

Cambridge International AS & A Level

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MATHEMATICS

9709/61

Paper 6 Probability & Statistics 2

May/June 2023

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
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INFORMATION

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This document has **12** pages. Any blank pages are indicated.

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1 In a certain country, 20 540 adults out of a population of 6 012 300 have a degree in medicine.

- (a)** Use an approximating distribution to calculate the probability that, in a random sample of 1000 adults in this country, there will be fewer than 4 adults who have a degree in medicine. [4]

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- (b)** Justify the approximating distribution used in part (a). [2]

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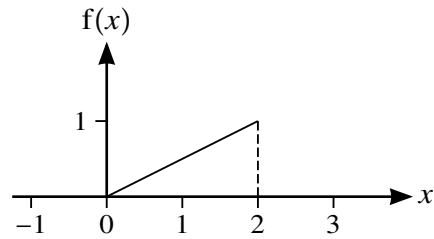
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2 (a)



The graph of the function f is a straight line segment from $(0, 0)$ to $(2, 1)$.

Show that f could be a probability density function.

[2]

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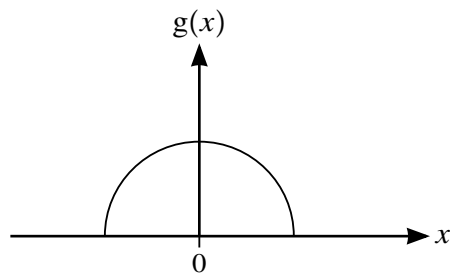
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(b)



The graph of the function g is a semicircle, centre $(0, 0)$, entirely above the x -axis.

Given that g is a probability density function, find the radius of the semicircle.

[2]

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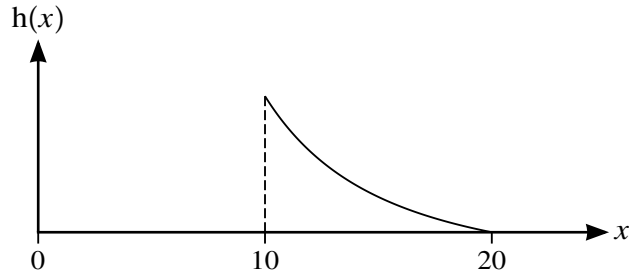
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(c)



The time, X minutes, taken by a large number of students to complete a test has probability density function h , as shown in the diagram.

- (i) Without calculation, use the diagram to explain how you can tell that the median time is less than 15 minutes. [1]

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It is now given that

$$h(x) = \begin{cases} \frac{40}{x^2} - \frac{1}{10} & 10 \leq x \leq 20, \\ 0 & \text{otherwise.} \end{cases}$$

- (ii) Find the mean time. [3]

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- 3 In the past, the annual amount of wheat produced per farm by a large number of similar sized farms in a certain region had mean 24.0 tonnes and standard deviation 5.2 tonnes. Last summer a new fertiliser was used by all the farms, and it was expected that the mean amount of wheat produced per farm would be greater than 24.0 tonnes. In order to test whether this was true, a scientist recorded the amounts of wheat produced by a random sample of 50 farms last summer. He found that the value of the sample mean was 25.8 tonnes.

Stating a necessary assumption, carry out the test at the 1% significance level.

[6]

This image shows a single page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- 4** A certain train journey takes place every day throughout the year. The time taken, in minutes, for the journey is normally distributed with variance 11.2.

- (a) The mean time for a random sample of n of these journeys was found. A 94% confidence interval for the population mean time was calculated and was found to have a width of 1.4076 minutes, correct to 4 decimal places.

Find the value of n .

[3]

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- (b) A passenger noted the times for 50 randomly chosen journeys in January, February and March.

Give a reason why this sample is unsuitable for use in finding a confidence interval for the population mean time.

[1]

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- (c) A researcher took 4 random samples and a 94% confidence interval for the population mean was found from each sample.

Find the probability that exactly 3 of these confidence intervals contain the true value of the population mean.

[2]

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- 5** Large packets of rice are packed in cartons, each containing 20 randomly chosen packets. The masses of these packets are normally distributed with mean 1010 g and standard deviation 3.4 g. The masses of the cartons, when empty, are independently normally distributed with mean 50 g and standard deviation 2.0 g.

(a) Find the variance of the masses of full cartons. [2]

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Small packets of rice are packed in boxes. The total masses of full boxes are normally distributed with mean 6730 g and standard deviation 15.0 g. The masses of the boxes and cartons are distributed independently of each other.

(b) Find the probability that the mass of a randomly chosen full carton is more than three times the mass of a randomly chosen full box. [5]

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- 6 A sample of 5 randomly selected values of a variable X is as follows:

1 2 6 1 a

where $a > 0$.

Given that an unbiased estimate of the variance of X calculated from this sample is $\frac{11}{2}$, find the value of a . [3]

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- 7 The number of accidents per week at a certain factory has a Poisson distribution. In the past the mean has been 1.9 accidents per week. Last year, the manager gave all his employees a new booklet on safety. He decides to test, at the 5% significance level, whether the mean number of accidents has been reduced. He notes the number of accidents during 4 randomly chosen weeks this year.

(a) State suitable null and alternative hypotheses for the test. [1]

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(b) Find the critical region for the test and state the probability of a Type I error. [6]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- (c) State what is meant by a Type I error in this context. [1]

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- (d) During the 4 randomly chosen weeks there are a total of 3 accidents.

State the conclusion that the manager should reach. Give a reason for your answer. [2]

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- (e) Assuming that the mean remains 1.9 accidents per week, use a suitable approximation to calculate the probability that there will be more than 100 accidents during a 52-week period. [4]

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9709/62

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May/June 2023

1 hour 15 minutes

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- 1 In a survey of 200 randomly chosen students from a certain college, 23% of the students said that they owned a car.

Calculate an approximate 93% confidence interval for the proportion of students from the college who own a car. [3]

This image shows a single page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- 2 (a)** The random variable W has a Poisson distribution.

State the relationship between $E(W)$ and $\text{Var}(W)$.

[1]

- (b) The random variable X has the distribution $B(n, p)$. Jyothi wishes to use a Poisson distribution as an approximate distribution for X .

Use the formulae for $E(X)$ and $\text{Var}(X)$ to explain why it is necessary for p to be close to 0 for this to be a reasonable approximation. [1]

[1]

- (c) Given that Y has the distribution $B(20\,000, 0.000\,07)$, use a Poisson distribution to calculate an estimate of $P(Y > 2)$. [3]

[3]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- 3 The masses, in kilograms, of newborn babies in country *A* are represented by the random variable X , with mean μ and variance σ^2 . The masses of a random sample of 500 newborn babies in this country were found and the results are summarised below.

$$n = 500 \quad \Sigma x = 1625 \quad \Sigma x^2 = 5663.5$$

- (a) Calculate unbiased estimates of μ and σ^2 . [3]

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

A researcher wishes to test whether the mean mass of newborn babies in a neighbouring country, B , is different from that in country A . He chooses a random sample of 60 newborn babies in country B and finds that their sample mean mass is 2.95 kg.

Assume that your unbiased estimates in part (a) are the correct values for μ and σ^2 . Assume also that the variance of the masses of newborn babies in country B is the same as in country A .

(b) Carry out the test at the 1% significance level. [5]

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- 4** The number, X , of books received at a charity shop has a constant mean of 5.1 per day.

- (a)** State, in context, one condition for X to be modelled by a Poisson distribution. [1]

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Assume now that X can be modelled by a Poisson distribution.

- (b) Find the probability that exactly 10 books are received in a 3-day period. [2]

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- (c) Use a suitable approximating distribution to find the probability that more than 180 books are received in a 30-day period. [4]

[illegible]

The number of DVDs received at the same shop is modelled by an independent Poisson distribution with mean 2.5 per day.

- (d) Find the probability that the total number of books and DVDs that are received at the shop in 1 day is more than 3. [3]

This image shows a single page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- 5** (a) Two random variables X and Y have the independent distributions $N(7, 3)$ and $N(6, 2)$ respectively. A random value of each variable is taken.

Find the probability that the two values differ by more than 2. [5]

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- Find the proportion of candidates who pass. [5]

[illegible]

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- 6** When a child completes an online exercise called a Mathlit, they might be awarded a medal. The publishers claim that the probability that a randomly chosen child who completes a Mathlit will be awarded a medal is $\frac{1}{3}$. Asha wishes to test this claim. She decides that if she is awarded no medals while completing 10 Mathlits, she will conclude that the true probability is less than $\frac{1}{3}$.

(a) Use a binomial distribution to find the probability of a Type I error. [2]

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The true probability of being awarded a medal is denoted by p .

(b) Given that the probability of a Type II error is 0.8926, find the value of p . [3]

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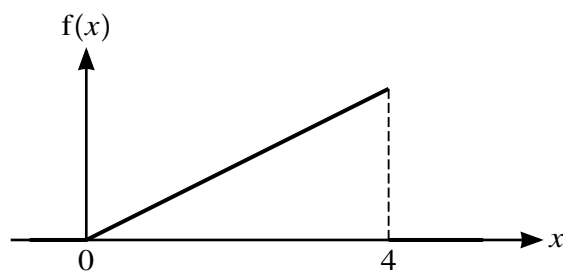
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7 (a)



The diagram shows the graph of the probability density function, f , of a random variable X which takes values between 0 and 4 only. Between these two values the graph is a straight line.

- (i) Show that $f(x) = kx$ for $0 \leq x \leq 4$, where k is a constant to be determined. [2]

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- (ii) Hence, or otherwise, find $E(X)$. [3]

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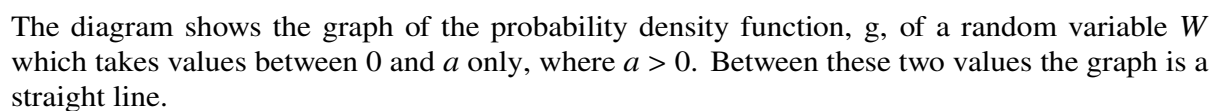
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Given that the median of W is 1, find the value of a . [3]

[illegible]

[illegible]

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- 1** A random variable X has probability density function f , where

$$f(x) = \begin{cases} \frac{3}{2}(1 - x^2) & 0 \leq x \leq 1, \\ 0 & \text{otherwise.} \end{cases}$$

Find $E(X)$.

[3]

[illegible]

- 2 A club has 264 members, numbered from 1 to 264. Donash wants to choose a random sample of members for a survey. In order to choose the members for the sample he uses his calculator to generate random digits. His first 20 random digits are as follows.

10612 11801 21473 22759

- (a) The numbers of the first two members in the sample are 106 and 121.

Write down the numbers of the next two members in the sample.

[2]

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- (b) To obtain the numbers for members after the 4th member, Donash starts with the second random digit, 0, and obtains the numbers 061 and 211.

Explain why this method will not produce a random sample.

[1]

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- 3** In a random sample of 100 students at Luciana's college, x students said that they liked exams. Luciana used this result to find an approximate 90% confidence interval for the proportion, p , of all students at her college who liked exams. Her confidence interval had width 0.15792.

(a) Find the two possible values of x .

[4]

[illegible]

Suzma independently took another random sample and found another approximate 90% confidence interval for p .

(b) Find the probability that neither of the two confidence intervals contains the true value of p . [1]

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- 4 The mass, in tonnes, of steel produced per day at a factory is normally distributed with mean 65.2 and standard deviation 3.6. It can be assumed that the mass of steel produced each day is independent of other days. The factory makes \$50 profit on each tonne of steel produced.

Find the probability that the total profit made in a randomly chosen 7-day week is less than \$22 000.

[6]

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- 5 Last year the mean time for pizza deliveries from Pete's Pizza Pit was 32.4 minutes. This year the time, t minutes, for pizza deliveries from Pete's Pizza Pit was recorded for a random sample of 50 deliveries. The results were as follows.

$$n = 50 \qquad \Sigma t = 1700 \qquad \Sigma t^2 = 59\,050$$

- (a) Find unbiased estimates of the population mean and variance. [3]

[illegible]

- (b) Test, at the 2% significance level, whether the mean delivery time has changed since last year. [5]

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- (c) Under what circumstances would it **not** be necessary to use the Central Limit Theorem in answering (b)? [1]

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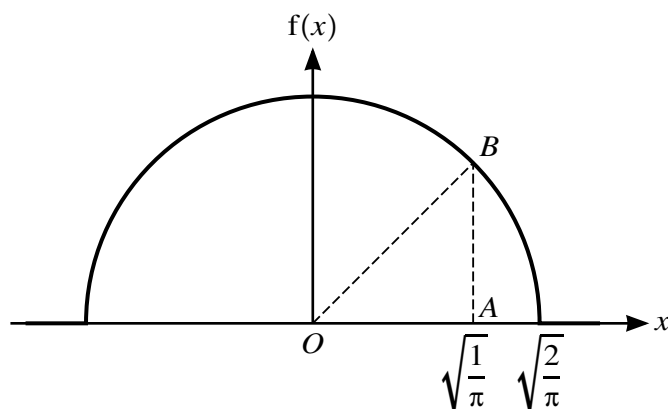
- 6** It is known that 1 in 5000 people in Atalia have a certain condition. A random sample of 12 500 people from Atalia is chosen for a medical trial. The number having the condition is denoted by X .

(a) Use an appropriate approximating distribution to find $P(X \leq 3)$. [3]

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(b) Find the values of $E(X)$ and $\text{Var}(X)$, and explain how your answers suggest that the approximating distribution used in (a) is likely to be appropriate. [2]

[illegible]



A random variable X has probability density function f , where the graph of $y = f(x)$ is a semicircle with centre $(0, 0)$ and radius $\sqrt{\frac{2}{\pi}}$, entirely above the x -axis. Elsewhere $f(x) = 0$ (see diagram).

- (a) Verify that f can be a probability density function. [2]

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A and B are the points where the line $x = \sqrt{\frac{1}{\pi}}$ meets the x -axis and the semicircle respectively.

- (b) Show that angle AOB is $\frac{1}{4}\pi$ radians and hence find $P\left(X > \sqrt{\frac{1}{\pi}}\right)$. [6]

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- 8 A new light was installed on a certain footpath. A town councillor decided to use a hypothesis test to investigate whether the number of people using the path in the evening had increased.

Before the light was installed, the mean number of people using the path during any 20-minute period during the evening was 1.01.

After the light was installed, the total number, n , of people using the path during 3 randomly chosen 20-minute periods during the evening was noted.

- (a) Given that the value of n was 6, use a Poisson distribution to carry out the test at the 5% significance level. [6]

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- (b) Later a similar test, at the 5% significance level, was carried out using another 3 randomly chosen 20-minute periods during the evening.

Find the probability of a Type I error.

[2]

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- (c) State what is meant by a Type I error in this context.

[1]

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- (d) State, in context, what further information would be needed in order to find the probability of a Type II error. Do not carry out any further calculation.

[2]

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