

Cambridge International A Level

MATHEMATICS**9709/31**

Paper 3 Pure Mathematics 3

May/June 2024**MARK SCHEME**Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

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This document consists of **15** printed pages.

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These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

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Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

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- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

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The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1	State correct unsimplified first two terms of the expansion of $(1-2x)^{\frac{1}{2}}$, e.g. $1 + \frac{1}{2}(-2x)$	B1	Symbolic coefficients are not sufficient. $1-x$
	State correct unsimplified term in x^2 , e.g. $\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)(-2x)^2}{2!}$	B1	Symbolic coefficients are not sufficient. $-\frac{1}{2}x^2$
	Obtain sufficient terms of the product of $(3+x)$ and the expansion up to the term in x^2	M1	
	Obtain final answer $3 - 2x - \frac{5}{2}x^2$	A1	
		4	

Question	Answer	Marks	Guidance
2	Use law of logarithm of a product (or quotient) on correct terms	*M1	
	Use correct method to eliminate logarithm	DM1	
	Obtain a correct quadratic in x , e.g. $x^2 - 5x - e^7 = 0$ (allow decimals)	A1	
	Obtain answer $x = 35.71$ only	A1	
		4	

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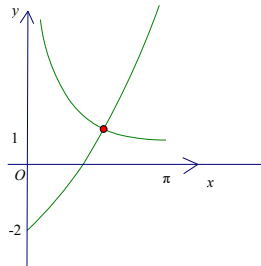
Question	Answer	Marks	Guidance
3	State or imply that $y \ln a = \ln b + \ln x$	B1	
	Carry out a completely correct method for finding $\ln a$ or $\ln b$	M1	E.g., from $\ln a = \ln b + 0.336$ $1.5 \ln a = \ln b + 1.31$.
	Obtain value $a = 7$	A1	
	Obtain value $b = 5$	A1	
		4	

Question	Answer	Marks	Guidance
4(a)	State or imply $r = 2$	B1	
	State or imply $\theta = -\frac{2}{3}\pi$	B1	
		2	
4(b)	State or imply $r = \frac{5}{2}$	B1FT	FT $\frac{5}{their\ 2}$.
	State or imply $\theta = \frac{5}{6}\pi$	B1FT	FT $\frac{1}{6}\pi - their - \frac{2}{3}\pi$.
		2	

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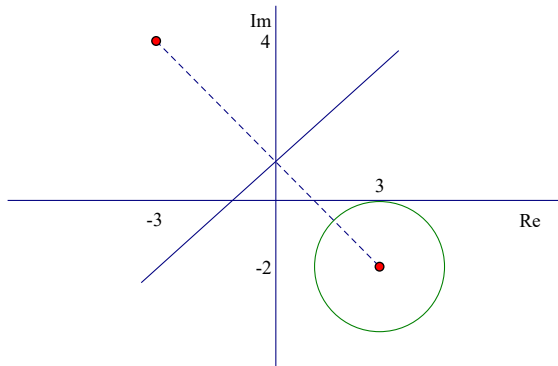
Question	Answer	Marks	Guidance
5	Use correct quotient (or product) rule	*M1	
	Obtain correct derivative $\frac{e^{\sin x} \cos^3 x - (-2e^{\sin x} \sin x \cos x)}{(\cos^2 x)^2}$ or equivalent	A1	
	Equate numerator to zero	DM1	
	Obtain equation in one unknown	DM1	E.g. $\sin^2 x - 2\sin x - 1 = 0$.
	Solve a 3 term quadratic in $\sin x$ to find a value for x	M1	
	Obtain a correct solution to the quadratic equation, e.g. 3.57°	A1	At least 3sf.
	Obtain a further correct solution, e.g. $x = 5.86^\circ$ and no others in the interval	A1FT	At least 3sf. FT $3\pi - \text{their } 3.57$.
	Alternative Method for the first 3 marks:		
	Take logarithms of both sides and simplify	(*M1)	$\ln y = \sin x - 2 \ln \cos x$ or equivalent.
	Obtain $\frac{1}{y} \frac{dy}{dx} = \cos x + 2 \frac{\sin x}{\cos x}$	(A1)	Or equivalent.
	Equate $\frac{dy}{dx}$ to zero	(DM1)	
	Continue as for the original		
		7	

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Question	Answer	Marks	Guidance
6(a)	Sketch a relevant graph, e.g. $y = e^x - 3$ Correct shape, correct vertical intercept	B1	
	Sketch a second relevant graph, e.g. $y = \operatorname{cosec} \frac{x}{2}$ (correct shape, minimum above the axis) and justify the given statement. Need to mark intersection with a dot, a cross, or say root at points of intersection, or equivalent	B1	
		2	
6(b)	Calculate the values of a relevant expression or pair of expressions at $x = 1$ and $x = 2$	M1	Use of degrees is M0.
	Complete the argument correctly with correct calculated values	A1	E.g. $-0.282 < 2.086$, $4.389 > 1.188$ $1 < 1.626$, $2 > 1.432$ $2.36 > 0$, $-3.2 < 0$ At least 2sf. Condone truncation.
		2	
6(c)	State $x = \ln \left(\operatorname{cosec} \frac{1}{2}x + 3 \right)$ and rearrange to the given equation $\operatorname{cosec} \frac{x}{2} = e^x - 3$	B1	AG. Or vice versa and obtain the iterative formula.
		1	
6(d)	Use the iterative formula correctly at least twice	M1	Use of degrees in M0 (might see 1.38....).
	Obtain final answer 1.50	A1	
	Show sufficient iterations to 4 dp to justify 1.50 to 4 dp or show there is a sign change in the interval (1.495, 1.505)	A1	1.5156, 1.4940, 1.4978, 1.4971.
		3	

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Question	Answer	Marks	Guidance
6(e)	4	B1	
		1	

Question	Answer	Marks	Guidance
7(a)	Show a circle centre $(3, -2)$	B1	
	Show a circle with radius 2 FT centre not at the origin	B1FT	
	Show the point representing $(-3, 4)$ or the midpoint $(0, 1)$	B1	
	Show the perpendicular bisector of the line joining $(-3, 4)$ and centre of the circle FT is on the position of $(-3, 4)$ and centre of the circle	B1FT	
		4	
7(b)	Carry out a correct method for finding the least value of $ z - w $	M1	(Distance $(3, -2)$ to $(0, 1)$) $- 2$.
	Obtain answer $\sqrt{18} - 2$ or $3\sqrt{2} - 2$	A1	
		2	

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Question	Answer	Marks	Guidance
8	State or imply $du = -\cos x \, dx$	B1	
	Use $\sin 2x = 2\sin x \cos x$ and write the integral in terms of u	*M1	
	Obtain $\pm 2 \int \frac{(1-u)}{\sqrt{u}} \, du$ or equivalent	A1	
	Integrate correctly to obtain $au^{\frac{1}{2}} + bu^{\frac{3}{2}}$	DM1	
	Obtain correct $-4u^{\frac{1}{2}} + \frac{4}{3}u^{\frac{3}{2}}$	A1	
	Correctly use limits $u = 2$ and 0 in an expression of the form $au^{\frac{1}{2}} + bu^{\frac{3}{2}}$ OR limits $x = \frac{3}{2}\pi$ and $\frac{1}{2}\pi$ in an expression of the form $a(1 - \sin x)^{\frac{1}{2}} + b(1 - \sin x)^{\frac{3}{2}}$	DM1	
	Obtain $\frac{8}{3} - \frac{4}{3}\sqrt{2}$	A1	
		7	

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Question	Answer	Marks	Guidance
9(a)	Carry out correct process for evaluating the scalar product of direction vectors, equate the result to zero and obtain given value of $a = 4$	B1	E.g. $2(3) + (-1)(-2) + a(-2) = 0$.
		1	
9(b)	Express general point of at least one line correctly in component form, i.e. $(1 + 2\lambda, -2 - \lambda, 3 + 4\lambda)$ or $(-1 + 3\mu, -1 - 2\mu, -1 - 2\mu)$	B1	The third component could be implied by a correct final answer.
	Equate at least two pairs of corresponding components and solve for λ or for μ	M1	
	Obtain $\lambda = -1$ or $\mu = 0$	A1	
	Obtain position vector of point of intersection is $-\mathbf{i} - \mathbf{j} - \mathbf{k}$	A1	
		4	
9(c)	Equate one component of l_1 to matching component of A and solve to find λ	M1	
	Use $\lambda = -3$ in equation of l_1 and show this gives position vector of A	A1	AG Or show $\lambda = -3$ for all three components equated.
		2	
9(d)	Method to find position vector of B	M1	E.g. $\pm 2 \times \text{their } (-\mathbf{i} - \mathbf{j} - \mathbf{k}) \pm (-5\mathbf{i} + \mathbf{j} - 9\mathbf{k})$
	Obtain position vector of B is $3\mathbf{i} - 3\mathbf{j} + 7\mathbf{k}$	A1	
		2	

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Question	Answer	Marks	Guidance
10(a)	Obtain $2 = \sec^2 y \frac{dy}{dx}$ or equivalent	B1	E.g. $2 \frac{dx}{dy} = \sec^2 y$ by differentiation with respect to y .
	Use $\sec^2 y = 1 + \tan^2 y$	M1	
	Replace $\tan y$ with $2x$ and rearrange to obtain given answer $\frac{dy}{dx} = \frac{2}{1+4x^2}$	A1	
		3	
10(b)	Integrate by parts and reach $ax^2 \tan^{-1} 2x + b \int \frac{x^2}{1+4x^2} dx$	*M1	
	Obtain $\frac{1}{2}x^2 \tan^{-1} 2x - \int \frac{x^2}{1+4x^2} dx$	A1	OE
	Reduce integral to expression of the form $\int m + \frac{n}{1+4x^2} dx$	M1	
	Complete integration and reach $px^2 \tan^{-1} 2x + qx + r \tan^{-1} 2x$	M1	
	Obtain $\frac{1}{2}x^2 \tan^{-1} 2x - \frac{1}{4}x + \frac{1}{8} \tan^{-1} 2x$	A1	OE
	Use limits of $x = \frac{1}{2}$ and $x = \frac{1}{2}\sqrt{3}$ in the correct order, having integrated twice	DM1	
	Obtain answer $\frac{5}{48}\pi - \frac{1}{8}\sqrt{3} + \frac{1}{8}$ or exact equivalent	A1	
		7	

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Question	Answer	Marks	Guidance
11(a)	State or imply equation of the form $\frac{dx}{dt} = kx(300 - x)$ and use $\frac{dx}{dt} = 0.2$ and $x = 1$	M1	M0 for verification.
	Obtain $k = \frac{1}{1495}$ and rearrange to the given answer	A1	$1495 \frac{dx}{dt} = x(300 - x).$
		2	

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Question	Answer	Marks	Guidance
11(b)	Separate variables correctly	B1	$\int \frac{1}{x(300-x)} dx = \int \frac{1}{1495} dt$
	Correct integration of t term	B1	E.g. obtain t or $\frac{t}{1495}$.
	State or imply partial fractions of the form $\frac{A}{x} + \frac{B}{300-x}$	B1	
	Correct method to find A or B	M1	$A = \frac{1}{300}$ and $B = \frac{1}{300}$. May see $A = B = \frac{1495}{300} = \frac{299}{60}$.
	Obtain terms $\frac{1495}{300} \ln x - \frac{1495}{300} \ln(300-x)$	A1	OE. May see $\frac{1}{300} \ln x - \frac{1}{300} \ln(300-x)$.
	Use $t = 0, x = 1$ to evaluate a constant or as limits in a solution containing terms of the form $\ln x, \ln(300-x)$ and t .	M1	
	Obtain correct answer in any form	A1	E.g. $\frac{1495}{300} [\ln x - \ln(300-x)] = t - \frac{1495}{300} \ln 299$.
	Use law of logarithms twice to obtain an expression for t	M1	
	Obtain final answer $t = \frac{299}{60} \ln \frac{299x}{300-x}$ or equivalent single logarithm	A1	
		9	

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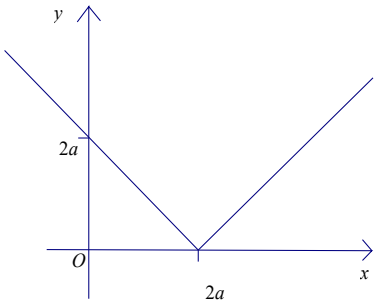
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Question	Answer	Marks	Guidance
1(a)		B1	Correct shape, roughly symmetrical. Both sections should be solid straight lines. If not drawn with a ruler the intention must be clear. Allow construction lines if dashed or clearly fainter. $2a$ marked on each axis (must be $2a$, not just 2). Needs to extend into negative x. If a is given a value, then B0. Ignore $y = 2x - 3a$ if seen.
		1	
1(b)	Solve linear equation or inequality to obtain critical value $x = \frac{5}{3}a$ or exact equivalent.	B1	Ignore $x = a$ if seen.
	Obtain $x < \frac{5}{3}a$ or exact equivalent	B1	Accept $x < \frac{10}{6}a$ or $(-\infty, \frac{5}{3}a)$. Must be strict inequality. Need a clear final solution: $x > a$ or $x < a$ must be rejected if seen as part of the working. Rejection can be implied, e.g. if only the correct inequality is underlined. B0 B0 if a is given a value.
Alternative Method for Question 1(b)			
	Solve quadratic equation $(2x - 3a)^2 = (x - 2a)^2$ to obtain critical value $x = \frac{5}{3}a$ or exact equivalent	(B1)	$(3x^2 - 8ax + 5a^2 = 0)$ Ignore $x = a$ if seen.
	Obtain $x < \frac{5}{3}a$ or exact equivalent	(B1)	Accept $x < \frac{10}{6}a$ or $(-\infty, \frac{5}{3}a)$. Must be strict inequality. Need a clear final solution: $x > a$ or $x < a$ must be rejected if seen as part of the working. Rejection can be implied, e.g. if only the correct inequality is underlined. B0 B0 if a is given a value.
		2	

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Question	Answer	Marks	Guidance
2	State or imply the form $A + \frac{B}{2x+3} + \frac{C}{x-4}$	B1	$\frac{Dx+E}{2x+3} + \frac{F}{x-4}$ and $\frac{P}{2x+3} + \frac{Qx+R}{x-4}$ are also valid.
	Use a correct method for finding a constant	M1	SC: If score B0, they can score M1 A1 for one correct constant. B0 M1 A0 available if they substitute two values to form simultaneous equations but get an incorrect answer, or they substitute one value and make an arithmetic error.
	Obtain one of $A = 3$, $B = -2$ and $C = 4$	A1	SC: If the horizontal equation is correct apart from an incorrect value for A , the other A marks may be available.
	Obtain a second value	A1	SC: If denominator factorised as $(x + \frac{3}{2})(x - 4)$ can score a maximum of B0 M1 A1 A1 A0 for a split involving 3 terms.
	Obtain a third value	A1	ISW Statement of the final split is not required.

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Question	Answer	Marks	Guidance
2	Alternative method for Question 2		
	Divide numerator by denominator	(M1)	
	Obtain $3 \left(+ \frac{Px+Q}{2x^2-5x-12} \right)$	(A1)	$\left(3 + \frac{6x+20}{(2x+3)(x-4)} \right)$
	State or imply the form $\frac{Px+Q}{2x^2-5x-12} = \frac{D}{2x+3} + \frac{E}{x-4}$	(B1)	Must deal with the 3 separately or include it correctly on both sides in their split.
	Obtain one of $D = -2$ and $E = 4$	(A1)	SC: If denominator factorised as $\left(x + \frac{3}{2}\right)(x-4)$, then can score a maximum of B0 M1 A1 A1 A0 for a split involving three terms.
	Obtain a second value	(A1)	ISW Statement of the final split is not required.
		5	

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Question	Answer	Marks	Guidance
3(a)	Use logarithms to obtain a correct expression without powers e.g. $(2y-1)\ln a = (x-y)\ln b$	B1	Could use logs to any base e.g. $2y-1 = (x-y)\log_a b$. Do not condone missing brackets unless recovered later.
	Separate terms and factorise to obtain $y(2\ln a + \ln b) = x\ln b + \ln a$	B1	Or equivalent, e.g. $y = x \frac{\ln b}{\ln a^2 b} + \frac{\ln a}{\ln a^2 b}$ or $y(2 + \log_a b) = x \log_a b + 1$.
	Clear explanation of linear form. From correct work only.	B1	E.g. equation matches the linear form $y = mx + c$ or $py = qx + r$. Condone if they compare with $y = mx + c$, but do not actually state that it must therefore be a straight line. Stating “this is a linear equation” without comparing to a relevant standard form scores B0. B0 if they have $m = \dots$ and $c = \dots$ correct but never actually mention $y = mx + c$.
		3	
3(b)	Use $a = b^3$ and log laws to simplify their equation	M1	$\left[y = x \frac{\ln b}{\ln b^7} + \frac{\ln b^3}{\ln b^7} \right]$ Denominator reduced to a single log term.
	Obtain $y = \frac{1}{7}x + \frac{3}{7}$	A1	Accept $y = \frac{x}{7} + \frac{3}{7}$ but not $y = \frac{x+3}{7}$.
	Alternative method for Question 3(b)		
	Use $a = b^3$ to obtain $b^{3(2y-1)} = b^{x-y}$ or equivalent	(M1)	Or $\log_a b = \frac{1}{3}$.
	Obtain $y = \frac{1}{7}x + \frac{3}{7}$	(A1)	Accept $y = \frac{x}{7} + \frac{3}{7}$ but not $y = \frac{x+3}{7}$.
		2	

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Question	Answer	Marks	Guidance
4	Substitute $y = 1$ and obtain $e^x = a$, where $a > 0$	M1	Must come from a quadratic in e^x . $(e^{2x} + e^x - 6 = 0)$ Ignore any negative solution.
	Obtain $e^x = 2$ only	A1	Or equivalent e.g. $x = \ln 2$. Condone $x = 0.693\dots$
	State or imply $\frac{d}{dx}(ye^{2x}) = 2ye^{2x} + e^{2x} \frac{dy}{dx}$	B1	Accept y' for $\frac{dy}{dx}$.
	State or imply $\frac{d}{dx}(y^2e^x) = y^2e^x + 2ye^x \frac{dy}{dx}$	B1	Accept y' for $\frac{dy}{dx}$.
	Differentiate RHS of given equation to obtain zero (could be implied by subsequent work), substitute for x and y and obtain $\frac{dy}{dx} = \dots$	M1	Independent. $\left[2 \times 4 + 4 \frac{dy}{dx} + 2 + 4 \frac{dy}{dx} = 0 \right]$ NB: Could also rearrange to obtain $\frac{dy}{dx}$ then substitute. Both steps are needed for M1.
	Obtain $-\frac{5}{4}$ or -1.25	A1	Correct answer from correct working only Accept $-\frac{10}{8}$. -1.2499 is A0.

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Question	Answer	Marks	Guidance
4	Alternative method for Question 4: Dividing through by e^x		
	Substitute $y = 1$ and obtain $e^x = a$, where $a > 0$	(M1)	Must come from a quadratic in e^x . $(e^{2x} + e^x - 6 = 0)$ Ignore any negative solution.
	Obtain $e^x = 2$ only	(A1)	Or equivalent e.g. $x = \ln 2$. Condone $x = 0.693\dots$
	State or imply $\frac{d}{dx}(y^2) = 2y \frac{dy}{dx}$	(B1)	Accept y' for $\frac{dy}{dx}$.
	State or imply $\frac{d}{dx}(ye^x) = ye^x + e^x \frac{dy}{dx}$	(B1)	Accept y' for $\frac{dy}{dx}$.
	Differentiate RHS of given equation to obtain $-6e^{-x}$, substitute for x and y and obtain $\frac{dy}{dx} = \dots$	(M1)	Independent. $\left[1 \times 2 + 2 \frac{dy}{dx} + 2 \frac{dy}{dx} = -\frac{6}{2} \right]$ NB: Could also rearrange to obtain $\frac{dy}{dx}$ then substitute. Both steps are needed for M1.
	Obtain $-\frac{5}{4}$ or -1.25	(A1)	Correct answer from correct working only Accept $-\frac{10}{8}$. -1.2499 is A0.
		6	

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Question	Answer	Marks	Guidance
5(a)	Calculate the value of a relevant expression or values of a pair of expressions at $x = 0.7$ and $x = 0.8$	M1	Allow if working with a smaller interval, e.g. (0.72, 0.78). Need all relevant values but condone one error. Pairings must be clear for solutions involving four values (do not accept embedded values). M0 if working in degrees e.g -1.94..., 1.04...
	Complete the argument correctly with correct calculated values. (can be using the equation in the rubric or the equation in (b) or equivalent)	A1	E.g. $4.95... > 4.26...$ and $4.06... < 4.49...$ $-0.439... < 0$, $0.690... > 0$ $1.4 < 1.5029...$, $1.6 > 1.449...$ $1.1 > 1$, $0.86 < 1$ $0.0515 > 0$, $-0.075 < 0$. Allow values rounded or truncated to 2sf.
		2	
5(b)	State $2x = \ln(5 + \cos 3x)$ and take exponential of both sides to obtain $e^{2x} = 5 + \cos 3x$	B1	Given answer requires fully correct working or work vice versa. If working in reverse, must get to the iterative formula, including subscripts.
		1	
5(c)	Use the iterative process correctly at least once	M1	M0 if working in degrees (e.g. values heading for 0.89...).
	Obtain final answer 0.740	A1	
	Show sufficient iterations to at least 5dp to justify 0.740 to 3dp, or show that there is a sign change in the interval (0.7395, 0.7405)	A1	E.g. 0.7, 0.75150, 0.73719, 0.74105, 0.74000, 0.74028 0.75, 0.73759, 0.74094, 0.74003, 0.74028 0.8, 0.72494, 0.74443, 0.73909, 0.74053, 0.74014, 0.74025 Allow recovery. Allow truncation or rounding and condone small differences in the final decimal place.
		3	

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Question	Answer	Marks	Guidance
6(a)	Use correct product rule	*M1	Or equivalent. Condone incorrect chain rule. M0 if a value is used for a (not equivalent work).
	Obtain correct derivative	A1	E.g. $\frac{dy}{dx} = -axe^{-ax} + e^{-ax}$
	Equate derivative to zero and solve for x	DM1	
	Obtain $x = \frac{1}{a}$, $y = \frac{1}{ae}$	A1	ISW Or exact equivalent.
		4	
6(b)	Use integration by parts to obtain $pxe^{-ax} + q \int e^{-ax} dx$	*M1	Condone sign error in parts formula and omission of dx . M0 if a value is used for a (not equivalent work).
	Obtain $-\frac{1}{a}xe^{-ax} + \frac{1}{a} \int e^{-ax} dx$	A1	OE
	Complete integration to obtain $-\frac{1}{a}xe^{-ax} - \frac{1}{a^2}e^{-ax}$	A1	OE
	Correct use of limits 0 and $\frac{2}{a}$ in an expression of the form $rx e^{-ax} + se^{-ax}$	DM1	$\left(\frac{-2}{a^2}e^{-2} - \frac{1}{a^2}e^{-2} + 0 + \frac{1}{a^2} \right)$
	Obtain $\frac{1}{a^2}(1 - 3e^{-2})$	A1	ISW Or simplified 2-term equivalent, e.g. $\frac{e^2 - 3}{a^2 e^2}$.
		5	

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Question	Answer	Marks	Guidance
7(a)	Factorise LHS using difference of 2 squares	*M1	$((\cos^2 \theta - \sin^2 \theta)(\cos^2 \theta + \sin^2 \theta))$
	Simplify	DM1	$\cos^2 \theta + \sin^2 \theta = 1$ must be seen or implied, e.g. $(\cos^2 \theta - \sin^2 \theta)(\cos^2 \theta + \sin^2 \theta) = (\cos^2 \theta - \sin^2 \theta)$.
	Obtain $\cos^4 \theta - \sin^4 \theta \equiv \cos 2\theta$ from correct working	A1	AG
	Alternative Method for Question 7(a)		
	Use of correct rearrangements of double angle formulae	(*M1)	E.g. $\left(\frac{1 + \cos 2\theta}{2}\right)^2 - \left(\frac{1 - \cos 2\theta}{2}\right)^2$ Only condone $\left(\frac{1 + \cos 2\theta}{2}\right)^2 - \left(\frac{\cos 2\theta - 1}{2}\right)^2$ if correct expression for $\sin^2 \theta$ seen.
	Expand and simplify	(DM1)	Collect like terms. Condone recovery from missing brackets.
	Obtain $\cos^4 \theta - \sin^4 \theta \equiv \cos 2\theta$ from correct working	(A1)	AG
	Alternative Method 2 for Question 7(a)		
	Correct use of Pythagoras	(*M1)	E.g. $(1 - \sin^2 \theta)^2 - \sin^4 \theta$ or $\cos^2 \theta(1 - \sin^2 \theta) - \sin^2 \theta(1 - \cos^2 \theta)$
	Expand and simplify	(DM1)	Condone recovery from missing brackets.
	Obtain $\cos^4 \theta - \sin^4 \theta \equiv \cos 2\theta$ from correct working	(A1)	AG
		3	

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Question	Answer	Marks	Guidance
7(b)	Use part (a) and correct double angle formula to obtain expression involving $\int \sin^2 2\theta d\theta$ or $\int \cos^2 2\theta d\theta$	M1	$\int \cos^4 \theta - \sin^4 \theta + 4\sin^2 \theta \cos^2 \theta d\theta = \int \cos 2\theta + \sin^2 2\theta d\theta$ Allow BOD for $2\sin^2 2\theta$ if $\sin 2\theta = 2\sin \theta \cos \theta$ seen.
	$\int \cos 2\theta d\theta = \frac{1}{2} \sin 2\theta$	B1	Seen or implied.
	Use of correct double angle formula on second part of the integral to obtain a form that can be integrated directly	M1	e.g. $\int \sin^2 2\theta d\theta = \int \frac{1 - \cos 4\theta}{2} d\theta$
	Obtain $\frac{1}{2}\theta - \frac{1}{8}\sin 4\theta$	A1	Condone a mixture of x and θ .
	Correct use of limits $\pm \frac{\pi}{8}$ in an expression of the form $p\theta + q\sin 2\theta + r\sin 4\theta$ and evaluate the trig	M1	$\left(2\left(\frac{1}{2} \times \frac{1}{\sqrt{2}} + \frac{\pi}{16} - \frac{1}{8}\right)\right)$
	Obtain $\frac{1}{2}\sqrt{2} + \frac{1}{8}\pi - \frac{1}{4}$	A1	ISW Or exact equivalent from exact working.
		6	

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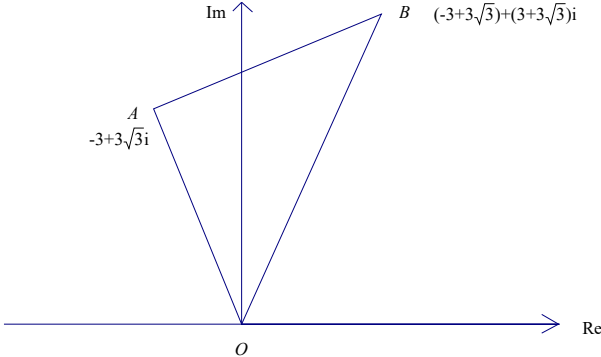
Question	Answer	Marks	Guidance
8(a)	Correct direction vector seen or implied ($\overrightarrow{BC} = 3\mathbf{i} + 3\mathbf{j} - 3\mathbf{k}$)	B1	Condone $\overrightarrow{BC} = -3\mathbf{i} - 3\mathbf{j} + 3\mathbf{k}$.
	Use a correct method to form a vector equation	M1	Allow for the RHS with no LHS.
	Obtain $\mathbf{r} = 5\mathbf{i} + 2\mathbf{j} + \lambda(\mathbf{i} + \mathbf{j} - \mathbf{k})$	A1	ISW Must have $\mathbf{r} = \dots$ or $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \dots$, not $l_1 = \dots$ Or, equivalent vector form, e.g. $\mathbf{r} = 8\mathbf{i} + 5\mathbf{j} - 3\mathbf{k} + \alpha(\mathbf{i} + \mathbf{j} - \mathbf{k})$ or $\mathbf{r} = 5\mathbf{i} + 2\mathbf{j} + \lambda(3\mathbf{i} + 3\mathbf{j} - 3\mathbf{k})$. Condone a column vector with $\mathbf{i}, \mathbf{j}, \mathbf{k}$.
		3	
8(b)	Use components to form two relevant equations in 2 unknowns For their l_1 B0 if they use the same unknown for both lines.	B1FT	Two components of $\begin{pmatrix} 5 + \lambda \\ 2 + \lambda \\ -\lambda \end{pmatrix} = \begin{pmatrix} -2 + 3\mu \\ 1 + \mu \\ 4 - 2\mu \end{pmatrix}$ seen or implied.
	Solve 2 relevant equations in 2 unknowns for λ or μ	M1	For <i>their</i> l_1 .
	Obtain $\lambda = 2$ or $\mu = 3$	A1	Or equivalent e.g. using $\overrightarrow{BC}$ as direction vector gives $\lambda = \frac{2}{3}$.
	Obtain $(7, 4, -2)$ No need to check the third equation – the question implies that the lines intersect.	A1	Accept position vector. Condone a column vector with $\mathbf{i}, \mathbf{j}, \mathbf{k}$. SC: B1 M1 A1 A1 if one component of their line is incorrect but they do not use that component.
		4	

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Question	Answer	Marks	Guidance
8(c)	State $AB = \sqrt{7^2 + 1^2 + 4^2} (= \sqrt{66})$	B1	Or $(AB)^2 = 66$ Condone a sign error in $\overline{AB}$.
	State $\overline{BD}$ in component form	B1	$\begin{pmatrix} -7 + 3r \\ -1 + r \\ 4 - 2r \end{pmatrix}$ or equivalent.
	$AB = BD \Rightarrow (3r - 7)^2 + (r - 1)^2 + (-2r + 4)^2 = 66$ $(14r^2 - 60r = 0)$	M1	Or equivalent equation in one unknown for their AB and <i>their</i> $\overline{BD} \neq \overline{OD}$. If you never see a correct form and they go direct to $9r^2 + 49 + r^2 + 1 \dots$ then M0.
	$\Rightarrow r = \frac{30}{7}$	A1	Correct only. Ignore $r = 0$ if seen.
	$\overline{OD} = \frac{76}{7}\mathbf{i} + \frac{37}{7}\mathbf{j} - \frac{32}{7}\mathbf{k}$	A1	Must be a vector. Condone if also have $\overline{OD} = \overline{OA}$.
		5	

Question	Answer	Marks	Guidance
9(a)	State $z\omega = (-3 + 3\sqrt{3}) + (3 + 3\sqrt{3})\mathbf{i}$	B1	Or exact equivalent with real and imaginary parts collected. Need brackets around the coefficient of $\mathbf{i}$. Allow for $a =$, $b =$ stated correctly.
		1	

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Question	Answer	Marks	Guidance
9(b)	Obtain $ z = \sqrt{2}$	B1	
	Obtain $\arg z = -\frac{\pi}{4}$ final answer	B1	
	Obtain $ \omega = 6$	B1	
	Obtain $\arg \omega = \frac{2\pi}{3}$ final answer	B1	
		4	
9(c)			Note: The question does not require the diagram. If they use $\frac{5\pi}{12}$ they need to demonstrate where it comes from. Complex number equivalent to AB is $3\sqrt{3} + 3i$.
	Show $ OA = AB = 6$, hence isosceles	B1	One mark for 'isosceles' and one mark for 'right angle'.
	$\angle AOB = \arg \omega - \arg z = -\arg z = \frac{\pi}{4}$ hence third angle is a right angle	B1	There will be alternatives e.g. use of Pythagoras (ratio of lengths is $1:1:\sqrt{2}$), expressing each number in "vector" form and using scalar product or explaining the effect of multiplying by $1 - i$.
		2	

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Question	Answer	Marks	Guidance
9(d)	$\arg z\omega = \arg z + \arg \omega \left(= \frac{2\pi}{3} - \frac{\pi}{4} = \frac{5\pi}{12} \right)$	M1	For showing correct use of their angles from part (b). Must demonstrate where $\frac{5\pi}{12}$ comes from.
	$\arg z\omega = \tan^{-1} \frac{3+3\sqrt{3}}{-3+3\sqrt{3}}$	M1	Correct method for <i>their</i> $z\omega$ from part (a). Must link to point B on diagram or to $\arg z\omega$. Need to see $\tan^{-1} \frac{3+3\sqrt{3}}{-3+3\sqrt{3}}$ or $\tan \theta = \frac{3+3\sqrt{3}}{-3+3\sqrt{3}}$ and not just $\tan^{-1} \frac{1+\sqrt{3}}{-1+\sqrt{3}}$.
	$\Rightarrow \tan\left(\frac{5}{12}\pi\right) = \frac{\sqrt{3}+1}{\sqrt{3}-1}$	A1	Obtain given answer from full and correct working.
		3	

Question	Answer	Marks	Guidance
10(a)	Use of correct chain rule (and correct quotient rule) and $\cos^{-3} \theta$	M1	Obtain $k \times (\cos \theta)^{-4} \times \sin \theta$ or equivalent.
	$\frac{dy}{d\theta} = -3 \times -\sin \theta (\cos \theta)^{-4} = 3 \sin \theta \sec^4 \theta$ Must be expressed in the given form	A1	Obtain given answer from full and correct working (signs must be shown), but condone $\frac{d}{d\theta}(\sec^3 \theta) = \dots$ and $y'(\theta)$.
		2	

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Question	Answer	Marks	Guidance
10(b)	Separate variables: $\int \frac{\sin \theta}{\cos^4 \theta} d\theta = \int \frac{(x+3)}{(x^2+9)} dx$	B1	Or $\int \frac{3 \sin \theta}{\cos^4 \theta} d\theta = \int \frac{3x+9}{x^2+9} dx$. Condone missing integral signs or missing dx or dθ, but not both.
	Obtain $p \sec^3 \theta (+A)$	B1	Correct form, p any constant but not 0.
	Use $\int \frac{3x+9}{x^2+9} dx = \int \left(\frac{3x}{x^2+9} + \frac{9}{x^2+9} \right) dx$ and obtain $q \ln(x^2+9)$ or $r \tan^{-1} \frac{x}{3} (+C)$.	*M1	Might have one third of both sides. Alt: substitute $x = 3 \tan \phi$ to obtain $q \int 1 + \tan \phi d\phi$; condone if have θ in place of φ in this method.
	Obtain $q \ln(x^2+9)$ and $r \tan^{-1} \frac{x}{3} (+C)$	DM1	Obtain $q(\phi \mp \ln(\cos \phi))$ OE.
	Obtain $\sec^3 \theta = \frac{3}{2} \ln(x^2+9) + 3 \tan^{-1} \frac{x}{3} (+C)$ or equivalent	A1	Or might see a third of both sides. Must have 2 different variables.
	Use $\theta = \frac{1}{3}\pi, x = 3$ in an equation including $p \sec^3 \theta, q \ln(x^2+9)$ and $r \tan^{-1} \frac{x}{3}$ to evaluate the constant of integration	M1	Or as limits in a definite integral. Limits for φ are 0 and $\frac{1}{4}\pi$.
	Obtain constant = $8 - \frac{3}{2} \ln 18 - \frac{3}{4}\pi$	A1	OE, e.g. 1.308... to at least 3sf.
	Obtain $\cos \theta = 0.601$	A1	Accept AWRT 0.601.
		8	

Cambridge International A Level

MATHEMATICS**9709/33**

Paper 3 Pure Mathematics 3

May/June 2024**MARK SCHEME**Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2024 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

This document consists of **19** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

PUBLISHED**Mathematics-Specific Marking Principles**

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1	Use law of the logarithm of product or quotient on each side	*B1	Allow logs to any base, as well as decimals, throughout. $\ln 8^3 + \ln 8^{-6x}$ and $\ln 4 + \ln 5^{-2x}$. Allow for $\ln \frac{8^3}{4}$ and $\ln 8^{6x} - \ln 5^{2x}$. $(3 - 6x) \ln 8$ and $\ln 4 + \ln 5^{-2x}$ gains next DB1 as well.
	Use law of logarithm of a power involving x on ONE side, e.g. $\ln 8^3 + (-)6x \ln 8$ or $(3 - 6x) \ln 8$ or $(9 - 18x) \ln 2$ or $\ln 4 - 2x \ln 5$	DB1	SC If *B0 DB0, then allow B1 (1/4) for a correct logarithm law seen anywhere.
	Obtain a correct linear equation in x , e.g. $(3 - 6x) \ln 8 = (9 - 18x) \ln 2 = \ln 4 - 2x \ln 5$	B1	If in decimals, allow small errors in 2 nd and 3 rd dp.
	Obtain answer $x = 0.524$	B1	3dp required. No working scores 0/4 marks. After *B1 DB1 to correct answer with no more log working seen, then SC B1 for $x = 0.524$. Maximum 3/4 possible.
	Alternative Method for Question 1		
	Use laws of indices to get to $a = b^{\pm 2x}$ or $c^{\pm x}$ in a correct form so now only ONE log power law required	(B2)	$(8^3/4)$ and $(5/8^3)^{-2x}$ or $(5^2/8^6)^{-x}$ opposite sides or $(4/8^3)$ and $(8^3/5)^{-2x}$ or $(8^6/5^2)^{-x}$ opposite sides
	Obtain a correct linear equation in x , e.g. $\ln \frac{8^3}{4} = 2x \ln \frac{8^3}{5}$	(B1)	$-2x \ln(5/8^3)$ or $2x \ln(8^3/5)$ or $x \ln(8^6/5^2)$ or $-x \ln(5^2/8^6)$. SC: If B0 then allow B1 (1/4) for a correct term seen anywhere. If in decimals, allow small errors in 2 nd and 3 rd dp.
	Obtain answer $x = 0.524$	(B1)	3dp required. No working scores 0/4 marks. From the first line to correct answer with no log working seen, then B2 and SC B1 for $x = 0.524$. Maximum 3/4 possible.

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Question	Answer	Marks	Guidance
1	Alternative Method 2 for Question 1		
	Use laws of indices to get to any correct form with indices combined so now TWO log power laws are required	(*B1)	Allow 2^{7-18x} and 5^{-2x} on opposite sides or 2^{9-18x} and $2^{2-4.64x}$ on opposite sides.
	Use law of logarithm of a power involving x on ONE side, e.g. $(7-18x)\ln 2 = \ln 5^{-2x}$ or $\ln 2^{7-18x} = -2x \ln 5$ or ... Allow $7-18x \ln 2$ or $9-18x \ln 2$	(DB1)	e.g. $(7-18x)\ln 2$ or $(9-18x)\ln 2$ or $-2x \ln 5$ or $(2-4.64x)\ln 2$ SC: If *B0 DB0 then allow B1 (1/4) for a correct term seen anywhere. E.g. any term in *B1 shown above.
	Obtain a correct linear equation in x , e.g. $(7-18x)\ln 2 = -2x \ln 5$ or $(9-18x)\ln 2 = (2-4.64x)\ln 2$	(B1)	If in decimals, allow small errors in 2 nd and 3 rd dp.
	Obtain answer $x = 0.524$	(B1)	3dp required. No working scores 0/4 marks. From the first line to correct answer with no log working seen, then *B1 and SC B1 for $x = 0.524$. Maximum 2/4 possible.
		4	

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Question	Answer	Marks	Guidance
2	Use correct product rule $\cos 2x$ may be $1 - 2\sin^2 x$ or ...	M1	$ae^{2x}\sin 2x + e^{2x}b\cos 2x$. Need a or $b = 2$. Allow M1 if only error is e^x instead of e^{2x} in one of terms, then maximum 1/5.
	Obtain correct derivative $2e^{2x}\sin 2x + 2e^{2x}\cos 2x$	A1	OE, e.g. $4e^{2x}\sin x \cos x + 2e^{2x}(\cos^2 x - \sin^2 x)$.
	Equate derivative of the form $ae^{2x}\sin 2x + e^{2x}b\cos 2x$ to 0 and solve for $2x$ or x using a correct method Note may have substituted for $\sin 2x$ and/or $\cos 2x$	M1	Obtain $2x = \tan^{-1}(-\text{their } b/\text{their } a)$ OE. Allow one slip in rearranging. Allow degrees. Variety of other methods available, such as solving quadratic equation in $\sin x$ or $\tan x$ e.g. $\tan^2 x - 2\tan x - 1 = 0$ leading to $x = \tan^{-1}(1 + \sqrt{2})$.
	Obtain $x = \frac{3}{8}\pi$ only or exact equivalent	A1	CWO 67.5° gets A0. Ignore any answers outside interval $0 \leq x \leq \frac{\pi}{2}$.
	Obtain $y = \frac{1}{2}\sqrt{2}e^{\frac{3}{4}\pi}$ only or exact simplified equivalent	A1	CWO, ISW. Not $\sin\left(\frac{3}{4}\pi e^{\frac{3}{4}\pi}\right)$. Ignore any answers using x outside interval $0 \leq x \leq \frac{\pi}{2}$.
		5	

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Question	Answer	Marks	Guidance
3	Square $x + iy$ obtaining three terms when simplified and equate real and imaginary parts to 24 and -7 respectively	M1	Having used $i^2 = -1$.
	Obtain equations $x^2 - y^2 = 24$ and $2xy = -7$	A1	Allow $2xyi = -7i$.
	Eliminate one variable by correct method and find a horizontal equation in the other	M1	All powers of x or y are positive and are in the numerator.
	Obtain $4x^4 - 96x^2 - 49 = 0$ or $4y^4 + 96y^2 - 49 = 0$ or 3-term equivalents	A1	
	Obtain answers $\frac{7\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$ and $-\frac{7\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$ or exact equivalents and no others	A1	E.g. $\pm \left(\frac{7\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i \right)$, but not $\pm \left(\frac{7\sqrt{2}}{2} \mp \frac{\sqrt{2}}{2}i \right)$ or $\left(\pm \frac{7\sqrt{2}}{2} \mp \frac{\sqrt{2}}{2}i \right)$. Allow coordinates or $x = \dots, y = \dots$ paired correctly. ISW converting to different form. Must simplify $\sqrt{49}$.
		5	

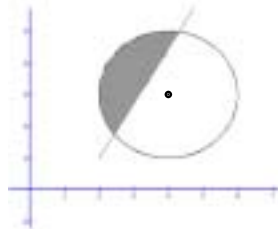
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Question	Answer	Marks	Guidance
4	State or imply that $\ln k + \ln y = cx$ or $\ln y = cx + \ln \frac{1}{k}$ etc.	B1	Allow $\ln k + \ln y = cx \ln e$
	Carry out a completely correct method for finding $\ln k$ or c	M1	Equations must have been formulated correctly.
	Obtain value $c = 0.80$	A1	AWRT Allow 0.8 for 0.80. Not a fraction. Accept in the equation $ky = e^{cx}$.
	Obtain value $k = 6.5$	A1	AWRT Not a fraction. Accept in the equation $ky = e^{cx}$.

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Question	Answer	Marks	Guidance
5	State or imply the form $A + \frac{B}{x-1} + \frac{C}{2x+1}$	B1	
	Use a correct method for finding a constant	M1	Correct appropriate method.
	Obtain one of $A = 3$, $B = 2$ and $C = -3$	A1	
	Obtain a second value	A1	
	Obtain a third value	A1	
	Alternative Method for Question 5		
	Divide numerator by denominator to reach $A = 3$	(M1)	May be implied by 3 $[+]$ $\frac{ax+b}{(x-1)(2x+1)}$ with a and b not both 0.
	Obtain $3 + \frac{x+5}{(x-1)(2x+1)}$	(A1)	
	State or imply the form $\frac{D}{x-1} + \frac{E}{2x+1}$	(B1)	
	Obtain one of $D = 2$ and $E = -3$	(A1)	
	Obtain a second value	(A1)	
		5	

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Question	Answer	Marks	Guidance
6(a)	Show a circle centre (4, 3) Allow dashes for coordinates on axes	B1	Note full circle is not required but must show centre and include relevant arc.
	Show a circle with radius 2. Can be implied by at least two of the points (2, 3), (6, 3), (4, 1) and (4, 5) being correct	B1FT	FT centre not at the origin.
	Point representing (2, 1)	B1	Half-line or ‘correct’ full line extending into the third quadrant implies point (2, 1).
	Show a half-line at their (2, 1) at an angle of $\frac{1}{3}\pi$, cutting top of circle between $x = 3$ and $x = 5$	B1FT	FT the point $(\pm 2, \pm 1)$ or $(\pm 1, \pm 2)$.
	Shade the correct region Needs correct half-line or “correct” full line extending into the third quadrant AND correct circle	B1	
		5	
6(b)	Carry out a correct method for finding the greatest value of $\arg z$ in the correct region in (a)	M1	E.g. $\sin^{-1}(2/\sqrt{25}) + \tan^{-1}(3/4)$ or $\sin^{-1}(2/\sqrt{25}) + \sin^{-1}(3/5)$. Or, e.g., substitute $y = kx$ in circle equation, solve when discriminant = 0, to get $\tan^{-1}\left(\frac{6 + \sqrt{21}}{6}\right)$.
	Obtain answer 1.06, or 1.05 or 1.055 or 1.056 or 60.4° or 60.5°	A1	The marks in (b) are available even if errors in (a). No working seen scores 0/2 marks.
		2	

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Question	Answer	Marks	Guidance
7(a)	Show $8 \times (-7)^3 + 54 \times (-7)^2 - 17 \times (-7) - 21 = 0$ This is sufficient if no errors seen. $[-2744 + 2646 + 119 - 21 = 0]$ Or complete division of $8x^3 + 54x^2 - 17x - 21$ by $x + 7$ to get quotient $8x^2 - 2x - 3$ and remainder of 0 Or state $(x + 7)(8x^2 - 2x - 3)$ is sufficient Factors must be stated again in (b) to collect marks there	B1	No errors allowed. Correct division: $ \begin{array}{r} 8x^2 \quad -2x \quad -3 \\ x+7 \overline{) 8x^3 + 54x^2 - 17x - 21} \\ \underline{8x^3 + 56x^2} \\ -2x^2 - 21 \\ \underline{-2x^2 - 14x} \\ -3x - 21 \\ \underline{-3x - 21} \\ 0 \end{array} $
		1	
7(b)	Commence division and reach partial quotient of the form $8x^2 \pm 2x$ or $8x^3 + 54x^2 - 17x - 21 = (x + 7)(Ax^2 + Bx + C)$ and reach $A = 8$ and $B = \pm 2$ or $C = -3$	M1	Condone no visible working.
	Obtain quotient $8x^2 - 2x - 3$ with no errors seen Stating $(x + 7)(8x^2 - 2x - 3)$ is sufficient	A1	Division can terminate with 0 or $-3x - 21$ stated once or twice. The working of division and finding quotient may be seen in (a) but results required here to collect marks.
		2	

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Question	Answer	Marks	Guidance
7(c)	Solve quadratic from (b) to obtain a value for $\theta = \cos^{-1}\left(\frac{-1}{2}\right)$ or $\cos^{-1}\left(\frac{3}{4}\right)$	M1	$(x+7)(8x^2-2x-3) = (x+7)(4x-3)(2x+1) = 0$ $x = \cos\theta = \frac{2 \pm \sqrt{4+96}}{16} = -\frac{1}{2}$ and $\frac{3}{4}$.
	Obtain one answer, e.g. $\theta = 120^\circ$	A1	
	Obtain three further answers, e.g. $\theta = 240^\circ, 41.4^\circ$ and 318.6° (condone 319°) and no others in the interval	A1	Accept more accurate answers. Answers in radians, maximum 2/3.
		3	

Question	Answer	Marks	Guidance
8(a)	State $R = \sqrt{12}$ or exact equivalent	B1	ISW
	Use trig formula to find α	M1	Allow $\alpha = 30^\circ$ or $\tan^{-1}\left(\frac{\pm\sqrt{3}}{3}\right)$ or $\cos^{-1}\left(\pm\frac{\sqrt{3}}{2}\right)$ or $\sin^{-1}\left(\pm\frac{1}{2}\right)$ Allow M1 if $-\tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$ etc. NB: If $\cos\alpha = 3$ and $\sin\alpha = \sqrt{3}$ seen then M0 A0.
	Obtain $\alpha = \frac{1}{6}\pi$	A1	CWO, so A0 if from $\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)$.
		3	

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Question	Answer	Marks	Guidance
8(b)	Express integral in the form $A \int \sec^2(2x + \dots) dx$ or $A \int \sec^2(2x - \dots) dx$	B1FT	FT α from (a).
	Integrate and reach $B \tan(2x + \dots)$ or $B \tan(2x - \dots)$	B1FT	FT α from (a). Where $B = A$ or $2A$ or $0.5A$.
	Obtain $\frac{1}{8} \tan(2x + \dots)$	B1FT	OE FT α from (a). Allow $\frac{1}{8}$ as $\frac{1}{4} \times \frac{1}{2}$. Coefficient must be correct.
	Use limits of $x = 0$ and $x = \frac{1}{12}\pi$ in the correct order in expression of form $B \tan(2x \pm \dots)$ so $B \tan\left(\frac{\pi}{6} + \dots\right) - B \tan(\dots)$ or $B \tan\left(\frac{\pi}{6} - \dots\right) - B \tan(-\dots)$	M1	Allow with tan still present. FT α from (a). SC: B1 $\frac{\sqrt{3}}{12}$ OE after $\frac{1}{8} \tan\left(2x + \frac{1}{6}\pi\right)$ with no working.
	Obtain answer $\frac{1}{12}\sqrt{3}$ or $\frac{1}{4\sqrt{3}}$ or $\frac{1}{\sqrt{48}}$ or single term exact equivalent	A1	$\frac{1}{8}(\sqrt{3} - \frac{1}{\sqrt{3}}) = \frac{1}{8}\left(\frac{3-1}{\sqrt{3}}\right)$ needs simplifying.
		5	Note: allow all marks in (b) even if $\alpha = \frac{1}{6}\pi$ found by an incorrect method in (a).

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Question	Answer	Marks	Guidance
9(a)	Obtain $\frac{dV}{dt} = [\pm] \frac{k}{t}$ or $\frac{dV}{dt} = [\pm] \frac{1}{kt}$	B1	
	Obtain $\frac{dV}{dx} = 20x - 3x^2$	B1	
	Correct use of chain rule involving k	M1	Use $\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt}$. Expressions for $\frac{dV}{dt}$ and $\frac{dV}{dx}$ must be seen to get M1.
	Obtain $\frac{dx}{dt} = [\pm] \frac{k}{t(20x - 3x^2)}$ or equivalent,	A1	If this expression is first seen with numerical values, allow A1 when their value of k is substituted back into the general expression.
	Use $t = \frac{1}{10}$, $x = \frac{1}{2}$ and $\frac{dx}{dt} = -\frac{20}{37}$ to obtain given answer which must be stated $\frac{dx}{dt} = -\frac{20}{37}$ needed to score final A1	A1	$\frac{dx}{dt} = \frac{-1}{2t(20x - 3x^2)}$ AG Need to at least see $-\frac{20}{37} = \frac{k}{\frac{1}{10}\left(10 - \frac{3}{4}\right)}$ if $\frac{k}{t}$ or $-\frac{20}{37} = \frac{-k}{\frac{1}{10}\left(10 - \frac{3}{4}\right)}$ if $-\frac{k}{t}$ in working for correct k . $\frac{dx}{dt} = \frac{20}{37}$ seen anywhere, then A0.
		5	

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Question	Answer	Marks	Guidance
9(b)	Separate variables correctly & integrate at least one side correctly	B1	
	Obtain terms $10x^2 - x^3$	B1	May see $-10x^2 + x^3$ if negative sign moved across or e.g. $20x^2 - 2x^3$ if 2 moved across. Allow $\frac{20x^2}{2} - \frac{3x^3}{3}$.
	Obtain term $\ln t$ with 'correct' coefficient from their separation of variables, for example $a \ln t$ for $\frac{a}{t}$.	B1FT	FT sign and position of 2 from their separation but B0 if error from later manipulation.
	Use $t = \frac{1}{10}$, $x = \frac{1}{2}$ to evaluate a constant or as limits in a solution containing terms of the form x^2 , x^3 and $\ln t$ (or $\ln 2t$)	M1	Allow numerical and sign errors and decimals. Allow if exponentiate before substitution, even if exponentiation done incorrectly, allow for c or e^c .
	Obtain correct answer in any form, for example $10x^2 - x^3 = -\frac{\ln t}{2} + \frac{19}{8} + \frac{\ln 0.1}{2}$	A1	$10x^2 - x^3 = -\frac{\ln 2t}{2} + \frac{19}{8} + \frac{\ln 0.2}{2}$ or $10x^2 - x^3 = -\frac{\ln t}{2} + 2.5 - 0.125 - 1.15\dots$ Allow 1.14 to 1.16 for 1.15 and allow 2.44 to 2.46 for 2.45
	Obtain answer $t = \frac{1}{10} e^{2x^3 - 20x^2 + \frac{19}{4}}$ or equivalent	A1	ISW Need $t = \dots\dots\dots$ E.g. $\frac{0.1}{e^{20x^2 - 2x^3 - \frac{19}{4}}}$, $\frac{e^{2x^3 + \frac{19}{4}}}{10e^{20x^2}}$, $\frac{1}{10} e^{\frac{19}{4}} e^{2x^3 - 20x^2}$. Allow decimals, allow 2.44 to 2.46 for 2.45, e.g. $e^{2x^3 - 20x^2 + 2.45}$. A0 if $e^{\frac{\ln 1}{10}}$ present in final answer.
		6	

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Question	Answer	Marks	Guidance
10(a)	Carry out correct process for evaluating the scalar product of direction vectors	*M1	$\begin{bmatrix} 3 \\ 4 \\ a \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix} \text{ or } \begin{bmatrix} 3 \\ 4 \\ a \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -2 \\ -2 \end{bmatrix}$ $3(-1) + 4(2) + 2a \text{ or } -3 + 8 + 2a \text{ or } 5 + 2a.$ Allow one slip in unsimplified form.
	Using the correct process for the moduli, divide the scalar product by the product of the moduli and equate to $\pm \frac{\sqrt{2}}{2}$, or equate the scalar product to the product of the moduli and $\pm \frac{\sqrt{2}}{2}$	*M1	*M1 marks independent of each other, so *M0 *M1 for failure to use both direction vectors, but must be using scalar product and same 2 vectors throughout. Allow $\frac{\sqrt{2}}{2}$ or $-\frac{\sqrt{2}}{2}$ throughout question.
	State a correct equation in any form, e.g. $\frac{5+2a}{3\sqrt{25+a^2}} = [\pm] \frac{\sqrt{2}}{2}$ Allow unsimplified as in guidance	A1	$\frac{5+2a}{\sqrt{9+16+a^2}\sqrt{1+4+4}} = [\pm] \frac{\sqrt{2}}{2} \text{ OE}$ E.g. $5+2a = [\pm] \frac{\sqrt{2}}{2} \sqrt{9+16+a^2}\sqrt{1+4+4}$ If moduli initially correct but later has errors, award A1 when using $\frac{\sqrt{2}}{2}$ or $\pm \frac{\sqrt{2}}{2}$ or $-\frac{\sqrt{2}}{2}$.
	Form a quadratic equation in a with 3 or more terms all on one side and solve for a . DM1 depends on BOTH *M1	DM1	Must square $(5+2a)$ to get 3 terms and must remove square roots from both terms on other side. $25 + 20a + 4a^2 = \frac{9}{2}(25 + a^2)$ $a^2 - 40a + 175 = 0$ hence $(a-5)(a-35) = 0$.
10(a)	Obtain $a = 5$ and $a = 35$	A2	A1 for each, working not needed if quadratic correct.
		6	

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Question	Answer	Marks	Guidance
10(b)	Express general point of at least one line correctly in component form, i.e. $\begin{pmatrix} 1 + 3\lambda \\ 1 + 4\lambda \\ 2a + a\lambda \end{pmatrix}$ or $\begin{pmatrix} -3 - \mu \\ -1 + 2\mu \\ 4 + 2\mu \end{pmatrix}$	B1	Often the third point on the line occurs after M1 A1 is gained.
	Equate at least two pairs of corresponding components and solve for λ or μ or a	M1	If solve for a first, they must have a complete method to eliminate both λ and μ . If using a to solve for λ or for μ , a must have been found from a valid method.
	Obtain $\lambda = -1$ or $\mu = -1$	A1	
	Obtain $a = 2$	A1	
	Obtain position vector of the point of intersection is $-2\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$ Two different answers for point of intersection scores A0 even if one is correct	A1	Accept coordinates, row or column, but not $(-2\mathbf{i}, -3\mathbf{j}, +2\mathbf{k})$ or $\begin{pmatrix} -2\mathbf{i} \\ -3\mathbf{j} \\ 2\mathbf{k} \end{pmatrix}$ but ISW after correct form seen.
		5	