simplified equivalent exact forms, e.g. $\frac{1}{2}\sqrt{840}$ .<br>Watch out for fortuitous answers from negative value of the cosine.<br>Do not accept an answer coming from $\sin\left(\cos^{-1}\sqrt{\frac{35}{59}}\right)$ with no evidence of the method of evaluation.<br>The answer needs to come from using angle <i>BAC</i> . |
|          |                                                                                                                                                                                                                                                                                         | <b>3</b>  |                                                                                                                                                                                                                                                                                                                                                 |

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| Question | Answer                          | Marks     | Guidance                                                                                                                                                                                 |
|----------|---------------------------------|-----------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 10(a)    | Obtain quotient $\frac{1}{4}$   | <b>B1</b> | Could be found by using long division or by writing $x^2 = q(1 + 4x^2) + r$ and comparing coefficients:<br>$1 = 4q, 0 = q + r$ .                                                         |
|          | Obtain remainder $-\frac{1}{4}$ | <b>B1</b> | Allow B1B1 if implied by correct division and no further working, but do not ISW.<br>Allow for a correct statement of the identity, but not for an incorrect statement of the remainder. |
|          |                                 | <b>2</b>  |                                                                                                                                                                                          |

| Question | Answer                                                                                                         | Marks        | Guidance                                                                                                                                                                                       |
|----------|----------------------------------------------------------------------------------------------------------------|--------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 10(b)    | Commence integration by parts and reach $Ax^2 \tan^{-1} 2x \pm \int x^2 \frac{B}{C + Dx^2} dx$                 | <b>*M1</b>   | OE                                                                                                                                                                                             |
|          | Obtain $\frac{1}{2}x^2 \tan^{-1} 2x - \int \frac{x^2}{1 + 4x^2} dx$                                            | <b>A1</b>    | OE                                                                                                                                                                                             |
|          | Integrate $\int \frac{x^2}{1 + 4x^2} dx$ to obtain an expression of the form $p \tan^{-1} 2x + qx$             | <b>DM1</b>   |                                                                                                                                                                                                |
|          | Complete integration and obtain $\frac{1}{2}x^2 \tan^{-1} 2x + \frac{1}{8} \tan^{-1} 2x - \frac{1}{4}x$        | <b>A1 FT</b> | OE<br>FT <i>their</i> constant quotient and remainder from <b>(a)</b> ,<br>and $\frac{k}{8} \tan^{-1} 2x - \frac{k}{4}x$ from <i>their</i> $\frac{kx^2}{1 + 4x^2}$ .                           |
|          | Substitute limits correctly in an expression of the form<br>$Fx^2 \tan^{-1} 2x + G \tan^{-1} 2x + Hx$          | <b>DM1</b>   | Need some evidence that they have considered the lower limit, e.g. sight of 0 in the working.<br>No need to evaluate trigonometry.<br>If in stages, then the 0 needs to be seen for each part. |
|          | Obtain answer $\frac{1}{16}\pi - \frac{1}{8}$ or exact one- or two-term equivalent with trigonometry evaluated | <b>A1</b>    |                                                                                                                                                                                                |
|          |                                                                                                                | <b>6</b>     |                                                                                                                                                                                                |

| Question | Answer                                                                          | Marks      | Guidance                                       |
|----------|---------------------------------------------------------------------------------|------------|------------------------------------------------|
| 11(a)    | Differentiate $\cos^2 x$ to obtain $-2\sin x \cos x$                            | <b>B1</b>  | OE<br>Could be stated as $-\sin 2x$ .          |
|          | Use correct product rule                                                        | <b>*M1</b> | With a '+' in the middle.                      |
|          | Obtain derivative $-10\sin 2x \sin x \cos x + 10\cos 2x \cos^2 x$               | <b>A1</b>  | OE                                             |
|          | Equate derivative to zero and obtain an equation in one trig function           | <b>DM1</b> | Trigonometry formulas used need to be correct. |
|          | Obtain $3 \tan^2 x = 1$ , $4 \sin^2 x = 1$ or $4 \cos^2 x = 3$ or $\cos 3x = 0$ | <b>A1</b>  | OE                                             |
|          | Obtain $x = \frac{1}{6}\pi$ only                                                | <b>A1</b>  |                                                |
|          | <b>Alternative Method for the Question 11(a)</b>                                |            |                                                |
|          | Use double angle formula to obtain $y = 10\sin x \cos^3 x$                      | <b>B1</b>  |                                                |
|          | Use correct product rule                                                        | <b>*M1</b> |                                                |
|          | Obtain derivative $10\cos^4 x - 30\sin^2 x \cos^2 x$                            | <b>A1</b>  | OE                                             |
|          | Equate derivative to zero and obtain an equation in one trig function           | <b>DM1</b> | Trigonometry formulas used need to be correct. |
|          | Obtain $3 \tan^2 x = 1$ , $4 \sin^2 x = 1$ or $4 \cos^2 x = 3$ or $\cos 3x = 0$ | <b>A1</b>  | OE                                             |
|          | Obtain $x = \frac{1}{6}\pi$ only                                                | <b>A1</b>  |                                                |

| Question | Answer                                                                             | Marks      | Guidance                        |
|----------|------------------------------------------------------------------------------------|------------|---------------------------------|
| 11(a)    | <b>Alternative Method 2 for the Question 11(a)</b>                                 |            |                                 |
|          | Use double angle formula to obtain $y = \frac{5}{2} \sin 2x (\cos 2x + 1)$         | <b>B1</b>  |                                 |
|          | Use double angle formula to obtain $y = \frac{5}{4} \sin 4x + \frac{5}{2} \sin 2x$ | <b>*M1</b> |                                 |
|          | Obtain derivative $5 \cos 4x + 5 \cos 2x$                                          | <b>A1</b>  |                                 |
|          | Equate derivative to zero and obtain an equation in $\cos 2x$                      | <b>DM1</b> | $2 \cos^2 2x + \cos 2x - 1 = 0$ |
|          | Obtain $\cos 2x = \frac{1}{2}$ only                                                | <b>A1</b>  |                                 |
|          | Obtain $x = \frac{1}{6} \pi$ only                                                  | <b>A1</b>  |                                 |
|          |                                                                                    | <b>6</b>   |                                 |

| Question | Answer                                                                                  | Marks      | Guidance                                                                                                                                                                                                                                                  |
|----------|-----------------------------------------------------------------------------------------|------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 11(b)    | $\frac{du}{dx} = -\sin x$                                                               | <b>B1</b>  | SOI                                                                                                                                                                                                                                                       |
|          | Reach an integral of the form $\int Au^3 du$                                            | <b>*M1</b> | OE<br>The question requires use of the substitution method.                                                                                                                                                                                               |
|          | Obtain $-10 \int u^3 du$                                                                | <b>A1</b>  | OE<br>Ignore limits, but check order of limits if no minus sign.                                                                                                                                                                                          |
|          | Substitute correct limits correctly in an expression of the form $Cu^4$ or $C \cos^4 x$ | <b>DM1</b> | $u = 1$ and $u = \frac{\sqrt{2}}{2}$<br>$x = 0$ and $x = \frac{1}{4}\pi$<br>$-10 \int_1^{\frac{1}{\sqrt{2}}} u^3 du$ or $10 \int_{\frac{1}{\sqrt{2}}}^1 u^3 du$<br>Allow the correct answer from the correct integration and relevant limits to imply M1. |
|          | Obtain answer $\frac{15}{8}$ or 1.875                                                   | <b>A1</b>  | WWW<br>ISW                                                                                                                                                                                                                                                |
|          |                                                                                         | <b>5</b>   |                                                                                                                                                                                                                                                           |

# Cambridge International A Level

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**MATHEMATICS****9709/33**

Paper 3 Pure Mathematics 3

**May/June 2025****MARK SCHEME**Maximum Mark: 75

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**Published**

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

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This document consists of **24** printed pages.

**PUBLISHED****Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

**GENERIC MARKING PRINCIPLE 1:**

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

**GENERIC MARKING PRINCIPLE 2:**

Marks awarded are always **whole marks** (not half marks, or other fractions).

**GENERIC MARKING PRINCIPLE 3:**

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

**GENERIC MARKING PRINCIPLE 4:**

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

**GENERIC MARKING PRINCIPLE 5:**

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

**GENERIC MARKING PRINCIPLE 6:**

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

**Mathematics-Specific Marking Principles**

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

**Annotations guidance for centres**

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

**Annotations**

| <b>Annotation</b>                                                                   | <b>Meaning</b>                         |
|-------------------------------------------------------------------------------------|----------------------------------------|
|    | More information required              |
|    | Accuracy mark awarded zero             |
|    | Accuracy mark awarded one              |
|    | Independent accuracy mark awarded zero |
|  | Independent accuracy mark awarded one  |
|  | Independent accuracy mark awarded two  |
|  | Benefit of the doubt                   |
|  | Blank Page                             |
|  | Incorrect                              |
| Dep                                                                                 | Used to indicate DM0 or DM1            |

| <b>Annotation</b>                                                                   | <b>Meaning</b>                                                                                                                             |
|-------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------|
| DM1                                                                                 | Dependent on the previous M1 mark(s)                                                                                                       |
|    | Follow through                                                                                                                             |
|    | Indicate working that is right or wrong                                                                                                    |
| Highlighter                                                                         | Highlight a key point in the working                                                                                                       |
|    | Ignore subsequent work                                                                                                                     |
|    | Judgement                                                                                                                                  |
|    | Judgement                                                                                                                                  |
|    | Method mark awarded zero                                                                                                                   |
|    | Method mark awarded one                                                                                                                    |
|    | Method mark awarded two                                                                                                                    |
|    | Misread                                                                                                                                    |
|    | Omission or Other solution                                                                                                                 |
| Off-page comment                                                                    | Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to. |
| On-page comment                                                                     | Allows comments to be entered in speech bubbles on the candidate response.                                                                 |
|  | Judgment made by the PE                                                                                                                    |
|  | Premature approximation                                                                                                                    |
|  | Special case                                                                                                                               |
|  | Indicates that work/page has been seen                                                                                                     |

| Annotation                                                                                  | Meaning                                |
|---------------------------------------------------------------------------------------------|----------------------------------------|
|  <b>SF</b> | Error in number of significant figures |
|            | Correct                                |
|  <b>TE</b> | Transcription error                    |
|  <b>XP</b> | Correct answer from incorrect working  |

**PUBLISHED****Mark Scheme Notes**

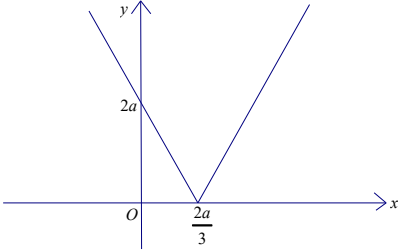
The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

**Types of mark**

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
  - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
  - The total number of marks available for each question is shown at the bottom of the Marks column.
  - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
  - Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

**Abbreviations**

|        |                                                                                                                                                                                       |
|--------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| AEF/OE | Any Equivalent Form (of answer is equally acceptable) / Or Equivalent                                                                                                                 |
| AG     | Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)                                                   |
| CAO    | Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)                                                                                           |
| CWO    | Correct Working Only                                                                                                                                                                  |
| ISW    | Ignore Subsequent Working                                                                                                                                                             |
| SOI    | Seen Or Implied                                                                                                                                                                       |
| SC     | Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance) |
| WWW    | Without Wrong Working                                                                                                                                                                 |
| AWRT   | Answer Which Rounds To                                                                                                                                                                |

| Question | Answer                                                                            | Marks     | Guidance                                                                                                                                                                               |
|----------|-----------------------------------------------------------------------------------|-----------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 1(a)     |  | <b>B1</b> | Symmetrical. In correct position.<br>Lines intended to be straight.<br>Must be in both first and second quadrants.<br>Key coordinates must be correct.<br>Ignore $y = x + 5a$ if seen. |
|          |                                                                                   | <b>1</b>  |                                                                                                                                                                                        |

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| Question | Answer                                                        | Marks     | Guidance                                                                                                                                                                            |
|----------|---------------------------------------------------------------|-----------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 1(b)     | Obtain critical value $\frac{7a}{2}$ from $x + 5a = 3x - 2a$  | <b>B1</b> | Allow if seen in an inequality.                                                                                                                                                     |
|          | Obtain critical value $-\frac{3a}{4}$ from $x + 5a = 2a - 3x$ | <b>B1</b> | Allow if seen in an inequality.                                                                                                                                                     |
|          | State final answer $-\frac{3a}{4} < x < \frac{7a}{2}$         | <b>B1</b> | <b>SC B1</b> only for $-\frac{3a}{4} < x < \frac{7a}{2}$ with <i>their a</i> from part (a).<br>Allow any equivalent notation.<br>Allow $-\frac{3a}{4} < x$ and $x < \frac{7a}{2}$ . |
|          | <b>Alternative Method for Question 1(b)</b>                   |           |                                                                                                                                                                                     |
|          | Solve quadratic equation $(3x - 2a)^2 = (x + 5a)^2$           | <b>M1</b> | $8x^2 - 22ax - 21a^2 = 0$                                                                                                                                                           |
|          | Obtain critical values $-\frac{3a}{4}$ and $\frac{7a}{2}$     | <b>A1</b> |                                                                                                                                                                                     |
|          | State final answer $-\frac{3a}{4} < x < \frac{7a}{2}$         | <b>A1</b> | <b>SC B1</b> only for $-\frac{3a}{4} < x < \frac{7a}{2}$ with <i>their a</i> from part (a).<br>Allow any equivalent notation.<br>Allow $-\frac{3a}{4} < x$ and $x < \frac{7a}{2}$ . |
|          |                                                               | <b>3</b>  |                                                                                                                                                                                     |

| Question | Answer                                                                                                        | Marks      | Guidance                                                  |
|----------|---------------------------------------------------------------------------------------------------------------|------------|-----------------------------------------------------------|
| 2        | Use the correct rule for logarithm of a power, product or quotient or an equivalent method using exponentials | <b>*M1</b> | Use of any correct law applied to original terms.         |
|          | Obtain an equation free of logarithms                                                                         | <b>A1</b>  | E.g. $\frac{(2x+3)^2}{2x+5} = 3x$ .                       |
|          | Form a 3-term quadratic from completely correct use of logarithms and solve for $x$                           | <b>DM1</b> | $2x^2 + 3x - 9 = 0$                                       |
|          | State final answer $\frac{3}{2}$ only                                                                         | <b>A1</b>  | OE<br>Rejection of negative value if given must be clear. |
|          |                                                                                                               | <b>4</b>   |                                                           |

| Question | Answer                                         | Marks      | Guidance                                                 |
|----------|------------------------------------------------|------------|----------------------------------------------------------|
| 3        | Use correct double angle formula               | <b>M1*</b> | $\frac{1}{2}(1 + \cos 10x)$                              |
|          | Obtain $\frac{3}{20} \sin 10x + \frac{3}{2} x$ | <b>A1</b>  | OE                                                       |
|          | Use correct limits correctly                   | <b>DM1</b> | $\frac{3}{20}(-0) + \frac{15\pi}{40} - \frac{12\pi}{40}$ |
|          | Obtain $\frac{3}{20} + \frac{3}{40} \pi$       | <b>A1</b>  | Or exact simplified equivalent.                          |
|          |                                                | <b>4</b>   |                                                          |

| Question | Answer                                                | Marks     | Guidance                                                                                                                          |
|----------|-------------------------------------------------------|-----------|-----------------------------------------------------------------------------------------------------------------------------------|
| 4(a)     | State $z_1 z_2 = r_1 r_2 e^{i(\theta_1 + \theta_2)}$  | <b>B1</b> | Allow equivalent forms.                                                                                                           |
|          | State correct conjugate of $z_1 z_2$ , $z_1$ or $z_2$ | <b>B1</b> | Allow equivalent forms.                                                                                                           |
|          | Obtain given result from correct working              | <b>B1</b> | Clear demonstration that the product of the conjugates is identical to the conjugate of the product.<br>Need to see a conclusion. |
|          |                                                       | <b>3</b>  |                                                                                                                                   |
| 4(b)     | State $(z^* =) 3e^{-\frac{1}{4}\pi i}$                | <b>B1</b> | Allow equivalent forms.<br>Allow $3e^{\frac{7}{4}\pi i}$ .                                                                        |
|          | Complete method to find both $b$ and $c$              | <b>M1</b> | E.g. find the product and sum of the roots, or<br>expand $(z - 3e^{-\frac{1}{4}\pi i})(z - 3e^{\frac{1}{4}\pi i})$ .              |
|          | Obtain $b = -3\sqrt{2}$ , $c = 9$                     | <b>A1</b> | OE<br>Allow $z^2 - 3\sqrt{2}z + 9 = 0$ .                                                                                          |
|          |                                                       | <b>3</b>  |                                                                                                                                   |

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| Question | Answer                                                                                 | Marks     | Guidance                                                |
|----------|----------------------------------------------------------------------------------------|-----------|---------------------------------------------------------|
| 5(a)     | State or imply $y + x \frac{dy}{dx}$ as the derivative of $xy$                         | <b>B1</b> |                                                         |
|          | State or imply $2ye^{-x} \frac{dy}{dx} - y^2 e^{-x}$ as the derivative of $y^2 e^{-x}$ | <b>B1</b> |                                                         |
|          | Equate attempted derivative to zero and solve for $\frac{dy}{dx}$                      | <b>M1</b> | Need implicit differentiation and attempt at a product. |
|          | Obtain $\frac{dy}{dx} = \frac{y^2 - ye^x}{xe^x + 2y}$ from correct working             | <b>A1</b> | AG<br>Need to see sufficient correct detail.            |
|          | <b>Alternative Method for Question 5(a)</b>                                            |           |                                                         |
|          | State or imply $xe^x \frac{dy}{dx} + y(xe^x + e^x)$ as the derivative of $xye^x$       | <b>B1</b> | Using $xye^x + y^2 - 4e^x = 0$ .                        |
|          | State or imply $2y \frac{dy}{dx} - 4e^x$ as the derivative of $y^2 - 4e^x$             | <b>B1</b> | Using $xye^x + y^2 - 4e^x = 0$ .                        |
|          | Equate attempted derivative to zero and solve for $\frac{dy}{dx}$                      | <b>M1</b> | Must make use of $4 - xy = y^2 e^{-x}$                  |
|          | Obtain $\frac{dy}{dx} = \frac{y^2 - ye^x}{xe^x + 2y}$ from correct working             | <b>A1</b> | AG<br>Need to see sufficient correct detail.            |
|          |                                                                                        | <b>4</b>  |                                                         |

| Question | Answer                         | Marks     | Guidance                           |
|----------|--------------------------------|-----------|------------------------------------|
| 5(b)     | Obtain one correct gradient    | <b>B1</b> | E.g. $\frac{1}{2}$ at $(0, 2)$ .   |
|          | Obtain second correct gradient | <b>B1</b> | E.g. $\frac{-3}{2}$ at $(0, -2)$ . |
|          |                                | <b>2</b>  |                                    |

| Question | Answer                                                       | Marks      | Guidance                                                                                                    |
|----------|--------------------------------------------------------------|------------|-------------------------------------------------------------------------------------------------------------|
| 6        | $\frac{(x+4)+iy}{x+i(y+4)} \times \frac{x-i(y+4)}{x-i(y+4)}$ | <b>*M1</b> | Multiply numerator (and denominator) by the conjugate of the denominator.                                   |
|          | Equate imaginary part of numerator to zero                   | <b>DM1</b> | Numerator<br>$= x(x+4) + y(y+4) + i[xy - (y+4)(x+4)]$                                                       |
|          | Obtain $x + y = -4$                                          | <b>A1</b>  | OE                                                                                                          |
|          | Correct use of modulus and solve for $x$ or $y$              | <b>DM1</b> | $x^2 + (-x-4)^2 = 10 \Rightarrow x^2 + 4x + 3 = 0$ or<br>$(-y-4)^2 + y^2 = 10 \Rightarrow y^2 + 4y + 3 = 0$ |
|          | $\Rightarrow z = -3 - i$                                     | <b>A1</b>  | One correct solution A1.<br><b>SC A1 A0</b> for both pairs of $x$ and $y$ correct but not in given form.    |
|          | or $z = -1 - 3i$                                             | <b>A1</b>  | Two solutions only.                                                                                         |

| Question | Answer                                                                     | Marks      | Guidance                                                                                                                                |
|----------|----------------------------------------------------------------------------|------------|-----------------------------------------------------------------------------------------------------------------------------------------|
| 6        | <b>Alternative Method for Question 6</b>                                   |            |                                                                                                                                         |
|          | $z + 4 = c(z + 4i), c \in \mathbb{R}$ or $x + iy + 4 = c(x + iy + 4i)$     | <b>*M1</b> | Equate to a real constant                                                                                                               |
|          | $x + 4 = cx$ and $y = cy + 4c$                                             | <b>DM1</b> | Equate real and imaginary parts                                                                                                         |
|          | $x = \frac{4}{c-1}, y = \frac{4c}{1-c}$ or $\frac{y}{x+4} = \frac{y+4}{x}$ | <b>A1</b>  |                                                                                                                                         |
|          | Correct use of modulus and solve for a value of $c$ , or $x$ or $y$        | <b>DM1</b> | $6c^2 + 20c + 6 = 0$ OE<br>$x^2 + (-x-4)^2 = 10 \Rightarrow x^2 + 4x + 3 = 0$ or<br>$(-y-4)^2 + y^2 = 10 \Rightarrow y^2 + 4y + 3 = 0.$ |
|          | $c = -\frac{1}{3}$ leading to $z = -3 - i$                                 | <b>A1</b>  | One correct solution A1.<br><b>SC A1 A0</b> for both pairs of $x$ and $y$ correct but not in given form.                                |
|          | $c = -3$ leading to $z = -1 - 3i$                                          | <b>A1</b>  | Two solutions only.                                                                                                                     |

| Question | Answer                                          | Marks      | Guidance                                                                                                          |
|----------|-------------------------------------------------|------------|-------------------------------------------------------------------------------------------------------------------|
| 6        | <b>Alternative Method 2 for Question 6</b>      |            |                                                                                                                   |
|          | $\arg(z + 4) = \arg(z + 4i)$                    | <b>*M1</b> | Use of $\arg\left(\frac{z + 4}{z + 4i}\right) = 0$ .                                                              |
|          | $\frac{y}{x + 4} = \frac{y + 4}{x}$             | <b>DM1</b> |                                                                                                                   |
|          | Obtain $x + y = -4$                             | <b>A1</b>  |                                                                                                                   |
|          | Correct use of modulus and solve for $x$ or $y$ | <b>DM1</b> | $x^2 + (-x - 4)^2 = 10 \Rightarrow x^2 + 4x + 3 = 0$ or<br>$(-y - 4)^2 + y^2 = 10 \Rightarrow y^2 + 4y + 3 = 0$ . |
|          | $\Rightarrow z = -3 - i$                        | <b>A1</b>  | One correct solution A1.<br><b>SC A1 A0</b> for both pairs of $x$ and $y$ correct but not in given form.          |
|          | or $z = -1 - 3i$                                | <b>A1</b>  | Two solutions only.                                                                                               |
|          |                                                 | <b>6</b>   |                                                                                                                   |

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| Question | Answer                                                                                                                                                                              | Marks        | Guidance                                                                                                                                                                                                                    |
|----------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 7(a)     | State or imply the form $\frac{A}{3a+2x} + \frac{B}{2a-x}$ and use a correct method to find a constant                                                                              | <b>M1</b>    |                                                                                                                                                                                                                             |
|          | Obtain one of $A = 3$ and $B = -1$                                                                                                                                                  | <b>A1</b>    | Allow M1 A1 if correct $A$ or $B$ found even if unwanted terms in the partial fractions expression.                                                                                                                         |
|          | Obtain the second value                                                                                                                                                             | <b>A1</b>    | ISW                                                                                                                                                                                                                         |
|          |                                                                                                                                                                                     | <b>3</b>     |                                                                                                                                                                                                                             |
| 7(b)     | Use a correct method to obtain the first two terms in the expansion of $(3a+2x)^{-1}$ , $\left(1+\frac{2x}{3a}\right)^{-1}$ , $(2a-x)^{-1}$ , or $\left(1-\frac{x}{2a}\right)^{-1}$ | <b>M1</b>    |                                                                                                                                                                                                                             |
|          | Obtain the correct unsimplified expansions in terms of $a$ , up to the term in $x^2$ .                                                                                              | <b>A2 FT</b> | A1 FT for each partial fraction.<br>Follow <i>their</i> $A, B$<br>$\frac{3}{3a}\left(1-\frac{2x}{3a}+\left(\frac{2x}{3a}\right)^2\right) \dots, -\frac{1}{2a}\left(1+\frac{x}{2a}+\left(\frac{x}{2a}\right)^2\right) \dots$ |
|          | Obtain final answer $\frac{1}{2a} - \frac{11}{12a^2}x + \frac{23}{72a^3}x^2$                                                                                                        | <b>A1</b>    | Ignore terms in higher powers of $x$ .<br>Do not ISW.<br>Allow reverse order.                                                                                                                                               |
|          |                                                                                                                                                                                     | <b>4</b>     |                                                                                                                                                                                                                             |
| 7(c)     | State $ x  < \frac{3}{2}a$                                                                                                                                                          | <b>B1</b>    | OE                                                                                                                                                                                                                          |
|          |                                                                                                                                                                                     | <b>1</b>     |                                                                                                                                                                                                                             |

| Question | Answer                                                                                                         | Marks | Guidance                                                                                                                                                                                 |
|----------|----------------------------------------------------------------------------------------------------------------|-------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 8(a)     | Express the left hand side in terms of $\sin \theta$ and $\cos \theta$                                         | M1    | $\frac{\cos^2 \theta}{\sin^2 \theta} - \frac{\sin^2 \theta}{\cos^2 \theta}$                                                                                                              |
|          | Combine to a single term and factorise the numerator                                                           | M1    | $\frac{(\cos^2 \theta - \sin^2 \theta)(\cos^2 \theta + \sin^2 \theta)}{\cos^2 \theta \sin^2 \theta}$                                                                                     |
|          | Use correct double angle formulae in numerator and denominator                                                 | M1    | $\frac{4 \cos 2\theta}{\sin^2 2\theta}$                                                                                                                                                  |
|          | Obtain $4 \cot 2\theta \operatorname{cosec} 2\theta$ from correct working                                      | A1    | $\frac{4 \cos 2\theta}{\sin 2\theta} \times \frac{1}{\sin 2\theta}$ or $\frac{4 \cos 2\theta}{\sin 2\theta \sin 2\theta}$ .                                                              |
|          | <b>Alternative Method for Question 8(a)</b>                                                                    |       |                                                                                                                                                                                          |
|          | Express the right hand side in terms of $\sin 2\theta$ and $\cos 2\theta$ or $\tan 2\theta$ and $\sin 2\theta$ | M1    | $\frac{4 \cos 2\theta}{\sin 2\theta} \times \frac{1}{\sin 2\theta}$ or $\frac{4}{\tan 2\theta} \times \frac{1}{\sin 2\theta}$<br>or $\frac{4 \cos 2\theta}{\sin 2\theta \sin 2\theta}$ . |
|          | Use correct double angle formulae in numerator and denominator                                                 | M1    | $\frac{(\cos^2 \theta - \sin^2 \theta)}{\sin^2 \theta \cos^2 \theta}$<br>or $\frac{4(1 - \tan^2 \theta)}{2 \tan \theta} \times \frac{1}{2 \sin \theta \cos \theta}$ OE                   |
|          | Split into 2 terms and simplify                                                                                | M1    | $\operatorname{cosec}^2 \theta - \sec^2 \theta$                                                                                                                                          |
|          | Obtain $\cot^2 \theta - \tan^2 \theta$ from correct working                                                    | A1    | $(1 + \cot^2 \theta) - (1 + \tan^2 \theta)$                                                                                                                                              |

| Question | Answer                                                                                  | Marks | Guidance                                                                                                                                                 |
|----------|-----------------------------------------------------------------------------------------|-------|----------------------------------------------------------------------------------------------------------------------------------------------------------|
| 8(a)     | <b>Alternative Method 2 for Question 8(a)</b>                                           |       |                                                                                                                                                          |
|          | Express the left hand side as a difference of 2 squares in terms of $\tan \theta$       | M1    | $\left(\frac{1}{\tan \theta} - \tan \theta\right) \times \left(\frac{1}{\tan \theta} + \tan \theta\right)$                                               |
|          | Use correct formula for $\tan 2\theta$                                                  | M1    | $\left(\frac{2}{\tan 2\theta}\right) \times \left(\frac{1}{\tan \theta} + \tan \theta\right)$                                                            |
|          | Use $1 + \tan^2 \theta = \sec^2 \theta$ and simplify using correct double angle formula | M1    | $\left(\frac{2}{\tan 2\theta}\right) \times \left(\frac{\sec^2 \theta}{\tan \theta}\right)$                                                              |
|          | Obtain $4 \cot 2\theta \operatorname{cosec} 2\theta$ from correct working               | A1    | $\left(\frac{2}{\tan 2\theta}\right) \times \left(\frac{2}{\sin 2\theta}\right)$                                                                         |
|          | <b>Alternative Method 3 for Question 8(a)</b>                                           |       |                                                                                                                                                          |
|          | Express the left hand side using appropriate identities                                 | M1    | $\cot^2 \theta = \operatorname{cosec}^2 \theta - 1$ and $\tan^2 \theta = \sec^2 \theta - 1$ leading to $\operatorname{cosec}^2 \theta - \sec^2 \theta$ . |
|          | Combine to a single term in terms of $\sin \theta$ and $\cos \theta$                    | M1    | $\frac{(\cos^2 \theta - \sin^2 \theta)}{\sin^2 \theta \cos^2 \theta}$                                                                                    |
|          | Use correct double angle formulae in numerator and denominator                          | M1    | $\frac{4 \cos 2\theta}{\sin^2 2\theta}$                                                                                                                  |
|          | Obtain $4 \cot 2\theta \operatorname{cosec} 2\theta$ from correct working               | A1    | $\frac{4 \cos 2\theta}{\sin 2\theta} \times \frac{1}{\sin 2\theta}$ or $\frac{4 \cos 2\theta}{\sin 2\theta \sin 2\theta}$ .                              |
|          |                                                                                         | 4     |                                                                                                                                                          |

| Question | Answer                                                                 | Marks      | Guidance |
|----------|------------------------------------------------------------------------|------------|----------|
| 8(b)     | Use the identity to obtain an expression in one trigonometric function | <b>*M1</b> |          |
|          | Obtain $\tan^2 2x = \frac{4}{5}$                                       | <b>A1</b>  | OE       |
|          | Obtain one solution e.g. $20.9^\circ$                                  | <b>DM1</b> | AWRT     |
|          | Obtain a second value e.g. $69.1^\circ$ and no extras in range         | <b>A1</b>  | AWRT     |
|          |                                                                        | <b>4</b>   |          |

| Question | Answer                                                                                                         | Marks     | Guidance                                                                                                                                                                                                      |
|----------|----------------------------------------------------------------------------------------------------------------|-----------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 9(a)     | Use a correct method to form an equation for the line through $B$ and $C$                                      | <b>M1</b> | E.g. $\mathbf{r} = \overrightarrow{OB} + \lambda \overrightarrow{BC}$ .                                                                                                                                       |
|          | Obtain $\mathbf{r} = \mathbf{i} + 3\mathbf{j} - 2\mathbf{k} + \lambda(\mathbf{i} - 4\mathbf{j} + 5\mathbf{k})$ | <b>A1</b> | OE<br>Must have $\mathbf{r}$ or component or column vector form.<br>E.g. $\mathbf{r} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k} + \lambda(-\mathbf{i} + 4\mathbf{j} - 5\mathbf{k})$ .<br>$l = \dots$ scores A0. |
|          |                                                                                                                | <b>2</b>  |                                                                                                                                                                                                               |

| Question | Answer                                                                                                                                           | Marks     | Guidance                                                                                                                                                                                                   |
|----------|--------------------------------------------------------------------------------------------------------------------------------------------------|-----------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 9(b)     | Find $\overrightarrow{AP}$ for a general point $P$ on $l$                                                                                        | <b>B1</b> | Allow unsimplified.<br>E.g. $\mathbf{j} - 2\mathbf{k} + \lambda(\mathbf{i} - 4\mathbf{j} + 5\mathbf{k})$ or<br>$\mathbf{i} - 3\mathbf{j} + 3\mathbf{k} + \mu(\mathbf{i} - 4\mathbf{j} + 5\mathbf{k})$ .    |
|          | Calculate the scalar product of $\overrightarrow{AP}$ (not $\overrightarrow{OP}$ ) and a direction vector for $l$ and equate the result to zero. | <b>M1</b> | E.g. $(\mathbf{j} - 2\mathbf{k} + \lambda(\mathbf{i} - 4\mathbf{j} + 5\mathbf{k})) \cdot (\mathbf{i} - 4\mathbf{j} + 5\mathbf{k}) = 0$<br>$\Rightarrow \lambda - 4(1 - 4\lambda) + 5(-2 + 5\lambda) = 0$ . |
|          | Obtain $\lambda = \frac{1}{3}$ or $\mu = -\frac{2}{3}$                                                                                           | <b>A1</b> | Or correct equivalents.                                                                                                                                                                                    |
|          | Obtain $\frac{4}{3}\mathbf{i} + \frac{5}{3}\mathbf{j} - \frac{1}{3}\mathbf{k}$                                                                   | <b>A1</b> | Or equivalent column vector.                                                                                                                                                                               |
|          |                                                                                                                                                  | <b>4</b>  |                                                                                                                                                                                                            |
| 9(c)     | Use a correct method to find <i>their</i> position vector of $D$                                                                                 | <b>M1</b> | E.g. $\overrightarrow{OD} = \overrightarrow{OA} + 2\overrightarrow{AP}$<br>Allow a slip in one component.                                                                                                  |
|          | Obtain $\frac{5}{3}\mathbf{i} + \frac{4}{3}\mathbf{j} - \frac{2}{3}\mathbf{k}$                                                                   | <b>A1</b> | Or equivalent column vector.                                                                                                                                                                               |
|          |                                                                                                                                                  | <b>2</b>  |                                                                                                                                                                                                            |

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| Question | Answer                                                                                                                                                | Marks      | Guidance                                                                                                                                |
|----------|-------------------------------------------------------------------------------------------------------------------------------------------------------|------------|-----------------------------------------------------------------------------------------------------------------------------------------|
| 10(a)    | Separate variables correctly                                                                                                                          | <b>B1</b>  |                                                                                                                                         |
|          | Use correct double angle formula to simplify integral in $y$                                                                                          | <b>*M1</b> | $\int \frac{\sin 4y}{\sin 2y} dy = \int 2 \cos 2y dy$                                                                                   |
|          | Obtain $\sin 2y$                                                                                                                                      | <b>A1</b>  |                                                                                                                                         |
|          | Commence integration by parts and obtain $px \cos 3x + q \int \cos 3x dx$                                                                             | <b>*M1</b> |                                                                                                                                         |
|          | Obtain $-\frac{1}{3}x \cos 3x + \frac{1}{3} \int \cos 3x dx$                                                                                          | <b>A1</b>  |                                                                                                                                         |
|          | Complete integration and obtain $-\frac{1}{3}x \cos 3x + \frac{1}{9} \sin 3x$                                                                         | <b>A1</b>  |                                                                                                                                         |
|          | Use $y = \frac{1}{12}\pi$ when $x = \frac{1}{2}\pi$ in an expression with $\sin 2y$ , $x \cos 3x$ and $\sin 3x$ to obtain the constant of integration | <b>DM1</b> | $\frac{1}{2} = 0 - \frac{1}{9} + c$                                                                                                     |
|          | Obtain $\sin 2y = -\frac{1}{3}x \cos 3x + \frac{1}{9} \sin 3x + \frac{11}{18}$                                                                        | <b>A1</b>  | OE ISW                                                                                                                                  |
|          |                                                                                                                                                       | <b>8</b>   |                                                                                                                                         |
| 10(b)    | Solve $\sin 2y = \frac{11}{18}$ to obtain one solution, e.g. 0.329                                                                                    | <b>M1</b>  | AWRT<br>Allow for <i>their</i> constant from a solution involving $\sin 2y$ , $x \cos 3x$ and $\sin 3x$ .<br>M0 for answers in degrees. |
|          | Obtain a second solution, e.g. 1.24, and no others in range                                                                                           | <b>A1</b>  | AWRT                                                                                                                                    |
|          |                                                                                                                                                       | <b>2</b>   |                                                                                                                                         |

| Question | Answer                                                                                           | Marks      | Guidance                                                                                                 |
|----------|--------------------------------------------------------------------------------------------------|------------|----------------------------------------------------------------------------------------------------------|
| 11(a)    | Use the correct product rule to differentiate                                                    | <b>*M1</b> | Could be working in terms of $a$ .<br>Obtain the form $\frac{p}{\sqrt{x}} \sin 2x + q\sqrt{x} \cos 2x$ . |
|          | Obtain $\frac{dy}{dx} = \frac{1}{2\sqrt{x}} \sin 2x + 2\sqrt{x} \cos 2x$                         | <b>A1</b>  |                                                                                                          |
|          | Equate the derivative to zero and form an equation without surds                                 | <b>DM1</b> | E.g. $\sin 2a + 4a \cos 2a = 0$ .<br>Could be working in terms of $x$ .                                  |
|          | Obtain $\tan 2a = -4a$ from correct work                                                         | <b>A1</b>  | AG                                                                                                       |
|          |                                                                                                  | <b>4</b>   |                                                                                                          |
| 11(b)    | Calculate the values of a relevant expression or pair of expressions at $x = 0.9$ and $x = 0.95$ | <b>M1</b>  | Allow smaller interval, provided it contains the root.<br>Must be working in radians.                    |
|          | Complete the argument correctly with correct calculated values                                   | <b>A1</b>  | E.g. $\begin{cases} \tan 1.8 + 3.6 = -0.686... < 0 \\ \tan 1.9 + 3.8 = 0.873... > 0 \end{cases}$         |
|          |                                                                                                  | <b>2</b>   |                                                                                                          |
| 11(c)    | State $a = \frac{1}{2}(\pi - \tan^{-1} 4a)$ and rearrange to $\pi - 2a = \tan^{-1} 4a$           | <b>B1</b>  | Or work from right to left.<br>Allow working in $x$ or $a$ .                                             |
|          | State $\tan(\pi - 2a) = 4a$ and rearrange to $\tan 2a = -4a$                                     | <b>B1</b>  | Allow working in $x$ or $a$ .                                                                            |
|          |                                                                                                  | <b>2</b>   |                                                                                                          |

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| Question | Answer                                                                                                                                         | Marks     | Guidance                                               |
|----------|------------------------------------------------------------------------------------------------------------------------------------------------|-----------|--------------------------------------------------------|
| 11(d)    | Use the iterative process correctly at least once                                                                                              | <b>M1</b> | M0 if working in degrees.                              |
|          | Obtain final answer 0.9183                                                                                                                     | <b>A1</b> |                                                        |
|          | Show sufficient iterations to at least 6 d.p. to justify 0.9183 to 4 d.p.<br>or show there is a sign change in the interval (0.91825, 0.91835) | <b>A1</b> | E.g.<br>0.9, 0.920872, 0.917944, 0.918347, 0.918292... |
|          |                                                                                                                                                | <b>3</b>  |                                                        |

# Cambridge International A Level

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**MATHEMATICS****9709/35**

Paper 3 Pure Mathematics 3

**May/June 2025****MARK SCHEME**Maximum Mark: 75

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Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

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This document consists of **31** printed pages.

**PUBLISHED****Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

**GENERIC MARKING PRINCIPLE 1:**

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

**GENERIC MARKING PRINCIPLE 2:**

Marks awarded are always **whole marks** (not half marks, or other fractions).

**GENERIC MARKING PRINCIPLE 3:**

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

**GENERIC MARKING PRINCIPLE 4:**

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

**GENERIC MARKING PRINCIPLE 5:**

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

**GENERIC MARKING PRINCIPLE 6:**

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

**Mathematics-Specific Marking Principles**

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

**Annotations guidance for centres**

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

**Annotations**

| <b>Annotation</b>                                                                   | <b>Meaning</b>                         |
|-------------------------------------------------------------------------------------|----------------------------------------|
|    | More information required              |
|    | Accuracy mark awarded zero             |
|    | Accuracy mark awarded one              |
|    | Independent accuracy mark awarded zero |
|  | Independent accuracy mark awarded one  |
|  | Independent accuracy mark awarded two  |
|  | Benefit of the doubt                   |
|  | Blank Page                             |
|  | Incorrect                              |
| Dep                                                                                 | Used to indicate DM0 or DM1            |

| <b>Annotation</b>                                                                   | <b>Meaning</b>                                                                                                                             |
|-------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------|
| DM1                                                                                 | Dependent on the previous M1 mark(s)                                                                                                       |
|    | Follow through                                                                                                                             |
|    | Indicate working that is right or wrong                                                                                                    |
| Highlighter                                                                         | Highlight a key point in the working                                                                                                       |
|    | Ignore subsequent work                                                                                                                     |
|    | Judgement                                                                                                                                  |
|    | Judgement                                                                                                                                  |
|    | Method mark awarded zero                                                                                                                   |
|    | Method mark awarded one                                                                                                                    |
|    | Method mark awarded two                                                                                                                    |
|    | Misread                                                                                                                                    |
|    | Omission or Other solution                                                                                                                 |
| Off-page comment                                                                    | Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to. |
| On-page comment                                                                     | Allows comments to be entered in speech bubbles on the candidate response.                                                                 |
|  | Judgment made by the PE                                                                                                                    |
|  | Premature approximation                                                                                                                    |
|  | Special case                                                                                                                               |
|  | Indicates that work/page has been seen                                                                                                     |

| Annotation                                                                        | Meaning                                |
|-----------------------------------------------------------------------------------|----------------------------------------|
|  | Error in number of significant figures |
|  | Correct                                |
|  | Transcription error                    |
|  | Correct answer from incorrect working  |

**PUBLISHED****Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

**Types of mark**

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
  - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
  - The total number of marks available for each question is shown at the bottom of the Marks column.
  - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
  - Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

**Abbreviations**

|        |                                                                                                                                                                                       |
|--------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| AEF/OE | Any Equivalent Form (of answer is equally acceptable) / Or Equivalent                                                                                                                 |
| AG     | Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)                                                   |
| CAO    | Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)                                                                                           |
| CWO    | Correct Working Only                                                                                                                                                                  |
| ISW    | Ignore Subsequent Working                                                                                                                                                             |
| SOI    | Seen Or Implied                                                                                                                                                                       |
| SC     | Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance) |
| WWW    | Without Wrong Working                                                                                                                                                                 |
| AWRT   | Answer Which Rounds To                                                                                                                                                                |

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| Question | Answer                                                                                                                       | Marks     | Guidance                                                                                                                                                                                                                                                                                                                                    |
|----------|------------------------------------------------------------------------------------------------------------------------------|-----------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 1        | Use law of logarithm of a product or quotient                                                                                | <b>M1</b> | $\ln 5 + \ln 6^{x-1}$ or $\ln 3^{4-2x} - \ln 5$ or $\ln 3^{4-2x} - \ln 6^{x-1}$<br>Allow logs to any base but must be consistent throughout the equation.                                                                                                                                                                                   |
|          | Use law of logarithm of a power twice                                                                                        | <b>M1</b> | $(4 - 2x)\ln 3$ and $(x - 1)\ln 6$ .<br>Omission of bracket(s) is an accuracy error if not corrected later.<br>Can have M0M1, i.e. go wrong with product but powers dealt with correctly. E.g. $(4 - 2x)\ln 3 = \ln 5 \times (x - 1)\ln 6$ , or $(4 - 2x)\ln 3 = (x - 1)\ln 30$ or $3^{3x} \times 2^x$ written as $3x\ln 3 \times x\ln 2$ . |
|          | Obtain a correct equation in any form,<br>e.g. $(4 - 2x)\ln 3 = \ln 5 + (x - 1)\ln 6$ or $1.10(4 - 2x) = 1.61 + 1.79(x - 1)$ | <b>A1</b> | May see $(4 - 2x)\ln 3$ written as $(2 - x)\ln 9$ .                                                                                                                                                                                                                                                                                         |
|          | Obtain $x = 1.15$                                                                                                            | <b>A1</b> | Must be 3 s.f.<br>If no working seen, no marks available.                                                                                                                                                                                                                                                                                   |
|          |                                                                                                                              | <b>4</b>  |                                                                                                                                                                                                                                                                                                                                             |

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| Question | Answer                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | Marks     | Guidance                                                                                                                                                                                               |
|----------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 2        | Reduce to an equation in a single trig function by use of correct trig formula(s)                                                                                                                                                                                                                                                                                                                                                                                                            | <b>M1</b> | E.g. $\operatorname{cosec}^2 \theta = \cot^2 \theta + 1$ .                                                                                                                                             |
|          | Obtain correct simplified solvable equation in one trig function with all terms on one side, any order, including $\theta$ , e.g. one of<br><br>$4\cot^2 \theta - 3\cot \theta - 1 = 0$ $\tan^2 \theta + 3\tan \theta - 4 = 0$<br>$34\sin^4 \theta - 49\sin^2 \theta + 16 = 0$ $34\cos^4 \theta - 19\cos^2 \theta + 1 = 0$<br>$\frac{34}{9}\cos^2 2\theta + \frac{10}{3}\cos 2\theta = 0$ $*\sqrt{34}\sin\left(2\theta - \tan^{-1}\left(\frac{5}{3}\right)\right) = 3$                       | <b>A1</b> | Other equations may be possible.<br><br>*For this equation, it is not necessary to have all terms on one side.                                                                                         |
|          | Solve <i>their</i> equation correctly, by formula, factorisation or calculator, to obtain two values for a trig function, e.g. one of<br><br>$\cot \theta = 1$ and $-\frac{1}{4}$ $0 \tan \theta = 1$ and $-4$<br>$\sin \theta = \pm \frac{4}{\sqrt{17}}$ and $\pm \frac{1}{\sqrt{2}}$ $\cos \theta = \pm \frac{1}{\sqrt{17}}$ and $\pm \frac{1}{\sqrt{2}}$<br>$\cos 2\theta = 0$ and $-\frac{15}{17}$ $*\sin\left(2\theta - \tan^{-1}\left(\frac{5}{3}\right)\right) = \frac{3}{\sqrt{34}}$ | <b>M1</b> | M0M1 is available if only sign slip(s) in trig formula(s).<br><br>Incorrect solvable equation from use of correct trig formula(s) can score M1M1.<br><br>*For this equation, only one value is needed. |
|          | Obtain two of answers, $\frac{1}{4}\pi$ , $-\frac{3}{4}\pi$ , $-1.33$ , $1.82$<br>Second M1 can be implied by $\frac{1}{4}\pi$ or $-\frac{3}{4}\pi$ , and $-1.33$ or $1.82$                                                                                                                                                                                                                                                                                                                  | <b>A1</b> | ISW<br>Accept $0.785$ for $\frac{1}{4}\pi$ and $-2.36$ for $-\frac{3}{4}\pi$ .<br>AWRT $-1.33$ , $1.82$ , $0.785$ , $-2.36$ .                                                                          |
|          | Obtain all answers $\frac{1}{4}\pi$ , $-\frac{3}{4}\pi$ , $-1.33$ , $1.82$ and no others in the interval                                                                                                                                                                                                                                                                                                                                                                                     | <b>A1</b> | ISW<br>Accept $0.785$ for $\frac{1}{4}\pi$ and $-2.36$ for $-\frac{3}{4}\pi$ .<br>AWRT $-1.33$ , $1.82$ , $0.785$ , $-2.36$ .<br>Ignore answers outside the given interval.                            |
|          |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | <b>5</b>  |                                                                                                                                                                                                        |

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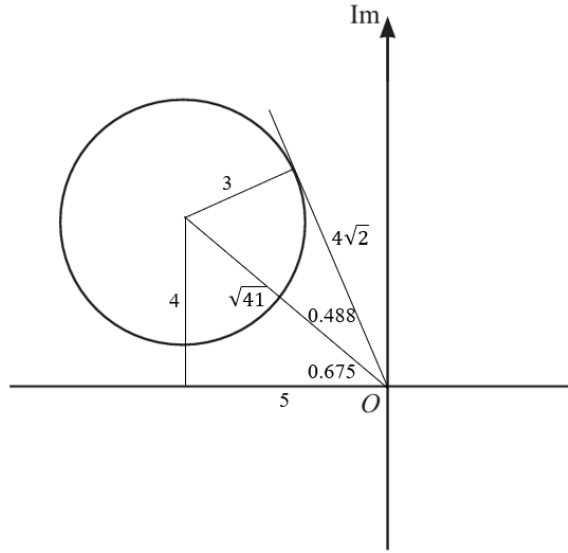
| Question | Answer                                                                                                                                                               | Marks     | Guidance                                                                                                                                                          |
|----------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 3(a)     | Correct modulus = $\frac{5}{6}$ or 0.833 or $\frac{5}{6} e^{i\theta}$ or $r = \frac{5}{6}$ only                                                                      | <b>B1</b> | Note 0.83 scores B0.<br>Do not ISW other than converting fractions to decimals.                                                                                   |
|          | Correct arg = $-2.75$ or $-\frac{11}{4}$ or $re^{-i2.75}$ or $\theta = -2.75$ only                                                                                   | <b>B1</b> | 0.25 – 3 must be evaluated.<br>B0 for $\theta = -2.75i$ .<br>Do not ISW other than converting fractions to decimals.                                              |
|          |                                                                                                                                                                      | <b>2</b>  |                                                                                                                                                                   |
| 3(b)     | Enlargement [scale] factor $\frac{1}{6}$<br>Allow expansion or stretch for enlargement<br>Do not allow e.g. ‘reduction’ or ‘shrink’ or ‘compression’ for enlargement | <b>B1</b> | Need both parts.<br>Not required to state centre (0, 0), but if incorrect then B0.                                                                                |
|          | Clockwise rotation of 3 [radians]<br>or rotation of $-3$ [radians], or [anticlockwise] rotation of $(2\pi - 3)$ [radians]                                            | <b>B1</b> | Need both parts.<br>Not required to state centre (0, 0), but if incorrect then B0.<br><br>If B0B0 scored, allow <b>SC B1</b> for enlargement AND rotation stated. |
|          |                                                                                                                                                                      | <b>2</b>  |                                                                                                                                                                   |

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| Question | Answer                                                                                                                                             | Marks      | Guidance                                                                                                                                                                                     |
|----------|----------------------------------------------------------------------------------------------------------------------------------------------------|------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 4        | Use the correct product rule<br>(Note: may start with $12x^3 \ln x$ )                                                                              | <b>*M1</b> | Attempt $\ln x^4 \frac{d}{dx}(3x^3) + 3x^3 \frac{d}{dx}(\ln x^4)$ with <i>their</i> derivatives.                                                                                             |
|          | Obtain the correct derivative in any form e.g. $9x^2 \ln x^4 + 12x^2$<br>(If starting with $12x^3 \ln x$ , should get e.g. $36x^2 \ln x + 12x^2$ ) | <b>A1</b>  | E.g. $9x^2 \ln x^4 + 3x^3 \times \frac{4x^3}{x^4}$ .<br>May see $\frac{d}{dx}(\ln x^4) = \frac{4}{x}$ .                                                                                      |
|          | Equate to zero and eliminate $\ln$ , e.g. $x^4 = e^{-\frac{4}{3}}$                                                                                 | <b>DM1</b> | E.g. $ax^4 = e^b$ , or $cx^3 = e^d$ or $x = e^f$ .<br>Allow this mark even if in decimals.<br>Allow <b>SCB1</b> if incorrect sign in product rule resulting in $x^4 = e^{\frac{4}{3}}$ , OE. |
|          | Obtain $x = e^{-\frac{1}{3}}$ only or exact simplified equivalent                                                                                  | <b>A1</b>  | ISW<br>$x = \left(e^{-\frac{4}{3}}\right)^{\frac{1}{4}}$ scores A0.<br>$x = \pm e^{-\frac{1}{3}}$ scores A0.<br>Answers with no working score no marks.                                      |
|          | Obtain $y = -\frac{4}{e}$ only or exact simplified equivalent                                                                                      | <b>A1</b>  | ISW<br>$y = 3e^{-1} \ln e^{-\frac{4}{3}}$ scores A0.<br>Answers with no working score no marks.                                                                                              |
|          |                                                                                                                                                    | <b>5</b>   |                                                                                                                                                                                              |

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| Question | Answer                                                                                     | Marks     | Guidance                                                                                                                            |
|----------|--------------------------------------------------------------------------------------------|-----------|-------------------------------------------------------------------------------------------------------------------------------------|
| 5(a)     | Attempt at method for calculating the distance from the origin to the centre of the circle | <b>M1</b> | E.g. $\sqrt{4^2 + 5^2}$ or $\sqrt{41}$ may be seen on diagram.                                                                      |
|          | Obtain distance $\sqrt{41} - 3$                                                            | <b>A1</b> | ISW<br>AWRT 3.4(0).<br>Need not identify as maximum and minimum, but if swapped, <b>M1 SCB1</b> max 2/3.                            |
|          | Obtain distance $\sqrt{41} + 3$                                                            | <b>A1</b> | ISW<br>AWRT 9.4(0).<br>Note: $\left(5 + \frac{\sqrt{3}}{2}\right)^2 + \left(4 + \frac{\sqrt{3}}{2}\right)^2 = 9.391$ scores M0A0A0. |
|          |                                                                                            | <b>3</b>  |                                                                                                                                     |

| Question | Answer                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | Marks       | Guidance                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |
|----------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 5(b)     | <p>Correct tangent and sight of correct trigonometry</p> <p>Identify or imply correct point on the circle and sight of</p> <p><math>\tan^{-1} \frac{4}{5}</math> or 0.674(7) or 38.7</p> <p>or <math>\tan^{-1} \frac{5}{4}</math> or 0.896 or 51.3</p> <p>or <math>\sin^{-1} \frac{3}{\sqrt{41}}</math> OE, or 0.487(6) or 27.9</p> <p>Allow <math>\sin^{-1} \frac{5 \text{ or } 4}{\sqrt{41}}</math> or <math>\cos^{-1} \frac{5 \text{ or } 4}{\sqrt{41}}</math> instead of inverse tan</p> <p>If no correct tangent on diagram:</p> <p><math>\tan^{-1} \frac{4}{5}</math> or <math>\tan^{-1} \frac{5}{4}</math> or <math>\tan^{-1} \frac{4}{-5}</math> (or above alternative trigonometry)</p> <p>AND <math>\sin^{-1} \frac{3}{\sqrt{41}}</math> OE B1FT</p> <p><math>\tan^{-1} \frac{4}{5} - \sin^{-1} \frac{3}{\sqrt{41}}</math> scores B1FT M0 A0</p> <p><math>\tan^{-1} \frac{4}{-5} - \sin^{-1} \frac{3}{\sqrt{41}}</math> scores B1FT M1 1.98 A1</p> <p><math>-2 \sin^{-1} \frac{3}{\sqrt{41}} - \tan^{-1} \frac{1}{5}</math> scores B0 M0 A0</p> | <b>B1FT</b> |  <p>FT <i>their</i> <math>\sqrt{41}</math> found from a correct method for B1 and M1 marks, e.g. by drawing correct tangent and sight of</p> <p><math>\tan^{-1} \frac{4}{5}</math> or <math>\tan^{-1} \frac{5}{4}</math> or <math>\sin^{-1} \frac{3}{\sqrt{41}}</math> OE.</p> <p>Note <math>\sin^{-1} \frac{3}{\sqrt{41}}</math> may be <math>\cos^{-1} \frac{4\sqrt{2}}{\sqrt{41}}</math> or <math>\tan^{-1} \frac{3}{4\sqrt{2}}</math> or <math>\tan^{-1} \frac{3\sqrt{2}}{8}</math>.</p> <p>SC Thinks min arg <math>z</math> is actually max arg <math>z</math>:</p> <p><math>\frac{\pi}{2} + \tan^{-1} \frac{5}{4} + \sin^{-1} \frac{3}{\sqrt{41}}</math> B1FT 2.95447° or 169.279° B1.</p> |

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| Question | Answer                                                                                                                         | Marks     | Guidance                                                                                                                                                                                                             |
|----------|--------------------------------------------------------------------------------------------------------------------------------|-----------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 5(b)     | Attempt at correct method to find the required argument,<br>e.g. $\pi - \tan^{-1} \frac{4}{5} - \sin^{-1} \frac{3}{\sqrt{41}}$ | <b>M1</b> | Or $\frac{\pi}{2} + \tan^{-1} \frac{5}{4} - \sin^{-1} \frac{3}{\sqrt{41}}$ .<br>FT <i>their</i> $\sqrt{41}$ found from a correct method for B1 and M1 marks.<br>Previous B1 may be implied by a correct method here. |
|          | Obtain 1.98 or 113.4                                                                                                           | <b>A1</b> | AWRT<br>E.g. 1.97923... or 113.401...                                                                                                                                                                                |
|          |                                                                                                                                | <b>3</b>  |                                                                                                                                                                                                                      |

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| Question | Answer                                                                                                                      | Marks     | Guidance                                                                                                                                                                                         |
|----------|-----------------------------------------------------------------------------------------------------------------------------|-----------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 6(a)     | Obtain $\frac{dy}{dt} = 3 \sec^2 3t$                                                                                        | <b>B1</b> | OE                                                                                                                                                                                               |
|          | Attempt chain rule on $2(\cos 3t)^{-1}$ for $\frac{dx}{dt}$ or attempt to differentiate $2 \sec 3t$                         | <b>M1</b> | Attempt $\frac{dx}{dt} = -2(\cos 3t)^{-2} \frac{d}{dt}(\cos 3t)$ with <i>their</i> derivative.                                                                                                   |
|          | Obtain $\frac{dx}{dt} = 6 \sec 3t \tan 3t$                                                                                  | <b>A1</b> | OE, e.g. $6(\cos 3t)^{-2} \sin 3t$ .<br>Allow unsimplified, e.g. $(-1) \times 2 \times (-1) \times 3$ for 6.                                                                                     |
|          | Use $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$ with <i>their</i> $\frac{dy}{dt}$ and <i>their</i> $\frac{dt}{dx}$ | <b>M1</b> | Expect, for example, $3 \sec^2 3t \times \frac{1}{6 \sec 3t \tan 3t}$<br>or $3 \sec^2 3t \times \frac{1}{6(\cos 3t)^{-2} \sin 3t}$ if correct.                                                   |
|          | Obtain $\frac{dy}{dx} = \frac{1}{2} \operatorname{cosec} 3t$                                                                | <b>A1</b> | WWW<br>Must be in this form of answer given in the question.<br>Not required to state $A = \frac{1}{2}$ .<br>Allow slips in notation for $\frac{dy}{dt}$ , $\frac{dx}{dt}$ and $\frac{dy}{dx}$ . |

| Question | Answer                                                                             | Marks     | Guidance                                                     |
|----------|------------------------------------------------------------------------------------|-----------|--------------------------------------------------------------|
| 6(a)     | <b>Alternative Method for Question 6(a)</b>                                        |           |                                                              |
|          | Convert to Cartesian form e.g. $1 + y^2 = \frac{x^2}{4}$                           | <b>B1</b> |                                                              |
|          | Correct use of implicit differentiation e.g. $\frac{d}{dy} y^2 = 2y \frac{dy}{dx}$ | <b>B1</b> |                                                              |
|          | Obtain $2y \frac{dy}{dx} = \frac{x}{2}$                                            | <b>B1</b> | OE                                                           |
|          | Convert to parametric form e.g. $2 \tan 3t \frac{dy}{dx} = \frac{2 \sec 3t}{2}$    | <b>M1</b> |                                                              |
|          | Obtain $\frac{dy}{dx} = \frac{1}{2} \operatorname{cosec} 3t$                       | <b>A1</b> | WWW<br>Must be in this form of answer given in the question. |
|          |                                                                                    | <b>5</b>  |                                                              |

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| Question | Answer                                                                                                                                                        | Marks     | Guidance                                                                                                                                                           |
|----------|---------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 6(b)     | Obtain $x = 2\sqrt{2}$ and $y = 1$ when $t = \frac{1}{12}\pi$                                                                                                 | <b>B1</b> | Must be exact.                                                                                                                                                     |
|          | Substitute $t = \frac{1}{12}\pi$ into $-1 \div \text{their } \frac{dy}{dx}$<br>Allow a small slip, but not with <i>their</i> coefficient $A$                  | <b>M1</b> | Expect gradient of normal $= -\sqrt{2}$ if correct.<br>Allow if in decimals.<br>If <i>their</i> coefficient $A$ is dealt with incorrectly M0, but allow second M1. |
|          | Form equation of the normal with <i>their</i> $(x, y)$ , found using $x = \frac{2}{\cos 3t}$ and<br>$y = \tan 3t$ , and $-1 \div \text{their } \frac{dy}{dx}$ | <b>M1</b> | E.g. $y - 1 = -\sqrt{2}(x - 2\sqrt{2})$ if correct or find $c$ in equation of line.<br>Allow M1 even if decimals.<br>M0 if using gradient of tangent.              |
|          | Obtain equation of normal $y = -\sqrt{2}x + 5$<br>Require $y = mx + c$ and exact $m$ and $c$                                                                  | <b>A1</b> | CAO<br>Accept $y = -\frac{2}{\sqrt{2}}x + 5$ .                                                                                                                     |
|          |                                                                                                                                                               | <b>4</b>  |                                                                                                                                                                    |

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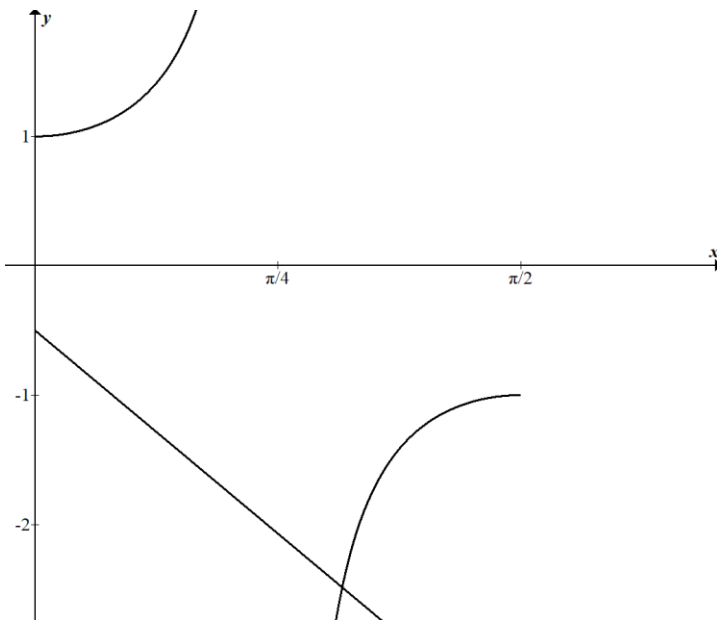
| Question | Answer                                                                                                          | Marks     | Guidance                                                     |
|----------|-----------------------------------------------------------------------------------------------------------------|-----------|--------------------------------------------------------------|
| 7(a)     | Differentiate to obtain $\frac{k_1}{1+Bx^2}$ or $\frac{k_2}{x^2+\frac{1}{B}}$                                   | <b>M1</b> | $k_1, k_2 \neq 1$ and $B \neq 1$ or 4.                       |
|          | Obtain $\frac{4}{1+16x^2}$                                                                                      | <b>A1</b> | OE, e.g. $\frac{1}{4} \times \frac{1}{x^2 + \frac{1}{16}}$ . |
|          | Set $\frac{dy}{dx} = \frac{1}{4}$ and obtain $x = \pm \frac{\sqrt{15}}{4}$ or $\pm \frac{\sqrt{15}}{\sqrt{16}}$ | <b>A1</b> | Or exact equivalent.                                         |
|          | <b>Alternative Method for Question 7(a)</b>                                                                     |           |                                                              |
|          | Rewrite as $\tan y = 4x$ and obtain $\sec^2 y = k \frac{dx}{dy}$                                                | <b>M1</b> |                                                              |
|          | Replace $\sec^2 y$ with $1 + \tan^2 y$ and $\tan y$ with $4x$ to obtain $1 + 16x^2 = 4 \frac{dx}{dy}$           | <b>A1</b> | OE                                                           |
|          | Set $\frac{dx}{dy} = 4$ and obtain $x = \pm \frac{\sqrt{15}}{4}$ or $\pm \frac{\sqrt{15}}{\sqrt{16}}$           | <b>A1</b> | Or exact equivalent.                                         |
|          |                                                                                                                 | <b>3</b>  |                                                              |

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| Question | Answer                                                                                                                                      | Marks       | Guidance                                                                                                                                                                                                                       |
|----------|---------------------------------------------------------------------------------------------------------------------------------------------|-------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 7(b)     | Commence integration by parts and reach $Ax \tan^{-1}(4x) \pm \int x \frac{B}{1+Cx^2} dx$                                                   | <b>*M1</b>  | OE<br>$A, B, C \neq 0$                                                                                                                                                                                                         |
|          | Obtain $x \tan^{-1}(4x) - \int \frac{4x}{1+16x^2} dx$                                                                                       | <b>A1</b>   | OE                                                                                                                                                                                                                             |
|          | Complete integration and obtain $x \tan^{-1}(4x) \pm \frac{B}{2C} \ln(1+16x^2)$                                                             | <b>A1FT</b> | OE SOI<br>FT <i>their B</i> and <i>their C</i> .<br>Expect $x \tan^{-1}(4x) - \frac{1}{8} \ln(1+16x^2)$ if correct.                                                                                                            |
|          | Substitute limits correctly in an expression of the form<br>$Ax \tan^{-1}(4x) - D \ln(1+Cx^2)$                                              | <b>DM1</b>  | $D \neq 0$<br>Allow omission of substitution of $x = 0$ .                                                                                                                                                                      |
|          | Obtain answer $\frac{1}{16} \pi - \frac{1}{8} \ln 2$ or exact equivalent                                                                    | <b>A1</b>   | ISW<br>Allow, for example, $\frac{1}{16} \pi + \frac{1}{4} \ln\left(\frac{1}{\sqrt{2}}\right)$ or<br>$\frac{1}{16} \pi + \frac{1}{4} \ln\left(\frac{\sqrt{2}}{2}\right)$ or $\frac{1}{16} \pi + \frac{1}{8} \ln \frac{1}{2}$ . |
|          | <b>Alternative Method for Question 7(b)</b>                                                                                                 |             |                                                                                                                                                                                                                                |
|          | Use substitution $4x = \tan y$ to find $\int Ay \sec^2 y dy$ and<br>commence integration by parts to reach $Ay \tan y \pm B \int \tan y dy$ | <b>*M1</b>  | OE<br>$A, B \neq 0$                                                                                                                                                                                                            |
|          | Obtain $\frac{1}{4} y \tan y - \frac{1}{4} \int \tan y dy$                                                                                  | <b>A1</b>   | OE                                                                                                                                                                                                                             |

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| Question | Answer                                                                                | Marks       | Guidance                                                                                                                                                                                                       |
|----------|---------------------------------------------------------------------------------------|-------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 7(b)     | Complete integration and obtain $Ay \tan y \mp B \ln \cos y$                          | <b>A1FT</b> | OE<br>FT <i>their</i> B.<br>Expect $\frac{1}{4}y \tan y + \frac{1}{4} \ln \cos y$ if correct.<br>Allow A1FT even if errors in substitution of limits into $\frac{1}{4}y \tan y$ if correct earlier.            |
|          | Substitute limits correctly in an expression of the form $Ay \tan y \pm B \ln \cos y$ | <b>DM1</b>  | Allow omission of substitution of $x = 0$ .                                                                                                                                                                    |
|          | Obtain answer $\frac{1}{16}\pi - \frac{1}{8} \ln 2$ or exact equivalent               | <b>A1</b>   | Allow e.g. $\frac{1}{16}\pi + \frac{1}{4} \ln \left( \frac{\sqrt{2}}{2} \right)$ or $\frac{1}{16}\pi - \frac{1}{4} \ln \left( \frac{2}{\sqrt{2}} \right)$ or $\frac{1}{16}\pi + \frac{1}{8} \ln \frac{1}{2}$ . |
|          |                                                                                       | <b>5</b>    |                                                                                                                                                                                                                |

| Question | Answer                                                                                            | Marks     | Guidance                                                                                                                                                                                                                                                                   |
|----------|---------------------------------------------------------------------------------------------------|-----------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 8(a)     | Sketch $y = \sec 2x$ for $0 \leq x \leq \frac{1}{2}\pi$                                           | <b>M1</b> | Need 1 or $-1$ and $\frac{1}{4}\pi$ or $\frac{1}{2}\pi$ .<br>Ignore regions outside $0 \leq x \leq \frac{1}{2}\pi$ .                                                                                                                                                       |
|          | Sketch $y = -2x - \frac{1}{2}$ for $0 \leq x \leq \frac{1}{2}\pi$ and justify the given statement | <b>A1</b> | Need a dot at the intersection of graphs, or dotted line parallel to the $y$ -axis from where graphs cross to the $x$ -axis, or state only one point of intersection OE.<br>Do not allow, e.g. ‘only one root’.<br>Ignore regions outside $0 \leq x \leq \frac{1}{2}\pi$ . |
|          |                                                                                                   |           | Diagram for reference<br>                                                                                                                                                              |
|          |                                                                                                   | <b>2</b>  |                                                                                                                                                                                                                                                                            |

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| Question | Answer                                                                                                                                                                                    | Marks     | Guidance                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |
|----------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 8(b)     | Calculate the values of a relevant expression or pair of expressions at $x = 0.8$ and $x = 1.2$<br><br>Can use smaller interval provided it contains root<br><br>M0 if working in degrees | <b>M1</b> | M1 two values attempted and at least one correct in $f(x) = \sec 2x + 2x + \frac{1}{2}$ :<br>$f(0.8) = -32.1 < 0$ , $f(1.2) = 1.54 > 0$ .<br><br>M1 four values attempted and at least three correct when comparing $\sec 2x$ and $-2x - \frac{1}{2}$ :<br><br>Using 0.8: $\sec 2x = -34.2$ $-2x - \frac{1}{2} = -2.1$ ,<br>so $-34.2 < -2.1$ .<br><br>Using 1.2: $\sec 2x = -1.36$ $-2x - \frac{1}{2} = -2.9$ ,<br>so $-1.36 > -2.9$ .<br><br>M1 two values attempted and at least one correct in $f(x) = \frac{1}{2} \cos^{-1} \frac{-2}{4x+1} - x$ :<br>$f(0.8) = 0.234 > 0$ , $f(1.2) = -0.239 < 0$ .<br><br>M1 four values attempted (must see 0.8 and 1.2 explicitly, not just embedded) and at least three correct when comparing $x$ and $\frac{1}{2} \cos^{-1} \frac{-2}{4x+1}$ :<br><br>Using 0.8: $\frac{1}{2} \cos^{-1} \frac{-2}{4x+1} = 1.03$ , so $0.8 < 1.03$ .<br><br>Using $x = 1.2$ $\frac{1}{2} \cos^{-1} \frac{-2}{4x+1} = 0.961$ , so $1.2 > 0.961$ |
|          | Complete the argument correctly with correct calculated values<br>< 0 and > 0 or change of sign is sufficient<br>For A1, answers must be correct to at least 2 sf                         | <b>A1</b> | If accurate to only 1sf, M1A0.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |
|          |                                                                                                                                                                                           | <b>2</b>  |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |

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| Question | Answer                                                                                                                                    | Marks     | Guidance                                                                                                                                                   |
|----------|-------------------------------------------------------------------------------------------------------------------------------------------|-----------|------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 8(c)     | Express $x_{n+1} = \frac{1}{2} \cos^{-1} \frac{-2}{4x_n + 1}$ as $x = \frac{1}{2} \cos^{-1} \frac{-2}{4x + 1}$                            | <b>M1</b> | Need consistent variable, could be $x_n$ or $x_{n+1}$ .                                                                                                    |
|          | Rearrange $x = \frac{1}{2} \cos^{-1} \frac{-2}{4x + 1}$ to $\sec 2x = -2x - \frac{1}{2}$ with full and correct working, no slips allowed  | <b>A1</b> | AG<br>Full working should include $\cos 2x = \frac{-2}{4x + 1}$ or<br>$-2x - \frac{1}{2} = \frac{1}{\cos 2x}$ or $\cos 2x = \frac{1}{-2x - \frac{1}{2}}$ . |
|          | <b>Alternative Method for Question 8(c)</b>                                                                                               |           |                                                                                                                                                            |
|          | Rearrange $\sec 2x = -2x - \frac{1}{2}$ to $x = \frac{1}{2} \cos^{-1} \frac{-2}{4x + 1}$ after full and correct working, no slips allowed | <b>M1</b> | Full working should include $\frac{-2}{4x + 1} = \frac{1}{\sec 2x}$ or<br>$\frac{-2}{4x + 1} = \cos 2x$ or $\cos 2x = \frac{1}{-2x - \frac{1}{2}}$ .       |
|          | Express $x = \frac{1}{2} \cos^{-1} \frac{-2}{4x + 1}$ as $x_{n+1} = \frac{1}{2} \cos^{-1} \frac{-2}{4x_n + 1}$                            | <b>A1</b> | AG                                                                                                                                                         |
|          |                                                                                                                                           | <b>2</b>  |                                                                                                                                                            |

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| Question | Answer                                                                                                                                   | Marks     | Guidance                                                                                                                                                                                                                                                                                                                                                           |
|----------|------------------------------------------------------------------------------------------------------------------------------------------|-----------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 8(d)     | Use the iterative formula correctly at least twice (consecutive) even if only 3 decimal places                                           | <b>M1</b> | M0 if first value is not between 0.8 and 1.2 inclusive.<br>M0 if working in degrees.                                                                                                                                                                                                                                                                               |
|          | Obtain final answer [ $x =$ or $\alpha =$ ] 0.992                                                                                        | <b>A1</b> | A0 if state $x_n = 0.992$ .                                                                                                                                                                                                                                                                                                                                        |
|          | Show sufficient iterations to at least 5 d.p. to justify 0.992 to 3 d.p. or show there is a sign change in the interval (0.9915, 0.9925) | <b>A1</b> | Iterations for each starting value:<br>0.8, 1.03356, 0.98547, 0.99373, 0.99226, 0.99252, 0.99247, 0.99248<br><br>0.9, 1.1030, 0.98938, 0.99303, 0.99238, 0.99250, 0.99248, 0.99248<br><br>1, 0.99116, 0.99271, 0.99244, 0.99249<br><br>1.1, 0.97510, 0.99560, 0.99193, 0.999258, 0.99246, 0.99248<br><br>1.2, 0.96143, 0.99813, 0.99148, 0.99265, 0.99245, 0.99248 |
|          |                                                                                                                                          | <b>3</b>  |                                                                                                                                                                                                                                                                                                                                                                    |

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| Question | Answer                                                                                                                                                                      | Marks     | Guidance                                                                                                                                                                                          |
|----------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 9(a)     | State or imply the form $A + \frac{B}{3x-2} + \frac{C}{x+6}$<br>Values for $A$ , $B$ and $C$ do not need to be substituted into this form to gain full marks                | <b>B1</b> | $\frac{Dx+E}{3x-2} + \frac{F}{x+6}$ and $\frac{P}{3x-2} + \frac{Qx+R}{x+6}$ B0<br>However, can recover all marks from these<br>$\frac{S}{3x-2} + \frac{T}{x+6}$ B0 and can only gain maximum M1A1 |
|          | Use a correct method for finding a constant                                                                                                                                 | <b>M1</b> |                                                                                                                                                                                                   |
|          | Obtain one of $A = 4$ , $B = 6$ and $C = -5$<br>or $F = -5$ or $P = 6$<br>or $S = 6$ or $T = -5$                                                                            | <b>A1</b> | Allow maximum M1A1 for one or more ‘correct’ values after B0, even if from $\frac{S}{3x-2} + \frac{T}{x+6}$ .<br>$D = 12$ $E = -2$ $F = -5$<br>$P = 6$ $Q = 4$ $R = 19$<br>$S = 6$ $T = -5$       |
|          | Obtain a second value from $\frac{Dx+E}{3x-2} = A + \frac{B}{3x-2}$ or $\frac{Qx+R}{x+6} = A + \frac{C}{x+6}$                                                               | <b>A1</b> |                                                                                                                                                                                                   |
|          | Obtain a third value                                                                                                                                                        | <b>A1</b> |                                                                                                                                                                                                   |
|          | <b>Alternative Method for Question 9(a)</b>                                                                                                                                 |           |                                                                                                                                                                                                   |
|          | Divide numerator by denominator and reach quotient of 4 and remainder of $Px + Q$                                                                                           | <b>M1</b> | Or by inspection<br>$P$ or $Q \neq 0$                                                                                                                                                             |
|          | Obtain $4 + \frac{-9x+46}{(3x-2)(x+6)}$ or $4 + \frac{-9x+46}{3x^2+16x-12}$                                                                                                 | <b>A1</b> |                                                                                                                                                                                                   |
|          | State or imply <i>their</i> remainder is of form $\frac{D}{3x-2} + \frac{E}{x+6}$<br>Values for $D$ and $E$ do not need to be substituted into this form to gain full marks | <b>B1</b> |                                                                                                                                                                                                   |

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| Question | Answer                                                                                                                                                                             | Marks       | Guidance                                                                                                                                                                                            |
|----------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 9(a)     | Obtain one of $D = 6$ , $E = -5$                                                                                                                                                   | <b>A1</b>   |                                                                                                                                                                                                     |
|          | Obtain a second value                                                                                                                                                              | <b>A1</b>   |                                                                                                                                                                                                     |
|          |                                                                                                                                                                                    | <b>5</b>    |                                                                                                                                                                                                     |
| 9(b)     | Use the correct method to find the first two unsimplified terms of the expansion of<br>$(3x - 2)^{-1}$ or $(x + 6)^{-1}$ or $(1 - \frac{3}{2}x)^{-1}$ or $(1 + \frac{1}{6}x)^{-1}$ | <b>M1</b>   | E.g. $-2^{-1} - 2^{-2}(3x)$ , or $6^{-1} - 6^{-2}(x)$ , or $1 + \frac{3}{2}x$ or $1 - \frac{1}{6}x$ .                                                                                               |
|          | Obtain correct unsimplified expansions up to the term in $x^2$ of each partial fraction<br>$B = 6$ $C = -5$ $A = 4$                                                                | <b>A1FT</b> | The FT is on $B$ and $C$ ,<br>e.g. $\frac{B}{-2} \left( 1 + \frac{3}{2}x + \left( \frac{3}{2}x \right)^2 \right) + \frac{C}{6} \left( 1 - \frac{1}{6}x + \left( \frac{1}{6}x \right)^2 \right)$ OE. |
|          |                                                                                                                                                                                    | <b>A1FT</b> |                                                                                                                                                                                                     |
|          | $\frac{1}{6} - \frac{157}{36}x - \frac{1463}{216}x^2$                                                                                                                              | <b>A1</b>   | OE<br>Do not ISW, e.g. multiplication through by 216.<br>Allow terms in any order.                                                                                                                  |
|          |                                                                                                                                                                                    | <b>4</b>    |                                                                                                                                                                                                     |

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| Question | Answer                                                                                                                                                                                                   | Marks     | Guidance                                                                                                                                                                |
|----------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 10(a)    | For reference $A = (2, -1, -6)$ , $B = (b, -2, 3)$ , $C = (-4, 5, -2)$                                                                                                                                   |           | Allow column vectors throughout. Allow coordinates throughout except in 10(b).<br>Allow <i>their</i> notation for vectors throughout                                    |
|          | Carry out a correct method for finding $\overrightarrow{AB}$ or $\overrightarrow{BC}$<br>Allow if use $\overrightarrow{BA}$ for $\overrightarrow{AB}$ or $\overrightarrow{CB}$ for $\overrightarrow{BC}$ | <b>M1</b> | E.g. $\overrightarrow{AB} = (b-2)\mathbf{i} + (-2+1)\mathbf{j} + (3-6)\mathbf{k}$<br>or $\overrightarrow{BC} = (-4-b)\mathbf{i} + (5-2)\mathbf{j} + (-2-3)\mathbf{k}$ . |
|          | Correct method to form equation with <i>their</i> $ \overrightarrow{AB}  = \text{their }  \overrightarrow{BC} $                                                                                          | <b>M1</b> | May see $\sqrt{(b-2)^2 + 1^2 + 9^2} = \sqrt{(-4-b)^2 + 7^2 + 5^2}$ allow one further slip.<br>Note that M0M1 is possible.                                               |
|          | Obtain $[b =] -\frac{1}{3}$                                                                                                                                                                              | <b>A1</b> |                                                                                                                                                                         |
|          |                                                                                                                                                                                                          | <b>3</b>  |                                                                                                                                                                         |

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| Question | Answer                                                                                                                                                                                                                                                                                                                                                                                                                                                      | Marks     | Guidance                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |
|----------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 10(b)    | Find $\overrightarrow{OA} + \text{their } \overrightarrow{BC}$ or $\overrightarrow{OC} - \text{their } \overrightarrow{AB}$<br>$\overrightarrow{OD} = \overrightarrow{OC} + \overrightarrow{BA}$ <b>M1</b><br>$\overrightarrow{OD} = \overrightarrow{OC} - \overrightarrow{BA}$ <b>M0</b><br><br>$\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{BC}$ <b>M1</b><br>$\overrightarrow{OD} = \overrightarrow{OA} - \overrightarrow{BC}$ <b>M0</b> | <b>M1</b> | E.g. $(2 - \frac{11}{3})\mathbf{i} + (-1 + 7)\mathbf{j} + (-6 - 5)\mathbf{k}$<br>or $(-4\mathbf{i} + 5\mathbf{j} - 2\mathbf{k}) - ((b - 2)\mathbf{i} + (-2 + 1)\mathbf{j} + (3 - -6)\mathbf{k})$<br>$= (-4 + \frac{7}{3})\mathbf{i} + (5 + 1)\mathbf{j} + (-2 - 9)\mathbf{k}$ .<br>May equate the midpoint of $AC$ and $BD$ to find $OD$ :<br>$\frac{1}{2}(\overrightarrow{OA} + \overrightarrow{OC}) = \frac{1}{2}(\overrightarrow{OB} + \overrightarrow{OD})$ .<br>Incorrect order of vertices scores M0. |
|          | Obtain $[\overrightarrow{OD} = ] -\frac{5}{3}\mathbf{i} + 6\mathbf{j} - 11\mathbf{k}$                                                                                                                                                                                                                                                                                                                                                                       | <b>A1</b> | Allow $\begin{pmatrix} -\frac{5}{3} \\ 6 \\ -11 \end{pmatrix}$ but not $\begin{pmatrix} -\frac{5}{3}\mathbf{i} \\ 6\mathbf{j} \\ -11\mathbf{k} \end{pmatrix}$ .<br>$\overrightarrow{OD} = +\frac{5}{3}\mathbf{i} - 6\mathbf{j} + 11\mathbf{k}$ scores <b>M1 A0</b> .<br>$\overrightarrow{OD} = (-\frac{5}{3}, 6, -11)$ scores <b>M1 A0</b> .                                                                                                                                                                |
|          |                                                                                                                                                                                                                                                                                                                                                                                                                                                             | <b>2</b>  |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |

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| Question | Answer                                                                                                                                                                                                            | Marks | Guidance                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |
|----------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 10(c)    | Carry out correct process for evaluating the scalar product of <i>their</i> $\pm \overrightarrow{BA}$ and <i>their</i> $\pm \overrightarrow{BC}$                                                                  | M1    | E.g. $\left(\frac{7}{3} \times -\frac{11}{3}\right) + (1 \times 7) + (-9 \times -5)$ ,<br>or $-\frac{77}{9} + 7 + 45$ or $\frac{391}{9}$ .                                                                                                                                                                                                                                                                                                                                                               |
|          | Using the correct process for the moduli, divide the scalar product by the product of the moduli for <i>their</i> pair of vectors and obtain cosine of angle (allow unsimplified form as in above scalar product) | M1    | E.g. cosine of angle = $\frac{\frac{391}{9}}{ \overrightarrow{AB}   \overrightarrow{BC} }$ or $\frac{\frac{391}{9}}{ \overrightarrow{AB}   \overrightarrow{AB} }$<br><br>$= \frac{\left(\frac{7}{3} \times -\frac{11}{3}\right) + (1 \times 7) + (-9 \times -5)}{\sqrt{\frac{49}{9} + 1 + 81} \sqrt{\frac{121}{9} + 49 + 25}}$ $\left( \frac{\frac{391}{9}}{\frac{787}{9}} = \frac{391}{787} = 0.4968 \right)$<br><br>$ \overrightarrow{AB}  =  \overrightarrow{BC} $ , so may be expressed differently. |
|          | Obtain answer 60.2 or 1.05                                                                                                                                                                                        | A1    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |
|          |                                                                                                                                                                                                                   | 3     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |

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| Question | Answer                                                                                                                                                   | Marks     | Guidance                                                                                                                                                                                                  |
|----------|----------------------------------------------------------------------------------------------------------------------------------------------------------|-----------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 11       | Separate variables correctly                                                                                                                             | <b>B1</b> | Sight of $\frac{1}{e^{3y}}$ sufficient for $f(y)$ in $f(y)dy = g(x)dx$ to obtain B1.                                                                                                                      |
|          | Obtain term $-\frac{1}{3}e^{-3y}$                                                                                                                        | <b>B1</b> |                                                                                                                                                                                                           |
|          | Separate fractions and obtain term $\frac{1}{2}\ln(x^2 + 3)$                                                                                             | <b>B1</b> |                                                                                                                                                                                                           |
|          | Separate fractions and obtain term of the form $a \tan^{-1} bx$                                                                                          | <b>M1</b> |                                                                                                                                                                                                           |
|          | Obtain term $-\frac{2}{\sqrt{3}} \tan^{-1} \frac{x}{\sqrt{3}}$                                                                                           | <b>A1</b> | OE                                                                                                                                                                                                        |
|          | Use $x = 0, y = 0$ to evaluate a constant or as limits in a solution containing terms of the form $a e^{\pm 3y}$ , $b \ln(x^2 + 3)$ and $c \tan^{-1} mx$ | <b>M1</b> |                                                                                                                                                                                                           |
|          | Obtain correct solution in any form relating $x$ and $y$                                                                                                 | <b>A1</b> | E.g. $-\frac{1}{3}e^{-3y} = \frac{1}{2}\ln(x^2 + 3) - \frac{2}{\sqrt{3}} \tan^{-1} \frac{x}{\sqrt{3}} - \frac{1}{3} - \frac{1}{2}\ln 3$ .<br>Constant $-\frac{1}{3} - \frac{1}{2}\ln 3$ may be $-0.883$ . |
|          | Obtain $-0.331$                                                                                                                                          | <b>A1</b> | OE<br>AWRT. E.g. $-0.33084\dots$<br>Note A0A1 is possible.                                                                                                                                                |
|          |                                                                                                                                                          | <b>8</b>  |                                                                                                                                                                                                           |