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9709/31

May/June 2024

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
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- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

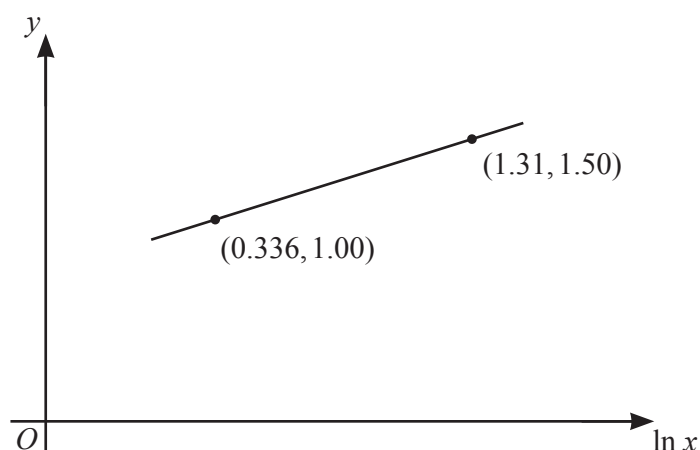
This document has **20** pages. Any blank pages are indicated.

- 1 Expand $(3+x)(1-2x)^{\frac{1}{2}}$ in ascending powers of x , up to and including the term in x^2 , simplifying the coefficients. [4]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- 2 Solve the equation $\ln(x-5) = 7 - \ln x$. Give your answer correct to 2 decimal places. [4]

[illegible]



The variables x and y satisfy the equation $a^y = bx$, where a and b are constants. The graph of y against $\ln x$ is a straight line passing through the points $(0.336, 1.00)$ and $(1.31, 1.50)$, as shown in the diagram.

Find the values of a and b . Give each value correct to the nearest integer. [4]

[illegible]

4 The complex number u is given by $u = -1 - i\sqrt{3}$.

- (a) Express u in the form $r(\cos \theta + i \sin \theta)$, where $r > 0$ and $-\pi < \theta \leq \pi$. Give the exact values of r and θ . [2]

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The complex number v is given by $v = 5\left(\cos \frac{1}{6}\pi + i \sin \frac{1}{6}\pi\right)$.

- (b) Express the complex number $\frac{v}{u}$ in the form $re^{i\theta}$ where $r > 0$ and $-\pi < \theta \leq \pi$. [2]

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- 5** The equation of a curve is $y = \frac{e^{\sin x}}{\cos^2 x}$ for $0 \leq x \leq 2\pi$.

Find $\frac{dy}{dx}$ and hence find the x -coordinates of the stationary points of the curve. [7]

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page, typical of notebook or legal stationery. There are no margins, text, or other markings on the page.

- 6 (a)** By sketching a suitable pair of graphs, show that the equation $\operatorname{cosec} \frac{1}{2}x = e^x - 3$ has exactly one root, denoted by α , in the interval $0 < x < \pi$. [2]

- (b) Verify by calculation that α lies between 1 and 2. [2]

[illegible]

- (c) Show that if a sequence of values in the interval $0 < x < \pi$ given by the iterative formula

$$x_{n+1} = \ln(\operatorname{cosec} \frac{1}{2}x_n + 3)$$

converges, then it converges to α .

[1]

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- (d) Use this iterative formula with an initial value of 1.4 to determine α correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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- (e) State the minimum number of calculated iterations needed with this initial value to determine α correct to 2 decimal places. [1]

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- 7 (a) On a single Argand diagram sketch the loci given by the equations $|z - 3 + 2i| = 2$ and $|w - 3 + 2i| = |w + 3 - 4i|$ where z and w are complex numbers. [4]

- (b)** Hence find the least value of $|z - w|$ for points on these loci. Give your answer in an exact form. [2]

This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

Give your answer in the form $a+b\sqrt{2}$ where a and b are rational numbers to be determined. [7]

[illegible]

- 9 The equations of two straight lines l_1 and l_2 are

$$l_1: \mathbf{r} = \mathbf{i} - 2\mathbf{j} + 3\mathbf{k} + \lambda(2\mathbf{i} - \mathbf{j} + a\mathbf{k}) \quad \text{and} \quad l_2: \mathbf{r} = -\mathbf{i} - \mathbf{j} - \mathbf{k} + \mu(3\mathbf{i} - 2\mathbf{j} - 2\mathbf{k}),$$

where a is a constant.

The lines l_1 and l_2 are perpendicular.

- (a) Show that $a = 4$. [1]

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The lines l_1 and l_2 also intersect.

- (b) Find the position vector of the point of intersection. [4]

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The point A has position vector $-5\mathbf{i} + \mathbf{j} - 9\mathbf{k}$.

- (c) Show that A lies on l_1 . [2]

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page, providing a template for handwriting practice or general writing. There are no margins, text, or other markings on the page.

The point B is the image of A after a reflection in the line l_2 .

- (d)** Find the position vector of B . [2]

[illegible]

10 (a) Given that $2x = \tan y$, show that $\frac{dy}{dx} = \frac{2}{1+4x^2}$. [3]

[illegible]

(b) Hence find the exact value of $\int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} x \tan^{-1}(2x) dx$. [7]

[illegible]

- 11** In a field there are 300 plants of a certain species, all of which can be infected by a particular disease. At time t after the first plant is infected there are x infected plants. The rate of change of x is proportional to the product of the number of plants infected and the number of plants that are **not** yet infected. The variables x and t are treated as continuous, and it is given that $\frac{dx}{dt} = 0.2$ and $x = 1$ when $t = 0$.

(a) Show that x and t satisfy the differential equation

$$1495 \frac{dx}{dt} = x(300 - x). \quad [2]$$

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(b) Using partial fractions, solve the differential equation and obtain an expression for t in terms of a single logarithm involving x . [9]

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[illegible]

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9709/32

May/June 2024

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

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This document has **20** pages. Any blank pages are indicated.

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- 1 (a) Sketch the graph of $y = |x - 2a|$, where a is a positive constant.

[1]

- (b) Solve the inequality $2x - 3a < |x - 2a|$.

[2]

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2 Express $\frac{6x^2-9x-16}{2x^2-5x-12}$ in partial fractions.

[5]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

3 The variables x and y satisfy the equation $a^{2y-1} = b^{x-y}$, where a and b are constants.

(a) Show that the graph of y against x is a straight line. [3]

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(b) Given that $a = b^3$, state the equation of the straight line in the form $y = px + q$, where p and q are rational numbers in their simplest form. [2]

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- 4 The equation of a curve is $ye^{2x} + y^2e^x = 6$.

Find the gradient of the curve at the point where $y = 1$.

[6]

[illegible]

- 5 (a)** It is given that the equation $e^{2x} = 5 + \cos 3x$ has only one root.

Show by calculation that this root lies in the interval $0.7 < x < 0.8$. [2]

- (b)** Show that if a sequence of values in the interval $0.7 < x < 0.8$ given by the iterative formula

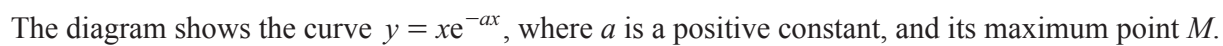
$$x_{n+1} = \frac{1}{2} \ln(5 + \cos 3x_n)$$

converges then it converges to the root of the equation in part (a). [1]

[illegible]

- (c) Use this iterative formula to determine the root correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

[illegible]



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- This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

(b) Find the exact value of $\int_0^{\frac{2}{a}} x e^{-ax} dx$.

[5]

[illegible]

7 (a) Show that $\cos^4 \theta - \sin^4 \theta \equiv \cos 2\theta$.

[3]

[illegible]

(b) Hence find the exact value of $\int_{-\frac{1}{8}\pi}^{\frac{1}{8}\pi} (\cos^4 \theta - \sin^4 \theta + 4 \sin^2 \theta \cos^2 \theta) d\theta$. [6]

[illegible]

- 8** The points A , B and C have position vectors $\overrightarrow{OA} = -2\mathbf{i} + \mathbf{j} + 4\mathbf{k}$, $\overrightarrow{OB} = 5\mathbf{i} + 2\mathbf{j}$ and $\overrightarrow{OC} = 8\mathbf{i} + 5\mathbf{j} - 3\mathbf{k}$, where O is the origin. The line l_1 passes through B and C .

- (a) Find a vector equation for l_1 . [3]

[illegible]

The line l_2 has equation $\mathbf{r} = -2\mathbf{i} + \mathbf{j} + 4\mathbf{k} + \mu(3\mathbf{i} + \mathbf{j} - 2\mathbf{k})$.

- (b)** Find the coordinates of the point of intersection of l_1 and l_2 . [4]

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- (c) The point D on l_2 is such that $AB = BD$.

Find the position vector of D .

[5]

[illegible]

9 The complex numbers z and ω are defined by $z = 1 - i$ and $\omega = -3 + 3\sqrt{3}i$.

- (a) Express $z\omega$ in the form $a + bi$, where a and b are real and in exact surd form. [1]

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- (b) Express z and ω in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leq \pi$. Give the exact values of r and θ in each case. [4]

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- (c) On an Argand diagram, the points representing ω and $z\omega$ are A and B respectively.

Prove that OAB is an isosceles right-angled triangle, where O is the origin. [2]

- (d) Using your answers to part (b), prove that $\tan \frac{5}{12}\pi = \frac{\sqrt{3}+1}{\sqrt{3}-1}$. [3]

- 10 (a)** By writing $y = \sec^3 \theta$ as $\frac{1}{\cos^3 \theta}$, show that $\frac{dy}{d\theta} = 3 \sin \theta \sec^4 \theta$. [2]

[illegible]

- (b)** The variables x and θ satisfy the differential equation

$$(x^2 + 9) \sin \theta \frac{d\theta}{dx} = (x + 3) \cos^4 \theta.$$

It is given that $x = 3$ when $\theta = \frac{1}{3}\pi$.

Solve the differential equation to find the value of $\cos \theta$ when $x = 0$. Give your answer correct to 3 significant figures. [8]

This image shows a full page of primary-ruled handwriting practice paper. It contains ten identical horizontal rows. Each row is defined by three parallel dashed lines: a solid top line, a dashed midline, and a solid bottom line. The rows are evenly spaced across the entire page, leaving margins at the top and bottom. There is no text or other markings on the page.

This image shows a full page of a worksheet designed for handwriting practice. It consists of approximately 20 horizontal rows. Each row is defined by two parallel dotted lines, creating a series of uniform gaps for writing. The entire page is otherwise blank, with no text or other markings.

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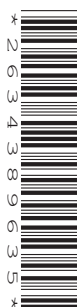
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**MATHEMATICS****9709/33**

Paper 3 Pure Mathematics 3

May/June 2024**1 hour 50 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
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INFORMATION

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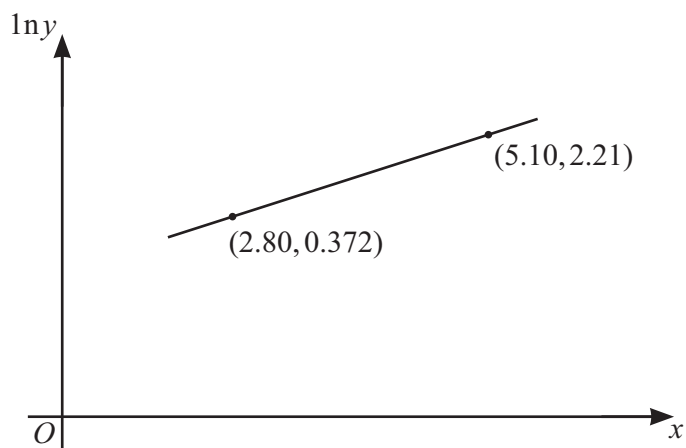


2 Find the exact coordinates of the stationary point of the curve $y = e^{2x} \sin 2x$ for $0 \leq x \leq \frac{1}{2}\pi$. [5]

This image shows a full page of white paper with horizontal dotted lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



4



The variables x and y satisfy the equation $ky = e^{cx}$, where k and c are constants. The graph of $\ln y$ against x is a straight line passing through the points $(2.80, 0.372)$ and $(5.10, 2.21)$, as shown in the diagram.

Find the values of k and c . Give each value correct to 2 significant figures.

[4]

This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

[illegible]



7 Let $f(x) = 8x^3 + 54x^2 - 17x - 21$.

(a) Show that $x + 7$ is a factor of $f(x)$. [1]

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(b) Find the quotient when $f(x)$ is divided by $x + 7$. [2]

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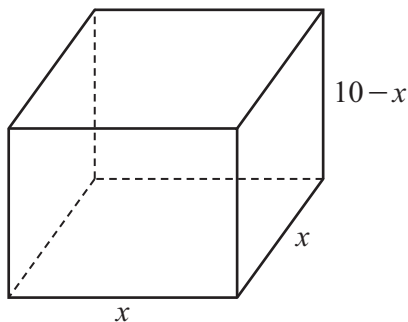


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- This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the entire width of the page. There are no margins, text, or other markings present.

[illegible]



A container in the shape of a cuboid has a square base of side x and a height of $(10 - x)$. It is given that x varies with time, t , where $t > 0$. The container decreases in volume at a rate which is inversely proportional to t .

When $t = \frac{1}{10}$, $x = \frac{1}{2}$ and the rate of decrease of x is $\frac{20}{37}$.

- (a)** Show that x and t satisfy the differential equation

$$\frac{dx}{dt} = \frac{-1}{2t(20x - 3x^2)}. \quad [5]$$

[illegible]



[6]

[illegible]

$$\mathbf{r} = \mathbf{i} + \mathbf{j} + 2a\mathbf{k} + \lambda(3\mathbf{i} + 4\mathbf{j} + a\mathbf{k}) \quad \text{and} \quad \mathbf{r} = -3\mathbf{i} - \mathbf{j} + 4\mathbf{k} + \mu(-\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}),$$

(a) Given that the acute angle between the directions of these lines is $\frac{1}{4}\pi$, find the possible values of a . [6]

[illegible]



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This image shows a full page of a worksheet designed for handwriting practice. It consists of approximately 20 evenly spaced horizontal dotted lines across the entire page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.

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[9]

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[illegible]

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