

Cambridge International A Level

MATHEMATICS**9709/61**

Paper 6 Probability & Statistics 2

May/June 2024**MARK SCHEME**Maximum Mark: 50

Published

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This document consists of **12** printed pages.

PUBLISHED**Generic Marking Principles**

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GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

PUBLISHED**Mathematics-Specific Marking Principles**

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1	$\lambda = 7.2$	B1	
	$P(X > 3) = 1 - e^{-7.2}(1 + 7.2 + \frac{7.2^2}{2!} + \frac{7.2^3}{3!})$ or $1 - e^{-7.2}(1 + 7.2 + 25.92 + 62.21)$ or $1 - (0.0007466 + 0.005375 + 0.01935 + 0.04644)$	M1	Allow any λ . Allow one end error. Must see expression. Allow fully correct sigma notation.
	$= 0.928$ (3sf)	A1	SC 0.928 with no working seen scores B1 B1 .
		3	

Question	Answer	Marks	Guidance
2	$E(X_1 - 3X_2) = 31.2 - 3 \times 31.2$ [= -62.4]	B1	OE $E(3X_2 - X_1) = +62.4$
	$\text{Var}(X_1 - 3X_2) = 10.4^2 + 3^2 \times 10.4^2$ [= 1081.6]	B1	
	$\frac{0 - (-62.4)}{\sqrt{1081.6}}$ [= 1.897]	M1	Standardising (with attempt at E and Var, not just using 31.2 and 10.4).
	$1 - \Phi(1.897)$	M1	For area consistent with their working.
	$= 0.0289$ (3 sf)	A1	
		5	

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Question	Answer	Marks	Guidance
3(a)	$81.5 \pm z \times \frac{5.8}{\sqrt{20}}$	M1	For a correct expression (accept if only one side of the interval calculated). Any z (must be a z).
	$z = 2.326$	B1	
	78.5 to 84.5 (3sf)	A1	Must be an interval.
		3	
3(b)	Not true. C. I. is for mean time, not individual times.	B1	OE Both comments needed.
		1	
3(c)	$\frac{0.98^2}{1-0.02^2}$	M1	Attempt $\frac{P(\text{both contain } \mu)}{P(\text{at least one contains } \mu)}$ with numerator attempt 0.98^2 and denominator attempt involving 0.02. Must see their quotient.
	= 0.961 (3sf)	A1	NB: $[0.98^2 =]$ 0.9604 scores M0 A0.
		2	

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Question	Answer	Marks	Guidance
4(a)	$\text{Est}(\mu) = \frac{2044}{8} [=255.5]$	B1	Accept 3sf if nothing better seen.
	$\text{Est}(\sigma^2) = \frac{8}{7} \left(\frac{522348}{8} - 255.5^2 \right) \text{ or } \frac{1}{7} \left('522348' - \frac{'2044'^2}{8} \right)$	M1	Attempt to find Σx^2 and substitute in correct formula. May be implied by correct answer. Biased 13.25 scores M0.
	$= 15.1 \text{ (3 sf) or } \frac{106}{7}$	A1	OE
		3	
4(b)(i)	$H_0: \mu = 253$ $H_1: \mu > 253$	B1	Allow 'Population mean' but not just 'mean'.
	$\frac{\frac{25360}{100} - 253}{3.5 \div \sqrt{100}}$	M1	Standardising must have $\sqrt{100}$.
	$= 1.714$	A1	
	$1.714 > 1.645 \text{ or } 0.0432 < 0.05$	M1	OE
	[Reject H_0] There is sufficient evidence (at 5% level) to suggest [mean] mass is greater than 253	A1FT	OE FT <i>their</i> '1.714' in context, not definite, no contradictions. Accept critical value method of $253.57 < 253.60$ or $253.02 > 253$. Use of a two-tailed test scores B0 M1 A1 M1 A0 (comp with $\frac{0.025}{1.96}$).
		5	

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Question	Answer	Marks	Guidance
4(b)(ii)	Not true. Large sample, [so sample mean is approx normally distributed].	B1	OE Allow 'Not true. Large sample' or 'Not true. n is large' or 'Not true. CLT used'.
		1	

Question	Answer	Marks	Guidance
5(a)	Constant average rate	B1	OE Accept constant rate. Allow without context.
		1	
5(b)	$\lambda = 14.4$	B1	
	$e^{-14.4} \left(\frac{14.4^{13}}{13!} + \frac{14.4^{14}}{14!} + \frac{14.4^{15}}{15!} \right)$ or $e^{-14.4} (183837 + 189089 + 181526)$ or $(0.102469 + 0.105396 + 0.101181)$	M1	Poisson P(13, 14, 15). Expression must be seen. Allow one end error; allow any λ . Allow fully correct sigma notation.
	$= 0.309$ (3sf)	A1	SC: 0.309 with no working scores B1 B1 .
		3	
5(c)	$N(140 \times 1.2, 140 \times 1.2)$ or $N(168, 168)$	B1	Stated or implied.
	$\frac{149.5 - 168}{\sqrt{168}} [= -1.427]$	M1	Standardising using their mean and variance. Allow with wrong or no continuity correction.
	$\Phi(-1.427) = 1 - \Phi(1.427)$	M1	For area consistent with their working.
	$= 0.0768$ or 0.0767 (3sf)	A1	
		4	

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Question	Answer	Marks	Guidance
6(a)	$\frac{1}{4}\pi a^2 = 1$	M1	OE Attempt to set area = 1.
	$a = \frac{2}{\sqrt{\pi}}$	A1	AG Correct equation and correctly rearranged to $a = \dots$ No errors seen.
		2	
6(b)	$x^2 + y^2 = \left(\frac{2}{\sqrt{\pi}}\right)^2$	M1	Or $x^2 + y^2 = a^2$.
	$[y^2 = \frac{4}{\pi} - x^2]$		Must see at least one intermediate step
	$f(x) = \sqrt{\frac{4}{\pi} - x^2}$	A1	AG Convincingly rearranged to reach given answer. No errors seen.
		2	
6(c)	$\int_0^{\frac{2}{\sqrt{\pi}}} x\sqrt{\frac{4}{\pi} - x^2} dx$	B1	Correct expression for $E(X) = \int xf(x) dx$ with correct limits (accept limits 0 and a).
	$-\frac{1}{3} \left[\left(\frac{4}{\pi} - x^2 \right)^{\frac{3}{2}} \right]_0^{\frac{2}{\sqrt{\pi}}}$	M1	Integrate $xf(x)$ with any limits or none. Must reach expression of form: any constant $\times \left(\frac{4}{\pi} - x^2 \right)^{\frac{3}{2}}$.
		A1	Wholly correct integration and limits.
	$\frac{8}{3\sqrt{\pi^3}}$	A1	AG Correctly obtained with no errors seen.
		4	

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Question	Answer	Marks	Guidance
7(a)	$H_0: p = 0.3$ $H_1: p < 0.3$	B1	
	$B(31, 0.3), P(X \leq 4) =$ $0.7^{31} + 31 \times 0.7^{30} \times 0.3 + {}^{31}C_2 \times 0.7^{29} \times 0.3^2 + {}^{31}C_3 \times 0.7^{28} \times 0.3^3 +$ ${}^{31}C_4 \times 0.7^{27} \times 0.3^4$ $= 0.00001577 + 0.0002096 + 0.0013475 + 0.0055826 + 0.016748$	M1	No end errors.
	$= 0.0239$ (3sf)	A1	SC 0.0239 with no working scores B1 .
	'0.0239' < 0.05	M1	Valid comparison.
	[reject H_0] 'There is sufficient evidence (at 5% level) to support Rita's suspicion', or 'There is sufficient evidence to suggest the probability of seeing this type of bird has decreased'	A1FT	In context. Not definite. No contradictions. FT <i>their</i> 0.0239.
		5	
7(b)	$P(X \leq 5) = ['0.0239' + {}^{31}C_5 \times 0.7^{26} \times 0.3^5] = 0.0627$ [which is > 0.05]	B1FT	Attempt $P(X \leq 5)$. Only FT if > 0.05. Only FT <i>their</i> 0.0239 if $P(X \leq 4)$ attempted in (a) ; arithmetic error only.
	$P(\text{Type I error}) = '0.0239'$	B1FT	Only FT <i>their</i> 0.0239 if $P(X \leq 4)$ attempted in (a) ; arithmetic error only and <i>their</i> 0.0239 < 0.05.
		2	

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Question	Answer	Marks	Guidance
7(c)(i)	$[\lambda =] 3.65$	B1	Stated or implied.
	$e^{-3.65} \times \frac{3.65^4}{4!}$	M1	Must see expression. Any λ .
	$= 0.192$ (3sf)	A1	SC: Use of Binomial. 0.193 scores B1 . SC: 0.192 with no working scores B1 B1 .
		3	
7(c)(ii)	$n = 365 > 50$ $np = 3.65 < 5$ or $p = 0.01 < 0.1$	B1	Explicit. Both needed. Note: and ‘ n large, p small’ is insufficient.
		1	

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Question	Answer	Marks	Guidance
1(a)	N(145, 145)	B1	Stated or implied.
	$\pm \frac{150.5-145}{\sqrt{145}}$ [= ±0.457]	M1	Condone incorrect or omitted continuity correction.
	$\Phi(0.457)$	M1	For area consistent with their working.
	= 0.676 (3sf)	A1	SC: Unsupported answer of 0.676 scores B3 . Unsupported answer of 0.646 or 0.661 scores B2 . Unsupported answer of 0.6799 scores B1 .
		4	
1(b)	145 > 15	B1	Explicit. $\lambda > 15$ B0 if $\lambda = 145$ not stated. Accept $\geq$ Accept mean for λ .
		1	

Question	Answer	Marks	Guidance
2(a)	[567, 109], 665, 21	B2	B1 for each. Allow 021. If more than 2 answers given, count first two and ISW.
		2	

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Question	Answer	Marks	Guidance
2(b)	$\text{Est}(\mu) = \frac{610}{30} \text{ or } \frac{61}{3}$	B1	OE or 20.3.
	$\text{Est}(\sigma^2) = \frac{30}{29} \left(\frac{12405}{30} - \left(\frac{610}{30} \right)^2 \right) \text{ or } \frac{1}{29} \left(12405 - \frac{610^2}{30} \right)$	M1	Use of correct formula.
	$= 0.0575 \text{ (3sf)}$	A1	Accept $\frac{5}{87}$.
		3	
2(c)	Variance is [unrealistically] small so Henri has [probably] made a mistake/claim is [probably] correct	B1 FT	Need both parts. Need ‘small’ OE, not just < 0.1 . FT <i>their</i> < 0.1 variance value (not –ve), e.g. 0.0556 (if omit $\frac{30}{29}$). Accept ‘s.d. = 0.24 is small, so Henri has probably made a mistake’. Note: ‘mean is large/small’ scores B0, but ‘mean large compared to variance so Henri prob made a mistake’ scores B1.
		1	

Question	Answer	Marks	Guidance
3	$\frac{18}{50} - z \times \sqrt{\frac{\frac{18}{50} \times (1 - \frac{18}{50})}{50}} = 0.244$	M1	Use of correct equation.
	$z = 1.709 \text{ or } 1.708$	A1	Accept 1.71 if nothing better seen.
	$\Phi^{-1}('1.709') = 0.956 ; 1 - 2(1 - '0.956')$ [= 0.912]	M1	Attempt area above or below their 1.709 and use correct method to find α .
	$\alpha = 91$	A1	Allow $\alpha = 91\%$ 0.91 or 91.2 score A0.
		4	

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Question	Answer	Marks	Guidance
4(a)	0.5	B1	
		1	
4(b)	$E(X_1 - 2X_2 + 3) = 10 - 20 + 3 [= -7]$ or $E(2X_2 - X_1 - 3) = 20 - 10 - 3 [= 7]$	B1	Or equivalent using $X_1 - 2X_2 = 10 - 20 [= -10]$ or $2X_2 - X_1 = 20 - 10 [= +10]$.
	$\text{Var}(X_1 - 2X_2 + 3) = 12 + 2^2 \times 12 + 0$ [= 60]	B1	
	$\frac{0 - ('-7')}{\sqrt{'60'}}$ [= 0.904]	M1	Or numerator 3-'10' or -3-('10'), but not '-3 -10' (i.e. numerator must be '7' or '-7').
	$1 - \Phi('0.904')$	M1	For area consistent with their working.
	= 0.183	A1	
		5	

Question	Answer	Marks	Guidance
5(a)	$e^{-3.1}(1 + 3.1 + \frac{3.1^2}{2!} + \frac{3.1^3}{3!})$ or $e^{-3.1}(1 + 3.1 + 4.805 + 4.965)$ or $0.0450 + 0.1397 + 0.2165 + 0.22368$	M1	Condone one end error. Any λ . Accept fully correct Σ notation. Expression must be seen.
	= 0.625 (3sf)	A1	Correct answer with no working scores SC B1 .
		2	

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Question	Answer	Marks	Guidance
5(b)	$[\lambda] = 5.5$	B1	SOI
	$1 - e^{-5.5}(1 + 5.5 + \frac{5.5^2}{2!} + \frac{5.5^3}{3!} + \frac{5.5^4}{4!})$ or $1 - e^{-5.5}(1 + 5.5 + 15.125 + 27.7292 + 38.1276)$ or $1 - e^{-5.5}(0.004087 + 0.0224772 + 0.061812 + 0.113323 + 0.155819)$	M1	Condone one end error. Any λ . Accept fully correct Σ notation. Expression must be seen.
	$= 0.642$ or 0.643 (3sf)	A1	Correct answer with no working scores SC B1 B1 .
		3	
5(c)	$[P(X=3) \times P(Y=2)] = e^{-3.1} \times \frac{3.1^3}{3!} \times e^{-2.4} \times \frac{2.4^2}{2!}$ or $0.223676 \times 0.261267 [= 0.05844]$	M1	Find P(3 in first half AND 2 in second half). Must see expression.
	$[P(\text{total } 5)] = e^{-5.5} \times \frac{5.5^5}{5!}$ or 0.17140	M1	Use of 5.5 to find P(5).
	$P(\text{P(exactly 3 in 1st half given total 5)}) = \frac{P(\text{exactly 3 in 1st half and total 5})}{P(\text{total 5})}$	M1	Attempt at conditional probability; numerator = <i>their</i> 0.05844 and denominator = P(total 5) Note: $(\frac{3.1^3}{3!} \times \frac{2.4^2}{2!}) \div (\frac{5.5^5}{5!})$ scores M1 M1 M1.
	$[= \frac{0.05844}{0.17140}] = 0.341$ (3sf)	A1	
		4	

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Question	Answer	Marks	Guidance
6(a)	H ₀ : Population mean mass = 510 g H ₁ : Population mean mass < 510 g	B1	Allow ' μ ' but not just 'mean'.
	$\pm \frac{508 - 510}{10 \div \sqrt{120}}$	M1	Standardising must have $\sqrt{120}$.
	= $\pm$ -2.191 or -2.190	A1	
	-2.191 < -1.96 or 2.191 > 1.96 Area comparison: 0.0143 or 0.0142 < 0.025	M1	OE For valid comparison. Inequality sign the wrong way round scores M1 A0.
	[Reject H ₀] There is sufficient evidence to suggest that the [mean] mass has decreased	A1FT	OE In context (must be 'decreased' OE, not 'changed'); not definite. No contradictions. Condone 'there is sufficient evidence to support the inspector's claim'. NB: Accept alternative method using critical value (= 508.21) and comparison with 508. Condone 509.79 compared with 510. Two tail test scores maximum B0 M1 A1 M1 A0; must have comparison with 0.0125 or 2.24/2.241.
		5	

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Question	Answer	Marks	Guidance
6(b)	$\frac{cv - 510}{10 \div \sqrt{120}} = -1.96$	M1	Standardising to find critical value (must use 510 and $10 \div \sqrt{120}$). Accept ± 1.96 .
	$cv = 508.21$	A1	Accept 3 sf if nothing better seen. Note: cv could be found in (a).
	$z = \pm \frac{508.21 - 506}{10 \div \sqrt{120}} [= 2.421]$	M1	Standardising with their 508.21 and 506 (must use $10 \div \sqrt{120}$).
	$P(\bar{X} > 508.21 \mid \mu = 506) = 1 - \Phi(2.421)$	M1	For area consistent with their working.
	$= 0.0077 \text{ to } 0.0080 \text{ (2sf)}$	A1	Note: $\frac{510 - 506}{10 \div \sqrt{120}}$ scores max M0 A0 M1 M1 A0.
		5	

Question	Answer	Marks	Guidance
7(a)	$k \int_0^{\pi} (1 + \cos x) dx = 1$	M1	Attempt integrate $f(x)$ with correct limits and equate to 1.
	$k[x + \sin x]_0^{\pi} = 1$	A1	Correct integration.
	[e.g. $k(\pi + \sin \pi - (0 + 0)) = 1$], $k\pi = 1$, $k = \frac{1}{\pi}$	A1	AG Some evidence of substitution of limits, i.e. at least one interim step (e.g. $k\pi = 1$) as minimum requirement. Convincingly obtained; no errors seen.
		3	

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Question	Answer		Marks	Guidance
7(b)	$\frac{1}{\pi} [x + \sin x]_0^{0.83}$ or $\frac{1}{\pi} (0.83 + \sin 0.83)$		M1	Substitute correct limits into their integral. OR ₁ : integrate 0 to 0.83 and 0.84 to π . OR ₂ : use $g(m) = m + \sin m - (\pi/2)$ and find $g(0.83)$ and $g(0.84)$. OR ₃ : use $h(m) = m + \sin m$ and find $h(0.83)$ and $h(0.84)$. Both attempted.
	$= 0.499 \text{ (3 sf)}$		A1	OR ₁ : 0.499 and 0.496. OR ₂ : $g(0.83) = -0.00286/7$ and $g(0.84) = 0.0138/9$. OR ₃ : $h(0.83) = 1.57$ and $h(0.84) = 1.58$ or 1.59 . Both correct.
	‘0.499’ < 0.5 < ‘0.504’ hence $0.83 < \text{median} < 0.84$ Equivalent to $-0.000912 < 0 < 0.00441$ hence $0.83 < \text{median} < 0.84$		A1FT	FT their areas; dep 0.5 is between their areas OE. OR ₁ : $0.499 < 0.5$ and $0.496 < 0.5$, so $0.83 < m < 0.84$. OR ₂ : $g(0.83) > 0$ $g(0.84) < 0$ OE, so $0.83 < m < 0.84$. OR ₃ : $h(0.83) < \frac{\pi}{2}$ $h(0.84) > \frac{\pi}{2}$, so $0.83 < m < 0.84$. Both statements needed. Note: A score of M1 A0 A1FT is possible.
				If 0 scored, SC : $\frac{1}{\pi} (m + \sin m) = 0.5$ B1 and $m = 0.831$ to 0.832 , so $0.83 < m < 0.84$ B1 .
			3	

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Question	Answer	Marks	Guidance
7(c)	$\frac{1}{\pi} \int_0^{\pi} (x + x \cos x) dx$	M1*	Attempt integrate $xf(x)$. Ignore limits.
	$= \frac{1}{\pi} \left(\left[\frac{x^2}{2} \right]_0^{\pi} + [x \sin x]_0^{\pi} - \int_0^{\pi} (\sin x dx) \right)$	DM1	OE Attempt to integrate (using ‘parts’) with correct limits, reaching an expression of the form $ax^2 + uv - \int v du$. OR using parts to integrate $x(1+\cos x)$ reaching an expression of the form $uv - \int v du$ i.e. $\frac{1}{\pi} (x^2 + x \sin x - \int (x + \sin x) dx)$
	$= \frac{1}{\pi} \left(\frac{x^2}{2} + x \sin x + \cos x \right)$ or e.g. $\frac{\pi}{2} + \frac{1}{\pi} (0 - [-\cos x]_0^{\pi})$ or e.g. $\frac{\pi}{2} + \frac{1}{\pi} ((-1 - 1))$	A1	Integration fully correct.
	$= \frac{\pi}{2} - \frac{2}{\pi}$	A1	OE ISW after correct exact value seen. SC1: Unsupported answer of $\frac{\pi}{2} - \frac{2}{\pi}$ scores B3 . SC2: Unsupported answer of 0.934 scores B2 .
		4	

Cambridge International A Level

MATHEMATICS**9709/63**

Paper 6 Probability & Statistics 2

May/June 2024**MARK SCHEME**Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2024 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

This document consists of **12** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

PUBLISHED**Mathematics-Specific Marking Principles**

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1(a)	$[\lambda =] 4$	B1	
	$e^{-4} \left(\frac{4^2}{2!} + \frac{4^3}{3!} + \frac{4^4}{4!} \right)$ or $e^{-4} (8 + 10.67 + 10.67)$ or $0.1465 + 0.19537 + 0.19537$	M1	Allow one end error. Any λ . Expression must be seen.
	$= 0.537$ (3sf)	A1	SC B1 B1 for unsupported correct answer. SC B2 for use of Binomial leading to 0.537. Note: use of normal could score B1 only for mean = 4.
		3	
1(b)	$n = 4000 > 50$ and either $np = 4 < 5$ or $p = 0.001 < 0.1$	B1	Explicit values seen.
		1	

Question	Answer	Marks	Guidance
2(a)	$3.12 + z \times \frac{\sigma}{\sqrt{150}} = 3.23$	M1	OE – correct expression. Any z , but must be a z value.
	$z = 1.96$	B1	
	$\sigma = 0.687$ (3sf) [cm]	A1	
		3	
2(b)	Yes, because population [of widths] not given to be normally distributed	B1	Or ‘underlying distribution’ instead of population. Allow ‘yes, because population distribution not known’. Need both statements.
		1	

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Question	Answer	Marks	Guidance
3	Diff ~ N(5×50 – 10×26, 5×2.4 + 10×1.8) [= N(–10, 30)]	B1	For N and mean = ±(5×50 – 10×26) SOI
		B1	For var = 5×2.4 + 10×1.8 SOI
	$\frac{0 - (-10)}{\sqrt{30}}$ [= 1.826]	M1	Standardising with <i>their</i> values.
	1 – Φ('1.826')	M1	For area consistent with <i>their</i> values.
	= 0.0339 or 0.034[0] (3sf)	A1	
		5	

Question	Answer	Marks	Guidance
4	H ₀ : $p = 0.24$ H ₁ : $p < 0.24$	B1	
	$P(X \leq 2) = 0.76^{25} + 25 \times 0.76^{24} \times 0.24 + {}^{25}C_2 \times 0.76^{23} \times 0.24^2$ or 0.0010479 + 0.0082732 + 0.0313513	M1	Expression must be seen. No end errors.
	= 0.0407	A1	SC B1 for unsupported 0.0407.
	0.0407 < 0.05	M1	For valid comparison.
	[Evidence to reject H ₀ .] There is sufficient evidence to suggest that the support for the Today Party has decreased.	A1FT	FT <i>their</i> probability. In context, not definite, no contradictions. SC : if H ₁ : $p \neq 0.24$ and compare with 0.025; max B0 M1 A1 M1 A0 .
		5	

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Question	Answer	Marks	Guidance
5(a)	$\int_0^{\sqrt{2}} (ax - x^3) dx = 1$	M1	Attempted integration of f(x) and equated to 1.
	$\left[a \frac{x^2}{2} - \frac{x^4}{4} \right]_0^{\sqrt{2}} = 1$ $a - \frac{4}{4} = 1$	A1	Correct integration and substitute correct limits.
	$a = 2$	A1	AG Convincingly obtained and no errors seen.
		3	
5(b)	$\int_0^m (2x - x^3) dx = \quad \quad \quad = \frac{1}{2}$	M1	Attempt integrate f(x) with limits 0 to m (or m to $\sqrt{2}$) and equate to $\frac{1}{2}$.
	$m^2 - \frac{m^4}{4} = \frac{1}{2}$	A1	For correct quartic in any form.
	$m^4 - 4m^2 + 2 = 0 \Rightarrow m^2 = \frac{4 \pm \sqrt{16-8}}{2} \quad [= 2 \pm \sqrt{2}]$	M1	For solving their three term quartic to find m^2 .
	$m = \sqrt{2 - \sqrt{2}} \text{ or } 0.765 \text{ (3sf)}$	A1	
		4	

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Question	Answer	Marks	Guidance
5(c)	$\int_0^{\sqrt{2}} (2x^2 - x^4) dx$	M1	Attempt to integrate $xf(x)$. Ignore limits.
	$\left[\frac{2x^3}{3} - \frac{x^5}{5} \right]_0^{\sqrt{2}}$	A1	Correct integration and correct limits.
	$\left[= \frac{4\sqrt{2}}{3} - \frac{4\sqrt{2}}{5} \right] = \frac{8}{15}\sqrt{2}$	A1	OE For single exact term.
		3	

Question	Answer	Marks	Guidance
6(a)	$\hat{\mu} = \frac{307}{200}$ or 1.535	B1	Accept 3sf if nothing better seen (1.53 or 1.54).
	$\Sigma x^2 f = 627, \text{Est}(\sigma^2) = \frac{200}{199} \left(\frac{627}{200} - 1.535^2 \right)$ or $\frac{1}{199} \left('627' - \frac{307^2}{200} \right)$	M1	Use of a correct formula with <i>their</i> values.
	$= 0.783$	A1	AG Correctly obtained with no errors seen.
		3	
6(b)	$H_0: \mu = 1.65$ $H_1: \mu < 1.65$	B1	Accept ‘population mean’ but not just ‘mean’.
		1	

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Question	Answer	Marks	Guidance
6(c)	$\frac{1.535 - 1.65}{\sqrt{0.783/200}}$	M1	Standardising with <i>their</i> mean.
	= -1.838 or -1.84	A1*	
	$\Phi(0.05)$ and $\Phi(0.01)$ attempted	M1	Or $P(z < -1.838)$ attempted. SC: Condone $\Phi(0.025) = 2.807$ and $\Phi(0.005) = 3.291$ following two-tailed test in (b) .
	-1.645 > -1.838 > -2.326 [Hence significant at 5% but not 1% level]	DA1	AG = 0.033 and $0.05 > 0.033 > 0.01$ SC: use of 1.54 or 1.53 for the mean leading to $-1.645 > -1.758 > -2.326$ or $-1.645 > -1.918 > -2.326$ or $0.95 < 0.9606$ or $0.9724 < 0.99$ scores M1 M1 A1 . Accept use of critical value method $1.535 < 1.547$ or accept $1.65 > 1.638$.
		4	
6(d)	At the 1% level H_0 is not rejected Or a Type I error can only occur if H_0 is rejected.	B1	OE
		1	

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Question	Answer	Marks	Guidance
7(a)	$\lambda = 1.9 + 2.2 [= 4.1]$	B1	
	$e^{-4.1}(1 + 4.1 + \frac{4.1^2}{2!} + \frac{4.1^3}{3!})$ or $e^{-4.1}(1 + 4.1 + 8.405 + 11.487)$ or $0.01657 + 0.06795 + 0.13929 + 0.19037$	M1	Allow any λ . Allow one end error. Must see expression.
	$= 0.414$ (3sf)	A1	SC: unsupported answer 0.414 scores B1 B1 .
		3	
7(b)	$P(X + Y < 4 \text{ and } X = 2) = P(2, 0 \text{ or } 2, 1)$	M1	Stated or implied.
	$= e^{-1.9 \times \frac{1.9^2}{2}} (e^{-2.2} + e^{-2.2 \times 2.2})$ 0.0957] [=	M1	
	$P(X = 2 X + Y < 4) = \frac{0.0957}{0.414}$	M1	Attempt $\frac{P(X+Y < 4 \text{ and } X = 2)}{P(X+Y < 4)}$. Prob for denominator can be found in (a).
	0.231 (3sf)	A1	
		4	

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Question	Answer	Marks	Guidance
7(c)	$E(X + Y) = 4.1$ $\text{Var}(X + Y) = 4.1$ or $\text{Po}(246)$	B1	SOI
	Normal and var = $\frac{4.1}{60}$ Or normal and var = 246	M1	
	$\frac{4.0-4.1}{\sqrt{4.1 \div 60}}$ or totals method $\frac{240-246}{\sqrt{246}}$ or use of continuity correction	M1	No mixed methods. Or continuity correction: $\frac{4.0 - \frac{1}{120} - 4.1}{\sqrt{(4.1 \div 60)}}$ or $\frac{239.5-246}{\sqrt{246}}$. Condone incorrect continuity correction for M1.
	= -0.383 (3sf)	A1	= -0.414
	$\Phi(-0.383) = 1 - \Phi(0.383)$	M1	$\Phi(-0.414) = 1 - \Phi(0.414)$
	= 0.351 (3sf)	A1	= 0.340 or 0.339
		6	