

Cambridge International AS & A Level

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MATHEMATICS

9709/31

Paper 3 Pure Mathematics 3

May/June 2023

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.

1 Solve the equation

$$3e^{2x} - 4e^{-2x} = 5.$$

Give the answer correct to 3 decimal places.

[3]

This image shows a single page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- 2 (a) Sketch the graph of $y = |2x + 3|$.

[1]

- (b) Solve the inequality $3x + 8 > |2x + 3|$.

[3]

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- 3** Find the coefficient of x^3 in the binomial expansion of $(3+x)\sqrt{1+4x}$. [4]

[illegible]

- 4 (a) Show that the equation $\sin 2\theta + \cos 2\theta = 2 \sin^2 \theta$ can be expressed in the form

$$\cos^2 \theta + 2 \sin \theta \cos \theta - 3 \sin^2 \theta = 0. \quad [2]$$

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- (b) Hence solve the equation $\sin 2\theta + \cos 2\theta = 2 \sin^2 \theta$ for $0^\circ < \theta < 180^\circ$. [4]

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- 5** The equation of a curve is $x^2y - ay^2 = 4a^3$, where a is a non-zero constant.

(a) Show that $\frac{dy}{dx} = \frac{2xy}{2ay - x^2}$. [4]

This image shows a single page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- (b) Hence find the coordinates of the points where the tangent to the curve is parallel to the y-axis. [4]

[illegible]

- 6** Relative to the origin O , the points A , B and C have position vectors given by

$$\overrightarrow{OA} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 4 \\ 3 \\ 2 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 3 \\ -2 \\ -4 \end{pmatrix}.$$

The quadrilateral $ABCD$ is a parallelogram.

- (a) Find the position vector of D . [3]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- (b) The angle between BA and BC is θ .

Find the exact value of $\cos \theta$.

[3]

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- (c) Hence find the area of $ABCD$, giving your answer in the form $p\sqrt{q}$, where p and q are integers. [4]

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where $0 \leq x < \frac{1}{4}\pi$. It is given that $y = 0$ when $x = \frac{1}{6}\pi$.

Solve the differential equation to obtain the value of x when $y = \frac{1}{6}\pi$. Give your answer correct to 3 decimal places. [8]

[illegible]

8 Let $f(x) = \frac{3 - 3x^2}{(2x + 1)(x + 2)^2}$.

(a) Express $f(x)$ in partial fractions.

[5]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- (b)** Hence find the exact value of $\int_0^4 f(x) \, dx$, giving your answer in the form $a + b \ln c$, where a , b and c are integers. [5]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

9 The constant a is such that $\int_0^a x e^{-2x} dx = \frac{1}{8}$.

(a) Show that $a = \frac{1}{2} \ln(4a + 2)$.

[5]

This image shows a full page of a worksheet designed for handwriting practice. It consists of approximately 20 horizontal rows. Each row is defined by two parallel dotted lines, creating a series of uniform gaps for writing. The entire page is otherwise blank, with no margins, text, or other markings.

- (b) Verify by calculation that a lies between 0.5 and 1.

[2]

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- (c) Use an iterative formula based on the equation in (a) to determine a correct to 2 decimal places. Give the result of each iteration to 4 decimal places.

[3]

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10 The polynomial $x^3 + 5x^2 + 31x + 75$ is denoted by $p(x)$.

- (a)** Show that $(x + 3)$ is a factor of $p(x)$. [2]

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- (b)** Show that $z = -1 + 2\sqrt{6}i$ is a root of $p(z) = 0$. [3]

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- (c) Hence find the complex numbers z which are roots of $p(z^2) = 0$. [7]

[illegible]

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1 Solve the inequality $|5x - 3| < 2|3x - 7|$.

[4]

[illegible]

- 2** Solve the equation $\ln(2x^2 - 3) = 2 \ln x - \ln 2$, giving your answer in an exact form. [3]

[illegible]

- 3 (a) On an Argand diagram, sketch the locus of points representing complex numbers z satisfying $|z + 3 - 2i| = 2$. [2]

- (b) Find the least value of $|z|$ for points on this locus, giving your answer in an exact form. [2]

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- 4** Solve the equation $2 \cos x - \cos \frac{1}{2}x = 1$ for $0 \leq x \leq 2\pi$.

[5]

This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- 5 The complex number $2 + yi$ is denoted by a , where y is a real number and $y < 0$. It is given that $f(a) = a^3 - a^2 - 2a$.

(a) Find a simplified expression for $f(a)$ in terms of y . [3]

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(b) Given that $\operatorname{Re}(f(a)) = -20$, find $\arg a$. [3]

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6 The equation $\cot \frac{1}{2}x = 3x$ has one root in the interval $0 < x < \pi$, denoted by α .

(a) Show by calculation that α lies between 0.5 and 1.

[2]

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(b) Show that, if a sequence of positive values given by the iterative formula

$$x_{n+1} = \frac{1}{3} \left(x_n + 4 \tan^{-1} \left(\frac{1}{3x_n} \right) \right)$$

converges, then it converges to α .

[2]

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- (c) Use this iterative formula to calculate α correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

[illegible]

7 The equation of a curve is $3x^2 + 4xy + 3y^2 = 5$.

(a) Show that $\frac{dy}{dx} = -\frac{3x+2y}{2x+3y}$. [4]

[illegible]

- (b)** Hence find the exact coordinates of the two points on the curve at which the tangent is parallel to $y + 2x = 0$. [5]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- 8 (a) The variables x and y satisfy the differential equation

$$\frac{dy}{dx} = \frac{4 + 9y^2}{e^{2x+1}}.$$

It is given that $y = 0$ when $x = 1$.

Solve the differential equation, obtaining an expression for y in terms of x .

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- (b) State what happens to the value of y as x tends to infinity. Give your answer in an exact form. [1]

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9 Let $f(x) = \frac{2x^2 + 17x - 17}{(1 + 2x)(2 - x)^2}$.

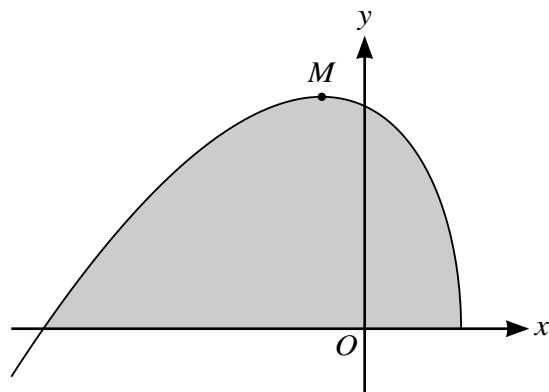
(a) Express $f(x)$ in partial fractions.

[5]

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(b) Hence show that $\int_0^1 f(x) \, dx = \frac{5}{2} - \ln 72$. [5]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



(a) Find the exact coordinates of M .

[5]

This image shows a full page of a handwriting practice worksheet. It consists of multiple sets of three horizontal dashed lines, providing a guide for letter height and placement. The lines are evenly spaced across the entire page, which is otherwise blank.

- (b) Using the substitution $u = 3 - 2x$, find by integration the area of the shaded region bounded by the curve and the x -axis. Give your answer in the form $a\sqrt{13}$, where a is a rational number. [5]

[illegible]

- 11** The points A and B have position vectors $\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$ and $2\mathbf{i} - \mathbf{j} + \mathbf{k}$ respectively. The line l has equation $\mathbf{r} = \mathbf{i} - \mathbf{j} + 3\mathbf{k} + \mu(2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k})$.

(a) Show that l does not intersect the line passing through A and B . [5]

[illegible]

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[illegible]

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- 1** Solve the equation $\ln(x + 5) = 5 + \ln x$. Give your answer correct to 3 decimal places. [4]

[illegible]

- 2** Find the quotient and remainder when $2x^4 - 27$ is divided by $x^2 + x + 3$. [3]

[illegible]

- 3 On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z - 3 - i| \leq 3$ and $|z| \geq |z - 4i|$. [4]

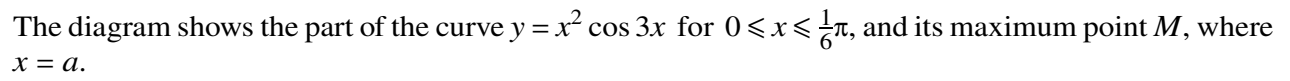
4 The parametric equations of a curve are

$$x = \frac{\cos \theta}{2 - \sin \theta}, \quad y = \theta + 2 \cos \theta.$$

Show that $\frac{dy}{dx} = (2 - \sin \theta)^2$.

[5]

[illegible]



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- (b)** Use an iterative formula based on the equation in **(a)** to determine a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

[illegible]

- 6 (a)** Express $3 \cos x + 2 \cos(x - 60^\circ)$ in the form $R \cos(x - \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. State the exact value of R and give α correct to 2 decimal places. [4]

[illegible]

(b) Hence solve the equation

$$3 \cos 2\theta + 2 \cos(2\theta - 60^\circ) = 2.5$$

for $0^\circ < \theta < 180^\circ$.

[4]

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7 (a) Use the substitution $u = \cos x$ to show that

$$\int_0^\pi \sin 2x e^{2\cos x} dx = \int_{-1}^1 2ue^{2u} du. \quad [4]$$

[illegible]

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8 The variables x and y satisfy the differential equation

$$\frac{dy}{dx} = \frac{y^2 + 4}{x(y + 4)}$$

for $x > 0$. It is given that $x = 4$ when $y = 2\sqrt{3}$.

Solve the differential equation to obtain the value of x when $y = 2$.

[8]

[illegible]

9 The lines l and m have equations

$$l: \mathbf{r} = a\mathbf{i} + 3\mathbf{j} + b\mathbf{k} + \lambda(c\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}),$$

$$m: \mathbf{r} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k} + \mu(2\mathbf{i} - 3\mathbf{j} + \mathbf{k}).$$

Relative to the origin O , the position vector of the point P is $4\mathbf{i} + 7\mathbf{j} - 2\mathbf{k}$.

- (a) Given that l is perpendicular to m and that P lies on l , find the values of the constants a , b and c . [4]

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- (b) The perpendicular from P meets line m at Q . The point R lies on PQ extended, with $PQ : QR = 2 : 3$.

Find the position vector of R .

[6]

[illegible]

10 Let $f(x) = \frac{21 - 8x - 2x^2}{(1 + 2x)(3 - x)^2}$.

(a) Express $f(x)$ in partial fractions.

[5]

[illegible]

- (b)** Hence obtain the expansion of $f(x)$ in ascending powers of x , up to and including the term in x^2 . [5]

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- 11** The complex number z is defined by $z = \frac{5a - 2i}{3 + ai}$, where a is an integer. It is given that $\arg z = -\frac{1}{4}\pi$.
- (a)** Find the value of a and hence express z in the form $x + iy$, where x and y are real. [6]

[illegible]

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- (b) Express z^3 in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leq \pi$. Give the simplified exact values of r and θ . [3]

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