



Cambridge International A Level

MATHEMATICS

9709/61

Paper 6 Probability & Statistics 2

May/June 2023

MARK SCHEME

Maximum Mark: 50

Published

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These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

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The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1(a)	$20540/6012300 = 0.0034163$	B1	
	$[1000 \times 0.0034163 = 3.4163]$		
	Po(3.4163)	B1	Could be implied by expression seen.
	$e^{-\text{their '3.4163'}}(1 + 3.4163 + \frac{3.4163^2}{2!} + \frac{3.4163^3}{3!})$ OR $e^{-\text{their '3.4163'}}(1 + 3.4163 + 5.8356 + 6.6453)$ or $0.03283 + 0.1122 + 0.1916 + 0.21819$	M1	Allow any λ . Allow with one end error. Must see expression.
	$= 0.555$ (3sf)	A1	CAO SC No working: B1 B1 (Po must be stated) B1 correct answer (max 3/4). SC Binomial: B1 B0 B1 correct answer (max 2/4).
		4	
1(b)	$n = 1000 > 50$	B1	Must show comparison with 50.
	$np = 3.4163 < 5$	B1	Must show comparison with 5.
		2	SC B1 : $n > 50$ (or n large), $np < 5$. SC B1 : n large, p small.

Question	Answer	Marks	Guidance
2(a)	$\frac{1}{2} \times 2 \times 1$ or $\int_0^2 \frac{1}{2} x dx = 1$, which is the correct area under a pdf.	B1	Calculation and result.
	$f(x) \geq 0$	B1	Condone $f(x) > 0$ or 'Line is above x-axis' OE.
		2	

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Question	Answer	Marks	Guidance
2(b)	$\frac{1}{2}\pi r^2 = 1$	M1	Area of semi-circle equated to 1 OE. Missing factor of $\frac{1}{2}$ gets M1A0.
	$r = \sqrt{\frac{2}{\pi}}$ or 0.798 (3sf)	A1	
		2	
2(c)(i)	Area to the left of 15 is greater than 0.5	B1	OE, e.g. ‘The distribution of X is skewed to the right / positively skewed, suggesting the median will be less than the mid-point of the interval.’ or ‘The distribution of X is skewed to the right / positively skewed’ or ‘It is a decreasing function suggesting the median will be less than the mid-point of the interval’.
		1	
2(c)(ii)	$\int_{10}^{20} \left(\frac{40}{x} - \frac{x}{10}\right) dx$	M1	Integration of $xh(x)$ attempted. Ignore limits.
	$\left[40 \ln x - \frac{x^2}{20}\right]_{10}^{20}$	A1	Correct integration and limits (can be implied by final answer).
	$= 40 \ln 2 - 15$ or 12.7 (3sf)	A1	
		3	

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Question	Answer	Marks	Guidance
3	Assume SD still = 5.2	B1	OE i.e. ‘Assume the SD remains unchanged’.
	$H_0: \mu = 24.0$ $H_1: \mu > 24.0$	B1	Or population mean; not just mean.
	$\frac{25.8-24.0}{\frac{5.2}{\sqrt{50}}}$	M1	For standardising (could be implied). Must have $\sqrt{50}$.
	= 2.448	A1	Or $P(\bar{X} > 25.8) = 0.0071$.
	‘2.448’ > 2.326	M1	Or $0.0071 < 0.01$. For valid comparison.
	[Reject H_0] There is evidence that (mean) amount of wheat is greater.	A1FT	OE. FT their z_{calc} . In context, not definite, eg not ‘Mean amount of wheat is greater’ No contradictions CV method: CV= 25.71 M1A1 25.71<25.8 M1 A1FT or CV=24.09 M1 A1 24.09>24 M1 A1FT.
		6	

Question	Answer	Marks	Guidance
4(a)	$z \times \sqrt{\frac{11.2}{n}} = 1.4076 \div 2$	M1	Any z, but must be a z.
	$z = 1.881$ or 1.882	B1	
	$[n = \left(\frac{1.881}{0.7038}\right)^2 \times 11.2]$ $n = 80$	A1	Must be a whole number.
		3	

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Question	Answer	Marks	Guidance
4(b)	Jan, Feb and March not typical of whole year.	B1	Or, e.g., weather is different at different times of year.
		1	
4(c)	$0.94^3 \times 0.06 \times 4$	M1	
	$= 0.199$ (3 sf)	A1	
		2	

Question	Answer	Marks	Guidance
5(a)	$2.0^2 + 20 \times 3.4^2$	M1	
	$= 235.2$	A1	
		2	
5(b)	$E(C - 3B) = 50 + 20 \times 1010 - 3 \times 6730$ or 60	B1	
	$\text{Var}(C - 3B) = '235.2' + 9 \times 15^2$ or 2260.2	M1	FT <i>their</i> values from (a) .
	$[C - 3B \sim N('60', '2260.2')]$ $= \frac{0 - 60}{\sqrt{2260.2}}$ $[= -1.262]$	M1	Standardising with their values (could be implied).
	$1 - \Phi(-1.262) = \Phi(1.262)$	M1	Probability area consistent with their values.
	$= 0.897$ (3 sf)	A1	
		5	

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Question	Answer	Marks	Guidance
6	$\frac{5}{4} \left(\frac{1+2^2+6^2+1+a^2}{5} - \left(\frac{1+2+6+1+a}{5} \right)^2 \right) = \frac{11}{2}$ or $\frac{1}{4} \left((42+a^2) - \frac{(10+a^2)}{5} \right) = \frac{11}{2}$	M1*	OE attempted or e.g., $\frac{42+a^2}{5} - \left(\frac{10+a}{5} \right)^2 = \frac{22}{5}$. Allow use of biased i.e., without $\frac{5}{4}$.
	$4a^2 - 20a + 0 = 0$ or $a^2 - 5a + 0 = 0$	DM1	Two- or three-term quadratic equation in a , with at least two terms correct.
	$a = 5$	A1	Ignore $a = 0$, if seen.
		3	

Question	Answer	Marks	Guidance
7(a)	$H_0: \lambda = 7.6$ [or 1.9] $H_1: \lambda < 7.6$ [or 1.9]	B1	Or Population mean = 7.6 or μ (not just ‘mean’). Or Population mean < 7.6 or μ .
		1	

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Question	Answer	Marks	Guidance
7(b)	Mean = 7.6	B1	Seen.
	$P(X \leq 2) = e^{-7.6} (1 + 7.6 + \frac{7.6^2}{2})$ [= 0.0188 or 0.0187]	M1	OE.
	$P(X \leq 3) = e^{-7.6} (1 + 7.6 + \frac{7.6^2}{2} + \frac{7.6^3}{3!})$ [= 0.0554 or 0.0553]	M1	OE. Expression must be seen in at least one probability calculation.
	0.0188 or 0.0187 and 0.0554 or 0.0553	A1	A1 for both values.
	Critical region is $X \leq 2$	A1	Dep on both M marks. SC No Poisson expression seen in either prob scores B1 for 0.0188 or 0.0187 and B1 for 0.0554 or 0.0553 and B1 for CR.
	$P(\text{Type I error}) = P(X \leq 2) = 0.0188 \text{ or } 0.0187 \text{ (3 sf)}$	B1FT	FT <i>their</i> $P(X \leq 2)$ or <i>their</i> CR.
		6	
7(c)	Concluding that the (mean) no. of accidents has reduced when it has not.	B1	OE. Must be in context. Accept: 'It is believed that the booklet has helped to improve safety when actually it has not'.
		1	
7(d)	3 not in critical region.	M1	FT their CR or $P(X \leq 3) = 0.0554 > 0.05$.
	No evidence mean number of accidents has decreased.	A1FT	In context. Cannot be a definite statement, e.g., 'mean number accidents has not decreased'.
		2	

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Question	Answer	Marks	Guidance
7(e)	N(98.8, 98.8)	B1	May be implied.
	$\frac{100.5 - 98.8}{\sqrt{98.8}} \quad [= 0.171]$	M1	For standardising (could be implied by correct answer). Allow with wrong or no continuity correction.
	$1 - \Phi(0.171)$	M1	For probability area consistent with their working.
	= 0.432 (3 sf)	A1	
		4	



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Question	Answer	Marks	Guidance
1	$0.23 \pm z \times \sqrt{\frac{0.23 \times (1-0.23)}{200}}$	M1	Expression of correct form. Any z , but $z = 0.8328$ scores B0M0.
	$z = 1.811$ or 1.812	B1	
	0.176 to 0.284 (3 sf)	A1	Must be an interval.
		3	

Question	Answer	Marks	Guidance
2(a)	$E(W) = \text{Var}(W)$.	B1	Allow ‘they are the same’ OE. Must be = not $\approx$ (and not both = and $\approx$). Condone $E(W) = \lambda$ and $\text{Var}(W) = \lambda$.
		1	
2(b)	$np \approx np(1-p)$, hence $1-p$ must be close to 1	B1	OE. Must see formulae and $q = 1-p$ must be seen or implied and conclusion made.
		1	

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Question	Answer	Marks	Guidance
2(c)	$\lambda = 1.4$	B1	Seen.
	$1 - e^{-1.4}(1 + 1.4 + \frac{1.4^2}{2})$ or $1 - e^{-1.4}(1 + 1.4 + 0.98)$ or $1 - (0.2466 + 0.3452 + 0.2417)$	M1	Allow any λ ; allow one end error. Expression must be seen (accept correct sigma notation).
	$= 0.167$ (3 sf) or 0.166	A1	Use of Binomial scores SCB1 for 0.167 or 0.166 . No working: 0.167 [or 0.166] SC B1 . Note: $\lambda=1.4$ and 0.167 with no working seen scores SC B1B1 . Use of Normal scores B0M0.
		3	

Question	Answer	Marks	Guidance
3(a)	$\text{Est}(\mu) = 3.25 = 13/4$ or $1625/500$	B1	
	$\text{Est}(\sigma^2) = \frac{500}{499}(\frac{5663.5}{500} - "3.25"{}^2)$ or $\frac{1}{499}\left(5663.5 - \frac{1625^2}{500}\right)$	M1	Expression of correct form.
	$= 0.766$ (3 sf) or $1529/1996$	A1	Biased variance of 0.7645 scores M0A0.
		3	

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Question	Answer	Marks	Guidance
3(b)	H_0 : Pop mean (or μ) = '3.25' H_1 : Pop mean (or μ) $\neq$ '3.25'	B1FT	Not just 'mean'. FT their 3.25 .
	$\frac{2.95 - 3.25}{\sqrt{0.766/60}}$	M1	Standardising with their values. Must have $\sqrt{60}$.
	= -2.655	A1	Or $P(\bar{X} < 2.95) = 0.0039$ or 0.00396 or 0.00397 . SC FT their biased $\text{est}(\sigma^2)$, i.e. 0.7645 to give $z = 2.658$ A1.
	'2.655' > 2.576 or '-2.655' < -2.576	M1	For valid comparison, e.g. 0.0039 or 0.00396 or 0.00397 < 0.005, or 0.0078 < 0.01, or 0.00792 < 0.01 .
	[Reject H_0] There is evidence that (mean) mass in (country B) is different (from country A).	A1FT	OE. Must be in context and not definite, e.g., not 'Mean mass is not different', No contradictions. Context needs either 'mass' or 'countries' OE.
			SC , Use of one-tail test. '2.655' > 2.326 or 0.0039 < 0.01 M1A0 (Max B0M1A1M1A0 3/5).
			Accept critical value method. Either: $X_{\text{crit}} = 2.959$ M1A1 2.95 < 2.959 M1A1FT with correct conclusion, or $X_{\text{crit}} = 3.241$ M1A1 3.25 > 3.241 M1A1FT with correct conclusion.
		5	

Question	Answer	Marks	Guidance
4(a)	Books received independently or singly or randomly.	B1	OE. Must be in context. If more than one condition given, ignore extras.
		1	

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Question	Answer	Marks	Guidance
4(b)	$e^{-15.3} \times \frac{15.3^{10}}{10!}$	M1	Allow incorrect λ .
	= 0.0439 (3sf)	A1	SC No working shown but correct answer seen scores B1.
		2	
4(c)	N(153, 153)	B1	Seen or implied.
	$\frac{180.5-153}{\sqrt{153}}$ [= 2.223]	M1	For standardising with their values (can be implied). Allow with wrong or missing continuity correction.
	$1 - \Phi('2.223')$	M1	For correct probability area consistent with their values.
	= 0.0131 (3sf)	A1	
		4	
4(d)	$(\lambda =) 5.1 + 2.5$ [= 7.6]	B1	Give at early stage (seen or implied).
	$1 - e^{-7.6} \left(1 + 7.6 + \frac{7.6^2}{2} + \frac{7.6^3}{3!}\right) = 1 - e^{-7.6}(1 + 7.6 + 28.88 + 73.16)$ = $1 - (0.0005005 + 0.003803 + 0.01445 + 0.03661)$	M1	Allow incorrect λ . Allow one end error. Must see an expression (accept correct sigma notation).
	= 0.945 (3sf)	A1	SC No working, 0.945 B1(could be implied) SC B1 .
		3	

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Question	Answer	Marks	Guidance
5(a)	$E(X - Y) = 1$ $\text{Var}(X - Y) = 5$	B1	Seen or implied, OE e.g. $X - Y - 2$.
	$\frac{2-1}{\sqrt{5}}$ [= 0.447] $\frac{-2-1}{\sqrt{5}}$ [= -1.342]	M1	Standardising with their values must come from a combination.
	$1 - \Phi(0.447)$ $\Phi(-1.342) = 1 - \Phi(1.342)$	M1	Correct probability area consistent with their values.
	= 0.327 or 0.328 = 0.0898 or 0.0899	A1	Seen or implied.
	Probability that difference is more than 2 = 0.417 (3 sf) or 0.418	A1	
		5	
5(b)(i)	$E(X) = 62 + 1.5(42)$ [= 125]	B1	OE.
	$\text{Var}(X) = 158 + 1.5^2 \times 108$ [= 401]	B1	Correct expression OE.
	$\frac{90 - 125}{\sqrt{401}}$ [= -1.748]	M1	Correct standardisation using their $E(X)$ and $\text{Var}(X)$. Must both be from a combination attempt. Ignore any attempted continuity correction.
	$\Phi(1.748)$	M1	Correct probability area consistent with their stated values.
	= 0.960 or 96.0% (3 sf)	A1	Allow 0.96 or 96%.
		5	
5(b)(ii)	Unlikely. A candidate who does well in Theory is likely to do well in Practical.	B1	Need both. Accept ‘unlikely’, ‘not independent’, ‘dependent’, ‘not realistic’, or similar; and accept ‘both testing knowledge from the same syllabus’, ‘theory and practical share same content’ or similar statement.
		1	

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Question	Answer	Marks	Guidance
6(a)	$(1 - \frac{1}{3})^{10}$	M1	
	$= 0.0173$ (3 sf)	A1	No working scores SC B1 .
		2	
6(b)	$1 - (1 - p)^{10} = 0.8926$	M1	Accept $1 - q^{10} = 0.8926$. Equation must be in p or in q but not both.
	$1 - p = 0.1074^{0.1}$ [= 0.800]	M1	For valid attempt to solve their (binomial) equation in p^{10} or q^{10} .
	$p = 0.200$ (3 sf) or 0.2	A1	
		3	

Question	Answer	Marks	Guidance
7(a)(i)	$\frac{1}{2} \times 4 \times a = 1$	M1	For use of area = 1 or let $f(x) = kx$ and attempt $\int_0^4 kx \, dx = 1$.
	$[a = \frac{1}{2}] \, f(x) = \frac{1}{8}x$	A1	$k \left[\frac{x^2}{2} \right]_0^4 = 1; 8k = 1; k = \frac{1}{8}$. $f(x) = \frac{1}{8}x$ or $k = \frac{1}{8}$.
		2	

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Question	Answer	Marks	Guidance
7(a)(ii)	$\int_0^4 x \times \frac{1}{8}x \, dx$	M1	Attempt to integrate $x \times \text{their } f(x)$. Ignore limits accept in terms of k .
	$\left[\frac{x^3}{24} \right]_0^4$	A1ft	Their integral and correct limits accept in terms of k .
	$= \frac{8}{3}$ or 2.67 (3 sf)	A1	Note: Final answer of $64k/3$ scores 2/3.
		3	
7(b)	$\frac{a-1}{a} = \frac{1}{\sqrt{2}}$	M1	Or attempt $\int_0^1 g(w)dw = \frac{1}{2}$ i.e. $\int_0^1 \left(\frac{2}{a} - \frac{2}{a^2}w \right)dw = \frac{1}{2}$, or integral from 1 to a . $g(w)$ must be linear of form $g(w) = mw (+c)$. Or area attempt: attempt to calculate heights using their linear equation ($h_1=2/a$ and $h_2=-2/a^2 + 2/a$) and use in either area trapezium = 0.5, or area trapezium = area small triangle or area small triangle = 0.5 . Area trapezium = $1/2 \times 1 (2/a + -2/a^2 + 2/a)$ Area triangle = $1/2(a-1)(-2/a^2 + 2/a)$ Note: alternative expression for $h_1 = (a-2)/(a-1)$.
	$a\sqrt{2} - \sqrt{2} = a$	A1	Or $a^2 - 4a + 2 = 0$. Any correct equation in a , a not in denominator.
	$a = 2 + \sqrt{2} = 3.41$	A1	
		3	



Cambridge International A Level

MATHEMATICS

9709/63

Paper 6 Probability & Statistics 2

May/June 2023

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1	$\frac{3}{2} \int_0^1 (x - x^3) dx$	M1	Attempt to integrate $xf(x)$; ignore limits.
	$= \frac{3}{2} \left[\frac{x^2}{2} - \frac{x^4}{4} \right]_0^1$	A1	Correct integration and limits.
	$= \frac{3}{8}$	A1	
		3	

Question	Answer	Marks	Guidance
2(a)	180, 227	B1	One correct. Ignore incorrect numbers.
		B1	Both correct and no extra numbers seen. (Allow other correct use of list of digits).
		2	
2(b)	These numbers are not independent of the previous numbers OR Only a finite number of digits used	B1	Already used these numbers, so therefore not random. Does not include numbers not in the list, therefore not random (not random or biased needs a reason).
		1	

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Question	Answer	Marks	Guidance
3(a)	$z = 1.645$	B1	
	$z \times \frac{\sqrt{\frac{x}{100} \times (1 - \frac{x}{100})}}{100} = 0.07896$	M1	OE. Equation of correct form. Accept $p = x/100$. Any z . Allow missing factor of 2.
	$[x(100 - x) = 100^3 \times 0.07896^2 \div 1.645^2]$ $x^2 - 100x + 2304 = 0$	A1	Any correct (likely scalar multiple) three-term quadratic equation in x or p with simplified coefficients. Accept $p^2 - p + 0.2304 = 0$ or $p(1-p) = 0.2304$.
	$x = 36$ or 64	A1	
		4	
3(b)	$0.1^2 = 0.01$	B1	Accept either.
		1	

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Question	Answer	Marks	Guidance
4	Method 1: Based on mass		
	Mean = $7 \times 65.2 = 456.4$	B1	
	Var = $7 \times 3.6^2 [= 90.72]$	M1	
	$22\,000/50 = 440$ used in standardising equation	M1	
	$\frac{'440' - '456.4'}{\sqrt{'90.72'}} [= -1.722]$ no mixed methods	M1	For standardising with their values. No mixed methods.
	$\Phi(-'1.722') = 1 - \Phi('1.722')$	M1	For correct probability area consistent with their values.
	= 0.0425 or 0.0426	A1	Note: accept alt method using per day. $N(65.2, \frac{3.6^2}{7})$. No mixed methods.
	Method 2: Based on profit		
	Mean = $7 \times 65.2 \times 50 = 22\,820$	B1	
	Var = 7×3.6^2	M1	
	Var = $50^2 \times '90.72' [= 226\,800]$	M1	
	$\frac{22\,000 - '22\,820'}{\sqrt{'226\,800'}} [= -1.722]$ no mixed methods	M1	For standardising with their values. No mixed methods.
	$\Phi(-'1.722') = 1 - \Phi('1.722')$	M1	For correct probability area consistent with their values.
	= 0.0425 or 0.0426	A1	
		6	

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Question	Answer	Marks	Guidance
5(a)	$\bar{x} = 1700/50 = 34$	B1	
	$\text{Est}(\sigma^2) = \frac{50}{49} \left(\frac{59050}{50} - 34^2 \right)$ or $\frac{1}{49} \left(59050 - \frac{1700^2}{50} \right)$	M1	$\text{Est}(\sigma^2) = \frac{59050}{50} - 34^2$ biased scores M0.
	$= 25.5$ (3 sf) or $\frac{1250}{49}$	A1	$= 25$ scores A0.
		3	
5(b)	H_0 : Population mean time = 32.4 H_1 : Population mean time $\neq$ 32.4	B1	Not just ‘mean’ but allow just ‘ μ ’.
	$\frac{34 - 32.4}{\frac{\sqrt{25.5}}{\sqrt{50}}}$	M1	Must have $\sqrt{50}$ and not 50. FT <i>their</i> mean and var. Can be implied.
	$= 2.24$ (3 sf)	A1	or $P(\bar{T} > 34) = 0.0125$. SC use of biased var (25) $z = 2.26$ or $p = 0.0119$, allow M1A1.
	‘2.24’ < 2.326	M1	Or $0.0125 > 0.01$ for a valid comparison.
	[Not reject H_0] Insufficient evidence that (mean) time has changed	A1FT	In context, not definite, e.g. not ‘Time not changed’. No contradictions. Note: accept CV method $x_{\text{cri}} = 34.06$ for M1A1. Compares $34 < 34.06$ for M1, conclusion for A1. Condone $x = 32.34$ M1A1: compares $32.4 > 32.34$ for M1, conclusion for A1.
		5	SC for using a one-tail method. Award max 3/5 (B0 M1 A1 M1 A0).

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Question	Answer	Marks	Guidance
5(c)	Distribution of times in the population is normal	B1	Accept answers with no context here. Accept underlying distribution for population.
		1	

Question	Answer	Marks	Guidance
6(a)	$X \sim \text{Po}(2.5)$	B1	SOI.
	$e^{-2.5}(1 + 2.5 + \frac{2.5^2}{2} + \frac{2.5^3}{3!})$	M1	Any λ . Allow one end error.
	$= 0.758$ (3 sf)	A1	SC use of binomial B1 for 0.758. SC when no working is shown, $X \sim \text{Po}(2.5)$ seen scores B1, 0.758 seen also scores B1.
		3	
6(b)	$E(X) = \frac{5}{2}$ or 2.5 , $\text{Var}(X) = \frac{4999}{2000}$ or 2.4995	*B1	Just an answer of 2.5 for the variance is not sufficient. However, 2.4995 is sufficient.
	These are almost equal	DB1	Condone ‘equal’.
		2	

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Question	Answer	Marks	Guidance
7(a)	$\frac{1}{2}\pi\left(\sqrt{\frac{2}{\pi}}\right)^2$	M1	
	= 1, which is the area under a PDF [and $f(x) \geq 0$]	A1	Result and statement are both needed.
		2	
7(b)	$\cos^{-1}\left(\frac{\sqrt{\frac{1}{\pi}}}{\sqrt{\frac{2}{\pi}}}\right) = \frac{\pi}{4}$	B1	AG. Accept alternative approaches, e.g. using Pythagoras, tangent, or isosceles right-angle triangles. Answer should be convincingly obtained and all correct.
	Area of sector = $\frac{1}{4}$	B1	
	Area of triangle $AOB = \frac{1}{2}OA \times OB = \frac{1}{2} \times \sqrt{\frac{1}{\pi}} \times \sqrt{\frac{2}{\pi} - \frac{1}{\pi}}$ or Area of triangle $AOB = \frac{1}{2}OA \times OB \times \sin(AOB) = \frac{1}{2} \times \sqrt{\frac{1}{\pi}} \times \sqrt{\frac{2}{\pi}} \sin \frac{\pi}{4}$	M1	Accept alternative approaches. Note: $AB = \sqrt{0.7979^2 - 0.5642^2}$ [= 0.5642] Allow values to 3sf.
	$\frac{1}{2\pi}$ or 0.1592	A1	
	‘ $\frac{1}{4}$ ’, ‘ $\frac{1}{2\pi}$ ’, or ‘0.25’ – ‘0.1592’	M1	Attempt area of sector – area of triangle AOB .
	$= \frac{1}{4} - \frac{1}{2\pi}$ or 0.0908 (3sf)	A1	

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Question	Answer	Marks	Guidance
7(b)	Alternative Method for Question Q7(b): Using integration		
	Find equation of curve $x^2 + y^2 = \frac{2}{\pi}$	M1	
	$y = \sqrt{\frac{2}{\pi} - x^2}$	A1	
	Attempt to integrate (any limits)	M1	
	Use of correct limits $\sqrt{\frac{1}{\pi}}$ to $\sqrt{\frac{2}{\pi}}$	B1	
	Correct integration with correct limits	A1	
	$= \frac{1}{4} - \frac{1}{2\pi}$ or 0.0908 (3sf)	A1	Correct final answer.
		6	

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Question	Answer	Marks	Guidance
8(a)	H ₀ : Pop mean no. people = 3.03 or 1.01 (per 20 min) H ₁ : Pop mean no. people > 3.03 or 1.01 (per 20 min)	B1	These must not just be ‘mean’, but allow just ‘ λ ’ or ‘ μ ’.
	Use of P ₀ (3.03)	M1	
	$= 1 - e^{-3.03} \left(1 + 3.03 + \frac{3.03^2}{2} + \frac{3.03^3}{3!} + \frac{3.03^4}{4!} + \frac{3.03^5}{5!} \right)$ $= 1 - e^{-3.03} (1 + 3.03 + 4.5905 + 4.6364 + 3.5120 + 2.128)$ $= 1 - (0.04832 + 0.1464 + 0.2218 + 0.2240 + 0.1697 + 0.1028)$	M1	Allow incorrect λ . Allow one end error. Must see Poisson expression used.
	= 0.0870 (3sf) [0.0869727]	A1	Allow 0.087 .
	0.0870 > 0.05	M1	For a valid comparison.
	(Do not reject H ₀) Insufficient evidence to believe (mean) number of people has increased	A1FT	Conclusion stated must be in context, not definite and include no contradictions (e.g. not ‘mean number people has not increased’).
		6	If only P(x = 6) award max 2/6 (single term not valid). SC No working B1 B2 M1 A1. Award maximum 5/6.

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Question	Answer	Marks	Guidance
8(b)	$"0.0869727" - e^{-3.03} \times \frac{3.03^6}{6!}$ or $0.869727 - e^{-3.03}(1.0748)$ or $0.869727 - 0.05193$ or $1 - e^{-3.03}(1 + 3.03 + \frac{3.03^2}{2} + \frac{3.03^3}{3!} + \frac{3.03^4}{4!} + \frac{3.03^5}{5!} + \frac{3.03^6}{6!})$	M1	OE. Must see Poisson expression (may be in part (a)).
	0.0350 or 0.0351	A1	Accept 0.035. SC no working seen, award B1 for 0.0350, 0.0351 or 0.035.
		2	
8(c)	Concluding that the (mean) number of people (using the path per 20 mins in the evening) has increased when it has not	B1	OE. Conclusion must be in context.
		1	
8(d)	A value for the true mean	B1	Allow without context for this mark.
	Number of people using the path per 20 mins in the evening.	B1	Condone equivalent comment on three randomly chosen 20-minute periods.
		2	