



Cambridge International A Level

MATHEMATICS

9709/31

Paper 3 Pure Mathematics 3

May/June 2023

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

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GENERIC MARKING PRINCIPLE 1:

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- the specific content of the mark scheme or the generic level descriptors for the question
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- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

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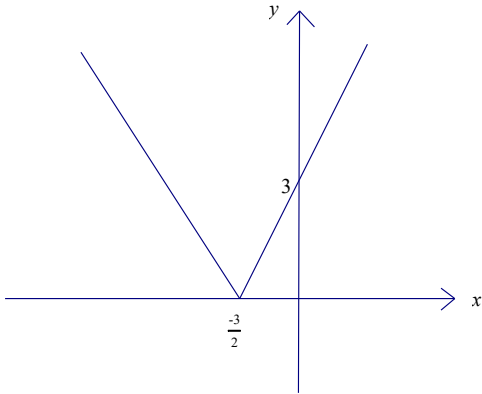
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- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
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Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1	$3(e^{2x})^2 - 5(e^{2x}) - 4 = 0$	B1	OE Form 3 term quadratic in e^{2x} .
	$e^{2x} = \frac{5 \pm \sqrt{73}}{6}, \quad x = \frac{1}{2} \ln \left(\frac{5 + \sqrt{73}}{6} \right)$	M1	Use correct method to solve for x .
	$x = 0.407$	A1	Only
		3	

Question	Answer	Marks	Guidance
2(a)		B1	Show a recognizable sketch graph of $y = 2x + 3$. (Ignore any attempt to sketch $y = 3x + 8$). Straight lines. Vertex in approximately correct position on x axis. Symmetry.
		1	

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Question	Answer	Marks	Guidance
2(b)	Find x -coordinate of intersection with $y = 3x + 8$	M1	
	Obtain $x = -\frac{11}{5}$	A1	
	State final answer $x > -\frac{11}{5}$ only	A1	$(x > -2.2)$ Do not condone $\geq$ for $>$.
	Alternative Method 1		
	Solve the linear inequality $3x + 8 > -(2x + 3)$, or corresponding linear equation	M1	
	Obtain critical value $x = -\frac{11}{5}$	A1	
	State final answer $x > -\frac{11}{5}$ only	A1	$(x > -2.2)$ Do not condone $\geq$ for $>$.
	Alternative Method 2		
	Solve the quadratic inequality $(3x + 8)^2 > (2x + 3)^2$, or corresponding quadratic equation	(M1)	$5x^2 + 36x + 55$.
	Obtain critical value $x = -\frac{11}{5}$	(A1)	Ignore -5 if seen.
	State final answer $x > -\frac{11}{5}$ only	(A1)	$(x > -2.2)$ Do not condone $\geq$ for $>$.
		3	

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Question	Answer	Marks	Guidance
3	State unsimplified term in x^3 , or its coefficient, in the expansion of $(1+4x)^{\frac{1}{2}}$	B1	$\frac{1}{2} \times \frac{-1}{2} \times \frac{-3}{2} (4x)^3 (=4)$ Must expand binomial coefficient.
	State unsimplified term in x^2 , or its coefficient, in the expansion of $(1+4x)^{\frac{1}{2}}$	B1	$\frac{1}{2} \times \frac{-1}{2} (4x)^2 (= -2)$ Must expand binomial coefficient.
	Multiply by $(3+x)$ and combine terms in x^3 , or their coefficients	M1	$(3 \times 4 - 1 \times 2)$ Allow if they expanded with x rather than $4x$.
	Obtain answer 10	A1	Accept $10x^3$
		4	

Question	Answer	Marks	Guidance
4(a)	Use correct double angle formulae	M1	e.g. $2\sin\theta\cos\theta + \cos^2\theta - \sin^2\theta = 2\sin^2\theta$
	Obtain $\cos^2\theta + 2\sin\theta\cos\theta - 3\sin^2\theta = 0$ from full and correct working	A1	AG Check conclusion is complete and matches the working.
		2	

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Question	Answer	Marks	Guidance
4(b)	Factorise to obtain $(\cos \theta - \sin \theta)(\cos \theta + 3 \sin \theta) = 0$	B1	OE
	Solve a quadratic in $\sin \theta$ and $\cos \theta$ to obtain a value for θ .	M1	$\tan \theta = 1$ or $\tan \theta = -\frac{1}{3}$.
	Obtain one correct value e.g. 45°	A1	
	Obtain a second correct value e.g. 161.6° and no others in the interval	A1	Mark answers in radians (0.785 and 2.82) as a misread. Accept awrt 161.6.
	Alternative Method 1		
	Obtain $3 \tan^2 \theta - 2 \tan \theta - 1 = 0$	B1	
	Solve a 3 term quadratic in $\tan \theta$ to obtain a value for θ .	M1	$\tan \theta = 1$ or $\tan \theta = -\frac{1}{3}$.
	Obtain one correct value e.g. 45°	A1	
	Obtain a second correct value e.g. 161.6° and no others in the interval	A1	Mark answers in radians (0.785 and 2.82) as a misread.
	Alternative Method 2		
	Obtain $(\cos \theta + \sin \theta)^2 = (2 \sin \theta)^2$	B1	
	Solve to obtain a value for θ .	M1	$\tan \theta = 1$ or $\tan \theta = -\frac{1}{3}$.
	Obtain one correct value e.g. 45°	A1	
	Obtain a second correct value e.g. 161.6° and no others in the interval	A1	Mark answers in radians (0.785 and 2.82) as a misread.
		4	

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Question	Answer	Marks	Guidance
5(a)	State or imply $2xy + x^2 \frac{dy}{dx}$ as derivative of x^2y	B1	Accept partial: $\frac{\partial}{\partial x} \rightarrow 2xy$.
	State or imply $2ay \frac{dy}{dx}$ as derivative of ay^2	B1	Accept partial: $\frac{\partial}{\partial y} \rightarrow x^2 - 2ay$.
	Equate attempted derivative to zero and solve for $\frac{dy}{dx}$	M1	
	Obtain answer $\frac{dy}{dx} = \frac{2xy}{2ay - x^2}$ from correct working	A1	AG
		4	
5(b)	State or imply $2ay - x^2 = 0$	*M1	
	Substitute into equation of curve to obtain equation in x and a or in y and a	DM1	e.g. $2ay^2 - ay^2 = 4a^3$ or $\frac{x^4}{2a} - \frac{x^4}{4a} = 4a^3$.
	Obtain one correct point	A1	e.g. $(2a, 2a)$.
	Obtain second correct point and no others	A1	e.g. $(-2a, 2a)$.
		4	SC: Allow A1 A0 for $x = \pm 2a$ or for $y = 2a$.

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Question	Answer	Marks	Guidance
6(a)	Obtain a vector for one side of the parallelogram	B1	e.g. $\overrightarrow{AB} = \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$ or $\overrightarrow{BC} = \begin{pmatrix} -1 \\ -5 \\ -6 \end{pmatrix}$.
	Correct method to obtain $\pm\overrightarrow{OD}$	M1	e.g. $\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{BC}$. MO if use $\overrightarrow{AB} = \overrightarrow{CD}$ or $\overrightarrow{BC} = \overrightarrow{DA}$.
	Obtain $\overrightarrow{OD} = \mathbf{i} - 4\mathbf{j} - 3\mathbf{k}$	A1	Any equivalent form. Accept coordinates.
		3	
6(b)	Using the correct process, evaluate the scalar product $\overrightarrow{BA} \cdot \overrightarrow{BC}$	M1	$(2 + 10 - 6)$ Scalar product of two relevant vectors. OE
	Using the correct process for the moduli, divide the scalar product by the product of the moduli.	M1	$\frac{2 + 10 - 6}{\sqrt{9} \times \sqrt{62}}$.
	Obtain answer $\frac{2}{\sqrt{62}}$	A1	ISW Or simplified equivalent i.e. $\frac{\sqrt{62}}{31}$.
		3	

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Question	Answer	Marks	Guidance
6(c)	State or imply $\sin \theta = \sqrt{\frac{58}{62}}$	B1 FT	Follow <i>their</i> $\cos \theta$.
	Use correct method to find the area of $ABCD$	M1	e.g. $2 \times \frac{1}{2} BA \times BC \sin \theta$. Condone decimals.
	Correct unsimplified expression for the area	A1 FT	e.g. $2 \times \frac{1}{2} \times 3 \times \sqrt{62} \times \sin \theta$. Condone decimals. Follow <i>their</i> sides and angle.
	Obtain answer $3\sqrt{58}$	A1	Correct only.
		4	

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Question	Answer	Marks	Guidance
7	Correct separation of variables	B1	$\int \sin^2 3y \, dy = \int 4 \sec 2x \tan 2x \, dx$ or equivalent. Condone missing integral signs or dx and dy.
	Integrate to obtain $k \sec 2x$	M1	
	Obtain $2 \sec 2x$	A1	
	Use double angle formula and integrate to obtain $py + q \sin 6y$	M1	Or two cycles of integration by parts.
	Obtain $\frac{1}{2}y - \frac{1}{12}\sin 6y$	A1	
	Use $y = 0, x = \frac{\pi}{6}$ in a solution containing terms $\lambda \sec 2x$ and $\mu \sin 6y$ to find the constant of integration	M1	
	Obtain $\frac{1}{2}y - \frac{1}{12}\sin 6y = 2 \sec 2x - 4$	A1	Or equivalent seen or implied by $\frac{\pi}{2} \left(-\frac{1}{12} \sin \pi \right) = 2 \sec 2x - 4$.
	Obtain $x = 0.541$	A1	From correct working (not by using the calculator to integrate).
		8	

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Question	Answer	Marks	Guidance
8(a)	State or imply the form $\frac{A}{2x+1} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$	B1	Accept $\frac{A}{2x+1} + \frac{Dx+E}{(x+2)^2}$.
	Use a correct method for finding a constant	M1	
	Obtain one of $A=1, B=-2, C=3$	A1	For alternative form: $A=1, D=-2, E=-1$.
	Obtain a second value	A1	
	Obtain the third value	A1	
		5	
8(b)	Integrate and obtain one of $\frac{1}{2}\ln(2x+1), -2\ln(x+2), \frac{-3}{x+2}$	B1 FT	The follow through is on <i>their</i> A, B, C .
	Obtain a second term	B1 FT	If the alternative form is used, then either need to use integration by parts or split the fraction further.
	Obtain the third term	B1 FT	
	Substitute limits correctly in an integral with at least two terms of the form $\frac{1}{2}\ln(2x+1), -2\ln(x+2)$ and $\frac{-3}{x+2}$ and subtract in correct order	M1	The terms used need to have been obtained correctly. Must be exact values, not decimals.
	Obtain $1 - \ln 3$	A1	
		5	

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Question	Answer	Marks	Guidance
9(a)	Commence integration and reach $pxe^{-2x} + q \int e^{-2x} dx$	*M1	OE
	Obtain $-\frac{1}{2}xe^{-2x} + \frac{1}{2} \int e^{-2x} dx$	A1	OE
	Complete integration and obtain $-\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x}$	A1	
	Use limits correctly and equate to $\frac{1}{8}$, having integrated twice	DM1	$-\frac{1}{2}ae^{-2a} - \frac{1}{4}e^{-2a} + \frac{1}{4} = \frac{1}{8}$.
	Obtain $a = \frac{1}{2} \ln(4a + 2)$ correctly	A1	AG
		5	
9(b)	Calculate the values of a relevant expression or pair of expressions at $a = 0.5$ and $a = 1$	M1	
	Justify the given statement with correct calculated values	A1	e.g. $0.5 < 0.69\dots$, $1 > 0.89\dots$ $0.193 > 0$, $-1.105 < 0$ $0.066 < 0.125$, $0.148 > 0.125$ if put limits in the integral. Condone if they use calculator for the definite integral.
		2	
9(c)	Use the iterative process $a_{n+1} = \frac{1}{2} \ln(4a_n + 2)$ correctly at least once.	M1	
	Obtain final answer 0.84	A1	
	Show sufficient iterations to at least 4 d.p. to justify 0.84 to 2 d.p. or show that there is a sign change in (0.835, 0.845)	A1	e.g. 0.75, 0.8047, 0.8261, 0.8343, 0.8373, 0.8385 1, 0.8959, 0.8599, 0.8469, 0.8420, 0.8402 .
		3	

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Question	Answer	Marks	Guidance
10(a)	Substitute $x = -3$ to obtain value of $p(-3)$	M1	
	Obtain $p(-3) = 0$ and hence given result	A1	
	Alternative method for Question 10(a)		
	Divide $p(x)$ by $(x + 3)$ to obtain quotient $x^2 \pm 2x + \dots$	M1	
	Obtain quotient $x^2 + 2x + 25$, with zero remainder and hence given result	A1	
		2	

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Question	Answer	Marks	Guidance
10(b)	Substitute $z = -1 + 2\sqrt{6}i$ and attempt expansions of z^2 and z^3	M1	$z^2 = -23 - 4\sqrt{6}i$, $z^3 = -1 + 6\sqrt{6}i + 72 - 48\sqrt{6}i$.
	Use $i^2 = -1$	M1	Seen at least once.
	Obtain $p(z) = 0$ and hence given result	A1	SC B1 if there is no evidence of working for the square or the cube. Total 1/3.
	Alternative Method 1		
	Use roots $z = -1 + 2\sqrt{6}i$ to form quadratic factor	M1	$z^2 + 2z + 25$.
	Divide $p(z)$ by <i>their</i> quadratic factor	M1	
	Obtain zero remainder and hence given result.	A1	
	Alternative Method 2		
	Set <i>their</i> quadratic factor from (a) equal to zero	M1	
	Solve for z	M1	Need to see method here as answer is given.
	Obtain $z = -1 + 2\sqrt{6}i$ (and $z = -1 - 2\sqrt{6}i$)	A1	
	Alternative Method 3		
	Substitute $z = -1 + 2\sqrt{6}i$ into <i>their</i> quadratic factor and attempt expansion of z^2	M1	
	Use $i^2 = -1$	M1	
	Obtain 0 and hence given result	A1	
		3	

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Question	Answer	Marks	Guidance
10(c)	State $z_1 = \sqrt{3}i$ and $z_2 = -\sqrt{3}i$	B1	
	Expand $(x + iy)^2 = -1 + 2\sqrt{6}i$ and compare real and imaginary parts	M1	Allow for use of $z^2 = -1 - 2\sqrt{6}i$.
	Obtain $x^2 - y^2 = -1$ and $xy = \sqrt{6}$	A1	
	Solve to obtain x and y	M1	
	Obtain $z_3 = \sqrt{2} + \sqrt{3}i$ and $z_4 = -\sqrt{2} - \sqrt{3}i$	A1	
	Use $z^2 = -1 - 2\sqrt{6}i$ to obtain z_5 and z_6	M1	Allow for use of $z^2 = -1 + 2\sqrt{6}i$.
	Obtain $z_5 = \sqrt{2} - \sqrt{3}i$ and $z_6 = -\sqrt{2} + \sqrt{3}i$	A1	
		7	



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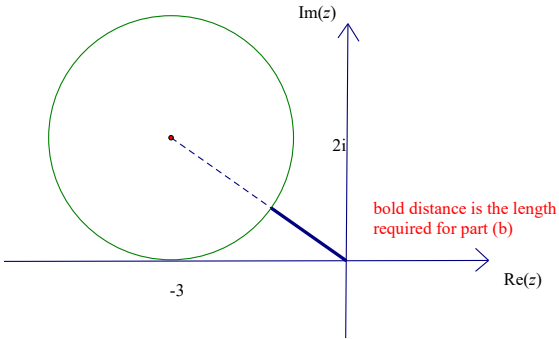
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WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1	State or imply non-modular inequality $(5x-3)^2 < 2^2(3x-7)^2$, or corresponding quadratic equation, or pair of linear equations $(5x-3) = \pm 2(3x-7)$	B1	$11x^2 - 138x + 187 > 0$.
	Solve a 3-term quadratic, or solve two linear equations for x	M1	If no working is shown, the M1 is implied by the correct roots for an incorrect quadratic.
	Obtain critical values $x = \frac{17}{11}$ and $x = 11$	A1	Accept 1.55 or better.
	State final answer $x < \frac{17}{11}$, $x > 11$	A1	Strict inequality required. In set notation, allow notation for open sets but not for closed sets e.g. accept $(-\infty, \frac{17}{11}) \cup (11, \infty)$ or $(-\infty, \frac{17}{11}[\cup]11, \infty)$ but not $(-\infty, \frac{17}{11}] \cup [11, \infty)$. Allow 'or' but not 'and'. Accept $\cup$. Final A0 for $\frac{17}{11} > x > 11$. Exact values expected but ISW if exact inequalities seen followed by decimal approx.
	Alternative Method for Question 1		
	Obtain critical value $x = 11$ from a graphical method, or by inspection, or by solving a linear equation or an inequality	B1	
	Obtain critical value $x = \frac{17}{11}$ similarly	B2	Accept decimal value.
	State final answer $x < \frac{17}{11}$, $x > 11$	B1	Strict inequality required. See notes above.
		4	

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Question	Answer	Marks	Guidance
2	Use law of the logarithm of a power, quotient or product	M1	Must be used correctly on a correct term. e.g. M1 for $2 \ln x = \ln x^2$ but M0 for $2 \ln x - \ln 2 = 2 \ln \frac{x}{2}$. M0 for $\ln(2x^2 - 3) = \ln 2x^2 - \ln 3$ $= \ln 2 + 2 \ln x - \ln 3$.
	Remove logarithms and obtain a correct equation in x	A1	e.g. $2x^2 - 3 = \frac{x^2}{2}$.
	Obtain final answer $x = \sqrt{2}$ only	A1	If $x = -\sqrt{2}$ is mentioned, it must be rejected.
		3	

Question	Answer	Marks	Guidance
3(a)		B1	Show a circle with centre $-3 + 2i$. Allow for a curved figure with 'centre' in roughly the correct position. Accept marks or numbers on axes, coordinates of centre shown. B0B1 available for axes the wrong way round (and M1 A1 in part (b)).
	Show a circle with radius 2	B1 FT	FT centre not at the origin. Allow 'near miss' on x axis. Different scales on axes require an ellipse for B1 B1. Scales on the axes and any label of the radius must be consistent for B1 B1. Correct circle shaded scores B1 B0.
		2	

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Question	Answer	Marks	Guidance
3(b)	Carry out a correct method for finding the least value of $ z $	M1	e.g. distance of centre from origin – radius or find point of intersection of circle and $3y = -2x$ and use Pythagoras. If they subtract the wrong way round M0. If their diagram is a reflection or a rotation of the correct diagram, M1 A1 is available (requires equivalent work). Any other circle M0.
	Obtain answer $\sqrt{13} - 2$ or $\sqrt{17 - 4\sqrt{13}}$	A1	Or exact equivalent e.g. $\sqrt{17 - \frac{26}{3}\sqrt{\frac{36}{13}}}$. Correct solution only. Allow A1 if exact answer seen and then decimal given.
		2	

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Question	Answer	Marks	Guidance
4	Use correct double angle formula to obtain an equation in $\cos\left(\frac{x}{2}\right)$ only	*M1	e.g. $2\left(2\cos^2\left(\frac{x}{2}\right)-1\right)-\cos\left(\frac{x}{2}\right)=1$.
	Obtain a 3 term quadratic in $\cos\left(\frac{x}{2}\right)$,	A1	e.g. $4\cos^2\left(\frac{x}{2}\right)-\cos\left(\frac{x}{2}\right)-3=0$. Allow $4\cos^2 u - \cos u - 3 = 0$. Condone $\frac{x}{2} = x$.
	Obtain $\cos\left(\frac{x}{2}\right) = -\frac{3}{4}$ and $\cos\left(\frac{x}{2}\right) = 1$	A1	Allow answer in u e.g. $(4\cos u + 3)(\cos u - 1)$ and condone $\frac{x}{2} = x$.
	Solve for the original x	DM1	Must see evidence of doubling, not halving.
	Obtain $x = 0$ and 4.84 and no others in the interval	A1	Ignore any answers outside interval. Accept AWR 4.84. Accept 1.54π . Must be in radians. 277.2 indicates M1 but is A0.
	Alternative Method for Question 4		
	Use correct double angle formula to obtain an equation in $\cos x$ only	*M1	e.g. $2\cos x - 1 = \sqrt{\frac{\cos x + 1}{2}}$.
	Obtain a 3 term quadratic in $\cos x$,	A1	e.g. $8\cos^2 x - 9\cos x + 1 = 0$.
	Obtain $\cos x = \frac{1}{8}$ and $\cos x = 1$	A1	
	Solve for x	DM1	
	Obtain answers $x = 0$ and 4.84 and no others in the interval	A1	Ignore any answers outside interval. Accept AWR 4.84. Must be in radians. 277.2 is A0.
		5	

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Question	Answer	Marks	Guidance
5(a)	Substitute $2 + yi$ in $a^3 - a^2 - 2a$ and attempt expansions of a^2 and a^3	M1	$a^2 = 4 + 4yi - y^2$ $a^3 = 8 + 12yi - 6y^2 - y^3i$. If using $a(a^2 - a - 2)$ must then expand fully. Must see working.
	Use $i^2 = -1$	M1	Seen at least once (e.g. in squaring).
	Obtain final answer $-5y^2 + (6y - y^3)i$	A1	Or simplified equivalent e.g. $6yi - 5y^2 - y^3i$. Do not ISW.
		3	No evidence of working for the square or the cube can score SC B1 for the correct answer.
5(b)	Equate <i>their</i> $-5y^2$ to -20 and solve for y	M1	Need to obtain a value for y . Available even if <i>their</i> y is not real.
	Obtain $y = -2$	A1	From correct work. Allow after incorrect $f(a)$ if the real part was correct. Condone ± 2 with positive not rejected.
	Obtain final answer $\arg a = -\frac{\pi}{4}$	A1	Correct only (must have rejected y positive). OE e.g. $-\frac{\pi}{4} \pm 2n\pi$. Accept $-0.785, 5.50$. Allow after incorrect $f(a)$ if the real part was correct. Accept degrees. Do not ISW.
		3	

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Question	Answer	Marks	Guidance
6(a)	Calculate the values of a relevant expression or pair of expressions at $x = 0.5$ and $x = 1$	M1	Need to evaluate at both points, but M1 still available if one value incorrect. Use of degrees is M0. Correct use of a smaller interval is M1. If using $g(x) - f(x)$, there needs to be a clear indication of the comparison being made e.g. by listing values in a table. Embedded values 0.5 and 1 are not sufficient. 3.92 and 1.83 alone are not sufficient.
	Complete the argument correctly with conclusion about change of sign or change of inequalities and with correct calculated values. Can all be in symbols – an explanation in words is not required.	A1	e.g. $3.92 > 1.5$, $1.83 < 3$ or $2.42 > 0$, $-1.17 < 0$.
		2	
6(b)	State $x = \frac{1}{3} \left(x + 4 \tan^{-1} \frac{1}{3x} \right)$	M1	Or rearrange $\cot\left(\frac{x}{2}\right) = 3x$ as far as $2x = 4 \tan^{-1}\left(\frac{1}{3x}\right)$
	Rearrange to the given equation $\cot\left(\frac{x}{2}\right) = 3x$ Need intermediate step between $\frac{x}{2} = \tan^{-1} \frac{1}{3x}$ and $\cot\left(\frac{x}{2}\right) = 3x$	A1	Or continue rearrangement to $x = \frac{1}{3} \left(x + 4 \tan^{-1} \frac{1}{3x} \right)$ and state iterative formula of $x_{n+1} = \frac{1}{3} \left(x_n + 4 \tan^{-1} \frac{1}{3x_n} \right)$ AG
		2	

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Question	Answer	Marks	Guidance
6(c)	Use the iterative process correctly at least once	M1	Obtain one value and substitute that back in to obtain a second value. Working in degrees is M0.
	Obtain final answer 0.79	A1	Must be to 2 d.p.
	Show sufficient iterations to at least 4 d.p. to justify 0.79 to 2 d.p. or show there is a sign change in the interval (0.785, 0.795)	A1	e.g. 1, 0.7623, 0.8037, 0.7921, 0.7951, 0.7943, 0.7945 or 0.5, 0.9506, 0.7665, 0.8024, 0.7924, 0.7950, 0.7944, 0.7945 or 0.75, 0.8076, 0.7911, 0.7954, 0.7943, 0.7946, 0.7945 . Condone truncation. Allow recovery. Condone minor differences in the final d.p.
		3	If they do the iteration in (b) but restate the conclusion here, no marks in (b) but could score 3/3 for (c) .

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Question	Answer	Marks	Guidance
7(a)	State or imply $6y \frac{dy}{dx}$ as the derivative of $3y^2$	B1	Allow y' for $\frac{dy}{dx}$ throughout. Accept $\frac{\partial f}{\partial x} = 6x + 4y$.
	State or imply $4x \frac{dy}{dx} + 4y$ as the derivative of $4xy$	B1	Accept $\frac{\partial f}{\partial y} = 4x + 6y$.
	Equate derivative of LHS to zero and solve for $\frac{dy}{dx}$	M1	Allow an extra $\frac{dy}{dx}$ in front of their differentiated equation. Allow if '= 0' is implied but not seen. Allow $\frac{dy}{dx} = - \frac{\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial y}}$
	Obtain $\frac{dy}{dx} = -\frac{3x+2y}{2x+3y}$	A1	AG – must come from correct working. The position of the negative must be clear.
		4	

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Question	Answer	Marks	Guidance
7(b)	Equate $\frac{dy}{dx}$ to -2 and solve for x in terms of y or for y in terms of x	*M1	Must be using the given derivative.
	Obtain $x = -4y$ or $y = -\frac{x}{4}$	A1	Seen or implied by correct later work.
	Substitute <i>their</i> $x = -4y$ or <i>their</i> $y = -\frac{x}{4}$ in curve equation	DM1	Allow unsimplified.
	Obtain $y = \pm \frac{1}{\sqrt{7}}$ or $x = \pm \frac{4}{\sqrt{7}}$	A1	Or exact equivalent. Or $x = \frac{4}{\sqrt{7}}$ and $y = -\frac{1}{\sqrt{7}}$ or exact equivalent.
	Obtain both pairs of values	A1	Or $x = -\frac{4}{\sqrt{7}}$ and $y = \frac{1}{\sqrt{7}}$ or exact equivalent. A1 A0 for incorrect final pairing.
		5	

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Question	Answer	Marks	Guidance
8(a)	Separate variables correctly	B1	$\int \frac{1}{4+9y^2} dy = \int e^{-(2x+1)} dx$. Condone missing integral signs or dx and dy missing.
	Obtain term $-\frac{1}{2}e^{-2x-1}$	B1	OE e.g. $-\frac{1}{2e}e^{-2x}$.
	Obtain term of the form $a \tan^{-1}\left(\frac{3y}{2}\right)$	M1	
	Obtain term $\frac{1}{6} \tan^{-1}\left(\frac{3y}{2}\right)$	A1	OE e.g. $\frac{1}{9} \times \frac{3}{2} \tan^{-1} \frac{3y}{2}$.
	Use $x = 1, y = 0$ to evaluate a constant or as limits in a solution containing or derived from terms of the form $a \tan^{-1}(by)$ and $ce^{\pm(2x+1)}$	M1	If they rearrange before evaluating the constant, the constant must be of the correct form.
	Obtain correct answer in any form	A1	e.g. $\frac{1}{6} \tan^{-1} \frac{3y}{2} = \frac{1}{2}e^{-3} - \frac{1}{2}e^{-(2x+1)}$.
	Obtain final answer $y = \frac{2}{3} \tan(3e^{-3} - 3e^{-2x-1})$	A1	OE Allow with $3e^{-3} = 0.149...$
		7	
8(b)	State that y approaches $\frac{2}{3} \tan(3e^{-3})$	B1 FT	Or exact equivalent. The FT is on correct work on a solution containing e^{-2x-1} . Condone $y = \dots$ Accept correct answer stated with minimal wording. 0.10032... is not exact so B0.
		1	

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Question	Answer	Marks	Guidance
9(a)	State or imply the form $\frac{A}{1+2x} + \frac{B}{2-x} + \frac{C}{(2-x)^2}$	B1	Alternative form: $\frac{A}{1+2x} + \frac{Dx+E}{(2-x)^2}$
	Use a correct method for finding a coefficient	M1	e.g. $A(2-x)^2 + B(1+2x)(2-x) + C(1+2x)$ $= 2x^2 + 17x - 17$ and compare coefficients or substitute for x . $A(2-x)^3 + B(1+2x)(2-x)^2 + C(1+2x)(2-x)$ $= 2x^2 + 17x - 17$ scores M0.
	Obtain one of $A = -4$, $B = -3$ and $C = 5$	A1	
	Obtain a second value	A1	
	Obtain the third value	A1	Extra term in partial fractions, then B0 unless recover at end. Allow the marks for any constants found correctly. Missing terms in partial fractions, B0 but M1A1 is available for a correct method that obtains at least one correct constant (e.g. cover-up rule) Max 2/5. Ignore any substitution back into their original expression. If alternative form used: $A = -4$, $D = 3$ and $E = -1$.
		5	

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Question	Answer	Marks	Guidance
9(b)	Integrate and obtain terms $-2\ln(1+2x) + 3\ln(2-x) + \frac{5}{2-x}$	B1FT	OE
		B1FT	The FT is on correct use of <i>their</i> A , B and C ; or on A , D and E .
		B1FT	If using the A , D , E form then B1 for the A term, but no further marks until partial fractions are used to split the second term or they use integration by parts to obtain $\frac{Dx+E}{2-x} - \int \frac{D}{2-x} dx$ for the 2 nd B1 and 3 rd B1 for correct completion. B0FT, B0FT, B0FT if they place <i>their</i> A , B , C with incorrect denominators.
	Substitute limits correctly in an integral with two terms (obtained correctly) of the form $a\ln(1+2x) + b\ln(2-x) + \frac{c}{2-x}$, where $abc \neq 0$	M1	Condone minor slips in substitution. Exact substitution required.
	Obtain answer $\frac{5}{2} - \ln 72$ after full and correct working	A1	AG – evidence of some correct work to combine or simplify logs is required e.g. allow from $-\ln 9 + \ln \frac{1}{8}$ or $-\ln 2^3 - \ln 3^2$.
		5	

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Question	Answer	Marks	Guidance
10(a)	Use the product rule correctly to obtain $p(x+5)(3-2x)^n + q(3-2x)^{\frac{1}{2}}$	*M1	Allow with incorrect chain rule. BOD over sign errors unless an incorrect rule is quoted.
	Obtain correct derivative in any form	A1	e.g. $-(x+5)(3-2x)^{-\frac{1}{2}} + (3-2x)^{\frac{1}{2}}$.
	Equate derivative to zero and obtain a linear equation	DM1	Allow with surd factor e.g. $(3-2x)^{-\frac{1}{2}}(-(x+5) + (3-2x)) = 0$.
	Obtain a correct linear equation.	A1	e.g. $-(x+5) + 3 - 2x = 0$.
	Obtain answer $\left(-\frac{2}{3}, \frac{13\sqrt{39}}{9}\right)$.	A1	Or exact equivalent e.g. $\left(-\frac{2}{3}, \frac{13\sqrt{13}}{3\sqrt{3}}\right)$ or $\left(-\frac{2}{3}, \frac{\sqrt{2197}}{\sqrt{27}}\right)$. Accept with x, y stated separately. ISW
	Alternative Method for Question 10(a)		
	Obtain y^2 and differentiate	*M1	Ignore <i>their</i> left hand side i.e. <i>their</i> $\frac{d}{dx}y^2$.
	Obtain correct derivative in any form	A1	e.g. $-6x^2 - 34x - 20$.
	Equate derivative to zero and solve for x	DM1	
	Obtain $-\frac{2}{3}$	A1	Ignore -5 if seen.
	Obtain answer $\left(-\frac{2}{3}, \frac{13\sqrt{39}}{9}\right)$ only	A1	Or exact equivalent e.g. $\left(-\frac{2}{3}, \frac{13\sqrt{13}}{3\sqrt{3}}\right)$ or $\left(-\frac{2}{3}, \frac{\sqrt{2197}}{\sqrt{27}}\right)$. ISW
		5	

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Question	Answer	Marks	Guidance
10(b)	Use the given substitution and reach $a \int \left(\frac{13}{2} - \frac{u}{2} \right) u^{\frac{1}{2}} du$	*M1	OE Need to see -2 or -½ used. Condone if du missing or the integral sign is missing. Allow M1A0 for complete substitution into $\int x\sqrt{3-2x} dx$ to obtain first term of the line below.
	Obtain correct integral $-\frac{1}{2} \int \left(\frac{13}{2} - \frac{u}{2} \right) u^{\frac{1}{2}} du$	A1	OE e.g. $-\frac{1}{2} \left[\int \frac{3-u}{2} \sqrt{u} du + 5 \int \sqrt{u} du \right]$. Ignore limits at this stage. Condone if du missing.
	$x = -5$ and $\frac{3}{2}$	B1	SOI e.g. by $u = 13$ and 0. In any order and at any stage.
	Use correct limits the right way round in an integral of the form $a \left(\frac{26}{3} u^{\frac{3}{2}} - \frac{2}{5} u^{\frac{5}{2}} \right)$	DM1	
	Obtain answer $\frac{169}{15} \sqrt{13}$ or $a = \frac{169}{15}$	A1	or exact equivalents.
		5	

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Question	Answer	Marks	Guidance																																
11(a)	Carry out correct method for finding a vector equation for AB	M1																																	
	Obtain $[\mathbf{r} =] \mathbf{i} + 2\mathbf{j} - 2\mathbf{k} + \lambda (\mathbf{i} - 3\mathbf{j} + 3\mathbf{k})$	A1	OE e.g. $\mathbf{r} = 2\mathbf{i} - \mathbf{j} + \mathbf{k} + \lambda (-\mathbf{i} + 3\mathbf{j} - 3\mathbf{k})$.																																
	Equate two pairs of components of general points on <i>their</i> AB and l and evaluate λ or μ	M1	$\begin{pmatrix} 1 + \lambda \\ 2 - 3\lambda \\ -2 + 3\lambda \end{pmatrix} = \begin{pmatrix} 1 + 2\mu \\ -1 - 3\mu \\ 3 + 4\mu \end{pmatrix}.$																																
	Obtain correct answer for λ or μ , e.g. $\lambda = -1, \mu = -2$	A1	Correct value from two correct component equations.																																
	Verify that all three equations are not satisfied and the lines fail to intersect ($\neq$ is sufficient justification e.g. $0 \neq -3$).	A1	Conclusion needs to follow correct values. Hybrid versions are possible e.g. using $\mathbf{j}$ and $\mathbf{k}$ to get one parameter and then $\mathbf{i}$ to obtain the other. or e.g. solving two pairs of simultaneous equations and showing that the results are not the same. Alternatives: <table><tr><td>A</td><td>λ</td><td>μ</td><td></td><td>B</td><td>λ</td><td>μ</td><td></td></tr><tr><td>$\mathbf{ij}$</td><td>2</td><td>1</td><td>$4 \neq 7$</td><td>$\mathbf{ij}$</td><td>1</td><td>1</td><td>$4 \neq 7$</td></tr><tr><td>$\mathbf{ik}$</td><td>5</td><td>5/2</td><td>$-13 \neq -17/2$</td><td>$\mathbf{ik}$</td><td>4</td><td>5/2</td><td>$-13 \neq -17/2$</td></tr><tr><td>$\mathbf{jk}$</td><td>-1</td><td>-2</td><td>$0 \neq -3$</td><td>$\mathbf{jk}$</td><td>-2</td><td>-2</td><td>$0 \neq -3$</td></tr></table>	A	λ	μ		B	λ	μ		$\mathbf{ij}$	2	1	$4 \neq 7$	$\mathbf{ij}$	1	1	$4 \neq 7$	$\mathbf{ik}$	5	5/2	$-13 \neq -17/2$	$\mathbf{ik}$	4	5/2	$-13 \neq -17/2$	$\mathbf{jk}$	-1	-2	$0 \neq -3$	$\mathbf{jk}$	-2	-2	$0 \neq -3$
	A	λ	μ		B	λ	μ																												
$\mathbf{ij}$	2	1	$4 \neq 7$	$\mathbf{ij}$	1	1	$4 \neq 7$																												
$\mathbf{ik}$	5	5/2	$-13 \neq -17/2$	$\mathbf{ik}$	4	5/2	$-13 \neq -17/2$																												
$\mathbf{jk}$	-1	-2	$0 \neq -3$	$\mathbf{jk}$	-2	-2	$0 \neq -3$																												
		5																																	

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Question	Answer	Marks	Guidance
11(b)	Find $\overrightarrow{AP}$ for a general point P on l , e.g. $-3\mathbf{j} + 5\mathbf{k} + \mu(2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k})$	B1	Or equivalent e.g. $\overrightarrow{PA} = -2\mu\mathbf{i} + (3\mu + 3)\mathbf{j} - (4\mu + 5)\mathbf{k}$.
	Calculate scalar product of <i>their</i> $\overrightarrow{AP}$ and a direction vector for l and equate the result to zero	M1	e.g. $4\mu + (9 + 9\mu) + (20 + 16\mu) = 0$. M0 if using $\overrightarrow{OP}$. M0 if using parallel line through A .
	Obtain $\mu = -1$	A1	
	Obtain answer $-\mathbf{i} + 2\mathbf{j} - \mathbf{k}$	A1	Accept coordinates in place of position vector.
	Alternative Method for Question 11(b)		
	Find $\overrightarrow{AP}$ for a general point P on l , e.g. $-3\mathbf{j} + 5\mathbf{k} + \mu(2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k})$	B1	Or equivalent e.g. $\overrightarrow{PA} = -2\mu\mathbf{i} + (3\mu + 3)\mathbf{j} - (4\mu + 5)\mathbf{k}$.
	Use Pythagoras and differentiate with respect to μ to obtain value of μ corresponding to minimum distance. (No need to prove it is a minimum)	M1	$\frac{d}{d\mu}(4\mu^2 + 9(\mu + 1)^2 + (4\mu + 5)^2) = 0$.
	Obtain $\mu = -1$	A1	
	Obtain answer $-\mathbf{i} + 2\mathbf{j} - \mathbf{k}$	A1	Accept coordinates in place of position vector.
		4	



Cambridge International A Level

MATHEMATICS

9709/33

Paper 3 Pure Mathematics 3

May/June 2023

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

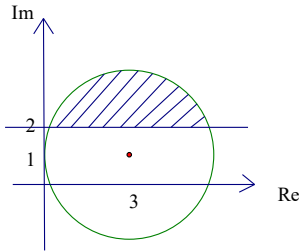
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Question	Answer	Marks	Guidance
1	Use exponentials or law for the logarithm of a product, quotient or power	M1*	$e^{\ln(5+x)} = e^{5+\ln x}$ insufficient. Need e.g. $\ln\left(\frac{x+5}{x}\right) = 5$ or $\ln(x+5) = \ln(e^5) + \ln x$ or $\ln(x+5) = \ln(e^5 x)$ or $x+5 = e^{5+\ln x}$ or $x+5 = e^5 e^{\ln x}$ and others.
	Correctly remove logarithms	DM1	
	Obtain a correct equation in x	A1	e.g. $\frac{x+5}{x} = e^5$ (or 148.4...) or $x+5 = xe^5$.
	Obtain 0.034	A1	CAO Final answer must be 3d.p.
		4	

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Question	Answer	Marks	Guidance
2	Divide to obtain quotient $2x^2 \pm 2x + k$ ($k \neq 0$)	M1	Obtain result in answer column, together with a linear polynomial or a constant as remainder. If correct: $ \begin{array}{r} \overline{2x^2 - 2x - 4} \\ x^2 + x + 3 \overline{2x^4} - 27 \\ \overline{2x^4 + 2x^3 + 6x^2} \\ \overline{-2x^3 - 6x^2} \\ \overline{-2x^3 - 2x^2 - 6x} \\ \overline{-4x^2 + 6x - 27} \\ \overline{-4x^2 - 4x - 12} \\ 10x - 15 \end{array} $
	Obtain [quotient] $2x^2 - 2x - 4$	A1	Allow unless quotient and remainder interchanged, then A0 A1.
	Obtain [remainder] $10x - 15$	A1	Allow $(x^2 + x + 3)(2x^2 - 2x - 4) + 10x - 15$.
	Alternative Method for Question 2		
	Expand $(x^2 + x + 3)(Ax^2 + Bx + C) + (Dx + E)$ and reach $A = 2$, $B = \pm 2$, $C = k$	M1	Solve all 3 equations for A , B and C , allow sign errors in establishing equations and in solving. If correct, $A = 2$, $A + B = 0$, $3A + B + C = 0$, $3B + C + D = 0$, $3C + E = -27$. Obtain result in answer column, together with a linear polynomial or a constant as remainder.
	Obtain [quotient] $2x^2 - 2x - 4$	A1	Allow unless quotient and remainder interchanged, then A0 A1.
	Obtain [remainder] $10x - 15$	A1	Allow $(x^2 + x + 3)(2x^2 - 2x - 4) + 10x - 15$.
		3	

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Question	Answer	Marks	Guidance
3	Show a circle with centre $3 + i$	B1	Must be some evidence of scale on both axes or centre stated as $3 + i$ or $(3, 1)$.
	Show a circle with radius 3 and centre not at the origin	B1	Must be some evidence that radius = 3 or stated $r = 3$
	Show the line $y = 2$	B1	Line $y = 2$ can be represented by 2 or correct dashes.
	Shade the correct region	B1	Line and circle must be correct.
		4	Scales may be replaced by dashes on axes for all marks. Correct figure, with no scale on either axis then allow 1/3 and the B1 for correct shaded region Max 2/4. If B0 above for line but relatively correct position then B1 for correct shaded region Max 3/4. Re and Im axes interchanged but clearly labelled, allow SCB1 for centre and radius of circle correct and SCB1 for line and shading correct Max 2/4.

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Question	Answer	Marks	Guidance
4	State $\frac{dy}{d\theta} = 1 - 2\sin\theta$	B1	Ignore left side throughout dx/dt , dy/dt , dx , dy but must see $\frac{dy}{dx}$ for final A1.
	Use correct quotient rule, or product rule if rewrite x as $\cos\theta(2 - \sin\theta)^{-1}$	M1	Incorrect formula seen M0 A0 otherwise BOD.
	Obtain $\frac{dx}{d\theta} = \frac{-(2 - \sin\theta)\sin\theta + \cos^2\theta}{(2 - \sin\theta)^2}$ o.e.	A1	$-\sin\theta(2 - \sin\theta)^{-1} - \cos\theta(2 - \sin\theta)^{-2}(-\cos\theta)$ or equivalent.
	Use $\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta}$	M1	$\left(\frac{dy}{dx} = (1 - 2\sin\theta) \div \frac{1 - 2\sin\theta}{(2 - \sin\theta)^2} \right)$. Allow M1 even if errors in both derivatives.
	Obtain $\frac{dy}{dx} = (2 - \sin\theta)^2$.	A1	AG – must see working in above cell to gain final A1. Allow $\cos^2\theta + \sin^2\theta = 1$ to be implied. x instead of θ or missing θ more than twice on right side then A0 final mark.
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Question	Answer	Marks	Guidance
5(a)	Use correct product rule	M1	$\frac{d}{dx}(x^2)\cos(3x) + x^2 \frac{d}{dx}(\cos 3x).$
	Obtain correct derivative in any form	A1	e.g. $2x \cos 3x - 3x^2 \sin 3x$.
	Equate derivative to zero and obtain $a = \frac{1}{3} \tan^{-1}\left(\frac{2}{3a}\right).$	A1	AG Condone $a = \frac{1}{3} \tan^{-1} \frac{2}{3a}$. Must at least reach expression $2x = 3x^2 \tan(3x)$ or better <u>before</u> final answer to gain A1. Final answer must be in terms of a . Can work with x and switch to a at very end. Look for $\frac{2}{3}a$ or $\frac{2}{3}x$ in working not immediately corrected or as penultimate line A0.
		3	
5(b)	Use the iterative process $a_{n+1} = \frac{1}{3} \tan^{-1}\left(\frac{2}{3a_n}\right)$ correctly at least twice during successive iterations in the numerous iterations	M1	Degrees 0/3.
	Obtain final answer 0.36	A1	Must be 2d.p.
	Show sufficient iterations to 4 or more d.p. to justify 0.36 to 2 d.p. or show there is a sign change in the interval (0.355, 0.365)	A1	Allow small errors in 4 th d.p. Allow errors at start if self corrects later.
	0.5 0.4 0.3 0.2 0.1 $\pi/6$ $\pi/12$ 0.3091 0.3435 0.3826 0.4264 0.4740 0.3017 0.3989 0.3789 0.3650 0.3499 0.3339 0.3176 0.3820 0.3439 0.3513 0.3566 0.3625 0.3688 0.3754 0.3502 0.3649 0.3619 0.3599 0.3576 0.3552 0.3526 0.3624 0.3567 0.3578 0.3604 0.3614 0.3576 0.3580	3	

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Question	Answer	Marks	Guidance
6(a)	Expand $\cos(x - 60^\circ)$ correctly and evaluate $3 \cos x + 2 \cos(x - 60^\circ)$ to obtain $4 \cos x + \sqrt{3} \sin x$ or unsimplified coefficients	B1	Need to see $A \cos x + B \sin x$ with A and B correct A may be 4 or $3 + 2 \cos 60$ and B may be $\sqrt{3}$ or $2 \sin 60$.
	State $R = \sqrt{19}$ [$R \cos \alpha = 4$ $R \sin \alpha = \sqrt{3}$]	B1 FT	Follow through <i>their</i> 4 and $\sqrt{3}$. If coefficients are 3 and 2 then B0. $R = \sqrt{19}$ from $R = 4.36$ B0 but 4.36 seen after $\sqrt{19}$ ISW.
	Use correct trig formulae for their expansion to find α e.g. $\alpha = \tan^{-1} \frac{\sqrt{3}}{4}$ or $\cos^{-1} \frac{4}{\sqrt{19}}$ or $\sin^{-1} \frac{\sqrt{3}}{\sqrt{19}}$	M1	If $\sin \alpha = \sqrt{3}$ $\cos \alpha = 4$ seen then M0 A0. If $\tan \alpha = 23.41^\circ$ M0 A0 but can recover if $\alpha = 23.41^\circ$ seen later. $\alpha = \tan^{-1} \frac{2}{3}$ M1 ($\alpha = 33.69^\circ$) but $\alpha = \tan^{-1} \frac{3}{2}$ M0
	Obtain $\alpha = 23.41^\circ$	A1	Allow if x instead of α .
		4	

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Question	Answer	Marks	Guidance
6(b)	$\cos^{-1}\left(\frac{2.5}{R}\right)$	B1 FT	SOI [55.0°]. Follow through <i>their</i> $\sqrt{19}$.
	Use a correct method to find a value of 2θ (not x) in the interval. Allow sign error in moving α to right side	M1	$2\theta = \cos^{-1}\left(\frac{2.5}{R}\right) + 23.41^\circ$ or $2\theta = 360^\circ - \cos^{-1}\left(\frac{2.5}{R}\right) + 23.41^\circ$ with R substituted.
	Obtain one correct answer e.g. 39.2°	A1	If working for M1 not seen then M1 implied by 39.2° or 164.2° Must be at least 1d.p.
	Obtain second correct answer e.g. 164.2° and no others in the interval	A1	Must be at least 1d.p. Ignore answers outside the given interval.
		4	

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Question	Answer	Marks	Guidance
7(a)	$\frac{du}{dx} = -\sin x$	B1	SOI
	Use double angle formula and substitute for x and dx throughout the integral	M1	All x 's must be removed, can be coefficient errors provided 2 seen in working.
	Obtain $\pm \int 2ue^{2u} du$	A1	Limits may be omitted, or left as 0 and π , during the change of variable stage.
	Justify new limits and obtain $\int_{-1}^1 2ue^{2u} du$ from correct working	A1	AG Must see $x = 0, u = 1$ and $x = \pi, u = -1$. Inequalities alone e.g. $0 \leq x \leq \pi$ and $1 \leq u \leq -1$ or $-1 \leq u \leq 1$ for limits are insufficient A0 If sign in expression and order of limits incorrect then A0. If negative sign is present in the integrand then this can be removed and limits introduced in correct order in a single step.
		4	

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Question	Answer	Marks	Guidance
7(b)	Commence integration and reach $ae^{2u} + b \int e^{2u} du$, where $ab \neq 0$, $b < 0$	M1*	Condone dx.
	Complete integration and obtain $ue^{2u} - \frac{1}{2}e^{2u}$	A1	OE Allow $(2u \frac{1}{2}e^{2u}) - \frac{1}{2}e^{2u}$.
	Use correct limits correctly in $cue^{2u} + de^{2u}$ having integrated twice or in $c \cos x e^{2 \cos x} + de^{2 \cos x}$	DM1	1 and -1 for u , 0 and π for x e.g. $ce^2 + de^2 - (-ce^{-2} + de^{-2})$. Not decimals. Allow one sign error at most in going from $cue^{2u} + de^{2u}$ or $c \cos x e^{2 \cos x} + de^{2 \cos x}$ to $ce^2 + de^2 - (-ce^{-2} + de^{-2})$. [$e^2 - \frac{1}{2}e^2 - (-e^{-2} - \frac{1}{2}e^{-2})$] Complete reversal of sign by converting back to $\cos x$ and not making $x = 0$ upper limit is DM0 A0.
	Obtain $\frac{1}{2}e^2 + \frac{3}{2}e^{-2}$	A1	ISW Or equivalent 2-term expression e.g. $\frac{e^4 + 3}{2e^2}$ or $\frac{1}{2}\left(e^2 + \frac{3}{e^2}\right)$.
		4	

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Question	Answer	Marks	Guidance
8	Separate the variables correctly	B1	$\frac{y+4}{y^2+4} dy = \frac{1}{x} dx$.
	Obtain $\ln x$	B1	
	Split the fraction and integrate to obtain $p \ln(y^2 + 4)$ or $q \tan^{-1} \frac{y}{2}$ correctly	*M1	Only following subdivision into $\frac{y}{y^2+4} + \frac{4}{y^2+4}$. If no subdivision seen then both terms $p \ln(y^2 + 4)$ and $q \tan^{-1} \frac{y}{2}$ must be present.
	Obtain $\frac{1}{2} \ln(y^2 + 4)$	A1	
	Obtain $2 \tan^{-1} \frac{y}{2}$	A1	
	Use $(4, 2\sqrt{3})$ in an expression containing at least 2 of $a \ln x$, $b \ln(y^2 + 4)$ and $c \tan^{-1} \frac{y}{2}$ to obtain constant of integration	DM1	Allow one sign or arithmetic error e.g. $\frac{2\pi}{3}$. May use $(4, 2\sqrt{3})$ and $(x, 2)$ as limits to find x for the final 3 marks.
	Correct solution (any form) e.g. $\frac{1}{2} \ln(y^2 + 4) + 2 \tan^{-1} \frac{y}{2} = \ln x + \frac{2\pi}{3}$ or $\frac{1}{2} \ln(y^2 + 4) + 2 \tan^{-1} \frac{y}{2} = \ln x + 2 \tan^{-1} \sqrt{3} + \frac{1}{2} \ln 16 - \ln 4$	A1	However solution not asked for so allow $\frac{1}{2} \ln 8 + 2 \tan^{-1} 1 = \ln x + 2 \tan^{-1} \sqrt{3} + \frac{1}{2} \ln 16 - \ln 4$.
	Obtain $\sqrt{8}e^{-\frac{1}{6}\pi}$ or 1.68 or more accurate or $2\sqrt{2}e^{-\frac{1}{6}\pi}$ or $\frac{\sqrt{8}}{e^{\frac{1}{6}\pi}}$ or $e^{0.516}$	A1	ISW Must remove $\ln$ so $x = e^{(\ln 2\sqrt{2} - \pi/6)}$ A0.

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Question	Answer	Marks	Guidance
8	Alternative method for first *M1 A1 A1		
	$p\left((y+4)\tan^{-1}\frac{y}{2}-\int\tan^{-1}\frac{y}{2}\mathrm{d}y\right)$	*M1	Allow sign error.
	$(y+4)\frac{1}{2}\tan^{-1}\frac{y}{2}-\frac{y}{2}\tan^{-1}\frac{y}{2}+\int\frac{y}{y^2+4}\mathrm{d}y$	A1	
	Obtain $2\tan^{-1}\frac{y}{2}+\frac{1}{2}\ln(y^2+4)$	A1	
		8	

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Question	Answer	Marks	Guidance
9(a)	Perform scalar product of direction vectors and set result equal to zero	M1	$2c + 6 + 4 = 0$.
	Use P to find the value of λ	M1	$3 - 2\lambda = 7 \Rightarrow \lambda = -2$ [$a + \lambda c = 4$, $b + 4\lambda = -2$]. Equation for line l may contain $-\lambda$ instead of $+\lambda$ leading to $\lambda = 2$ all marks available.
	Obtain $c = -5$ or $b = 6$	A1	
	$a = -6$, $b = 6$ and $c = -5$ all correct	A1	
		4	SC1: Use P to find the value of λ M1 Substitute $\lambda = -2$ into point P , so $a - 2c = 4$, and put $\mu = -1$ and $\lambda = -1$ into l so $a - c = -1$, then solve to obtain $a = -6$, $b = 6$ and $c = -5$. All 3 values correct A1. Max 2/4.

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Question	Answer	Marks	Guidance
9(b)	Find $\overrightarrow{PQ}$ (or $\overrightarrow{QP}$) for a general point Q on m $= \pm ((1 + 2\mu, 2 - 3\mu, 3 + \mu) - (a + \lambda c, 3 - 2\lambda, b + 4\lambda))$	B1	$\left[\overrightarrow{PQ} \text{ or } \overrightarrow{QP} = \pm \begin{pmatrix} -3 + 2\mu \\ -5 - 3\mu \\ 5 + \mu \end{pmatrix} \right]$ Could be <i>their</i> a, b, c and λ values provided M1 M1 gained in (a). Allow expression in answer column.
	Equate the scalar product of $\overrightarrow{PQ}$ (or $\overrightarrow{QP}$) and a direction vector for m to zero and obtain an equation in μ	M1*	$(2(-3 + 2\mu) - 3(-5 - 3\mu) + (5 + \mu)) = 0.$ Allow $\overrightarrow{PQ} = \overrightarrow{OQ} + \overrightarrow{OP}$ sign problem.
	Solve and obtain $\mu = -1$	A1	$PQ^2 = (-3 + 2\mu)^2 + (-5 - 3\mu)^2 + (5 + \mu)^2.$ $[= 14(\mu + 1)^2 + 45].$ Min when $\mu = -1$ or by differentiation.
	Obtain $\overrightarrow{OQ} = -\mathbf{i} + 5\mathbf{j} + 2\mathbf{k}$ or $\overrightarrow{PQ} = -5\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}$ Must be labelled correctly	A1	The working may be in (a) provided at least this result is used in (b) .
	Carry out a method to find the position vector of R Alternative method for DM1 $\overrightarrow{OR} = (4, 7, -2) + t(-5, -2, 4)$ $\overrightarrow{QR} = \overrightarrow{OR} - \overrightarrow{OQ}$ Solve $ \overrightarrow{QR} ^2 = \frac{9}{4} \overrightarrow{PQ} ^2$ or $ \overrightarrow{QR} = \frac{3}{2} \overrightarrow{PQ} $ $t = 2.5$	DM1	e.g. Use $\overrightarrow{OR} = \overrightarrow{OP} + \frac{5}{2} \overrightarrow{PQ}$ or $\overrightarrow{OR} = \overrightarrow{OQ} + \frac{3}{2} \overrightarrow{PQ}$ or $\overrightarrow{OR} = \frac{5}{2} \overrightarrow{OQ} - \frac{3}{2} \overrightarrow{OP} \text{ or } 2\overrightarrow{QR} = 2(\overrightarrow{OR} - \overrightarrow{OQ}) = 3 \overrightarrow{PQ}$ where $\overrightarrow{OR} = (x, y, z)$. $\overrightarrow{PQ}$ used in all these approaches, may be incorrect, must be in the correct direction, i.e. not using $\overrightarrow{QP}$ for $\overrightarrow{PQ}$.

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Question	Answer	Marks	Guidance
9(b)	Obtain $-\frac{17}{2}\mathbf{i} + 2\mathbf{j} + 8\mathbf{k}$ from correct working	A1	Accept coordinates. Don't accept $-\frac{17}{2}\mathbf{i} + \frac{4}{2}\mathbf{j} + \frac{16}{2}\mathbf{k}$.
		6	SC2 Equate lines, attempt to find $\mu = -1$ or $\lambda = -1$ M1* $\overrightarrow{OQ} = -\mathbf{i} + 5\mathbf{j} + 2\mathbf{k}$ A1. Attempt to find $\overrightarrow{OQ}$ using other parameter value DM1. $\overrightarrow{OQ} = -\mathbf{i} + 5\mathbf{j} + 2\mathbf{k}$ therefore intersect A1. Then use main scheme for the final DM1 A1.
			First DM1 A1 are available if they show the 3 coordinates are consistent for the 2 parameter values instead of attempting to find $\overrightarrow{OQ}$ using the other parameter value and then showing intersection

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Question	Answer	Marks	Guidance
10(a)	State or imply the form $\frac{A}{1+2x} + \frac{B}{3-x} + \frac{C}{(3-x)^2}$	B1	Alternative form: $\frac{A}{1+2x} + \frac{Dx+E}{(3-x)^2}$.
	Use a correct method to find a constant	M1	Incorrect format for partial fractions: Allow M1 and a possible A1 if obtain one of these correct values. Max 2/5 Allow M1 even if multiply up by $(1+2x)(3-x)^3$.
	Obtain one of $A = 2$, $B = 2$ and $C = -3$	A1	Alternative form: obtain one of $A = 2$, $D = -2$ and $E = 3$.
	Obtain a second value	A1	
	Obtain the third value	A1	Do not need to substitute values back into original form.
		5	If $\frac{A}{1+2x} + \frac{B}{3-x} + \frac{Cx+D}{(3-x)^2}$ B0 but M1 A1 for A, A1 for B and A1 for C and D. If $C = 0$ then recovers B1 from above.

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Question	Answer	Marks	Guidance
10(b)	Use a correct method to obtain the first two terms of one of the unsimplified expansions $(1+2x)^{-1}, \left(1-\frac{1}{3}x\right)^{-1}, \left(1-\frac{1}{3}x\right)^{-2} (3-x)^{-1}, (3-x)^{-2}$	M1	$(1+2x)^{-1} = 1 + (-1)(2x) + \dots$ $\left(1-\frac{1}{3}x\right)^{-1} = 1 + (-1)(-x/3) + \dots$ $\left(1-\frac{1}{3}x\right)^{-2} = 1 + (-2)(-x/3) + \dots$ $(3-x)^{-1} = 3^{-1} + (-1)3^{-2}(-x) \dots$ $(3-x)^{-2} = 3^{-2} + (-2)3^{-3}(-x) + \dots$
	Obtain the correct unsimplified expansions up to the term in x^2 for each partial fraction If correct, should be working with $\frac{2}{1+2x} + \frac{2}{3-x} - \frac{3}{(3-x)^2}$ or $\frac{2}{1+2x} + \frac{-2x+3}{(3-x)^2}$	A1 FT A1 FT A1 FT	Follow through on <i>their</i> A, B, C $A(1 + (-1)(2x) + ((-1)(-2)/2)(2x)^2 + \dots)$ $\frac{B}{3} (1 + (-1)(-x/3) + ((-1)(-2)/2)(-x/3)^2 + \dots)$ $\frac{C}{3^2} (1 + (-2)(-x/3) + ((-2)(-3)/2)(-x/3)^2 + \dots)$. Must be <i>their</i> coefficients from (a) but may be unsimplified expansions for FT marks. If correct, expect to see $2(1 - 2x + (2x)^2)$ or $2 - 4x + 8x^2$ $\frac{2}{3} \left(1 + \frac{x}{3} + \left(\frac{x}{3}\right)^2\right)$ or $\frac{2}{3} + \frac{2}{9}x + \frac{2}{27}x^2$ $-\frac{1}{3} \left(1 + \frac{2x}{3} + (3)\left(\frac{x}{3}\right)^2\right)$ or $-\frac{1}{3} - \frac{2}{9}x - \frac{x^2}{9}$.
	Obtain final answer $\frac{7}{3} - 4x + \frac{215}{27}x^2$	A1	Accept $2\frac{1}{3} - 4x + 7\frac{26}{27}x^2$. No ISW.

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Question	Answer	Marks	Guidance
10(b)	Alternative Method for Question 10(b)		
	For the form $\frac{A}{1+2x} + \frac{Dx+E}{(3-x)^2}$	M1*	For the first two terms of an expanded partial fraction, following their A, D, E .
	Obtain the correct unsimplified expansions up to the term in x^2 for each partial fraction	A1FT	$A(1 + (-1)(2x) + ((-1)(-2)/2)(2x)^2 + \dots) +$ $(Dx + E) \frac{1}{3^2} (1 + (-2)(-x/3) + ((-2)(-3)/2)(-x/3)^2 + \dots)$ $2(1 - 2x + (2x)^2 + \dots)$ $+ \frac{-2x+3}{3^2} (1 + \frac{2x}{3} + (3)\left(\frac{x}{3}\right)^2 + \dots).$
		A1FT	
	Multiply out fully	DM1	Provided $DE \neq 0$. Ignore cubic terms and above. Allow error in one term but all terms must be present. If correct, expect to see $2 - 4x + 8x^2 - \frac{2}{9}x - \frac{4}{27}x^2 + \frac{1}{3} + \frac{2}{9}x + \frac{1}{9}x^2$
	Obtain final answer $\frac{7}{3} - 4x + \frac{215}{27}x^2$	A1	Accept $2\frac{1}{3} - 4x + 7\frac{26}{27}x^2$. No ISW

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Question	Answer	Marks	Guidance
10(b)	Alternative Method for Question 10(b): Maclaurin's Series		
	Correct derivatives for $A(1+2x)^{-1}$, $B(3-x)^{-1}$ and $C(3-x)^{-2}$ $(-1)(2)A(1+2x)^{-2}$, $(-1)(-1)B(3-x)^{-2}$ and $(-2)(-1)C(3-x)^{-3}$	B1 FT	
	One of following $(-2)(2)(-1)(2)A(1+2x)^{-3}$, $(-2)(-1)(-1)(-1)B(3-x)^{-3}$ and $(-3)(-1)(-2)(-1)C(3-x)^{-4}$	B1 FT	
	All correct	B1 FT	
	Substitute in $f(0) + xf'(0) + \frac{x^2}{2} f''(0)$	M1	
	Obtain final answer $\frac{7}{3} - 4x + \frac{215}{27}x^2$	A1	Accept $2\frac{1}{3} - 4x + 7\frac{26}{27}x^2$. No ISW
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Question	Answer	Marks	Guidance
11(a)	Multiply numerator and denominator by $(3 - ai)$	M1	Must perform complete multiplications but need not simplify i^2 . Can have errors but no term duplicated or missing. $\frac{(5a - 2i)(3 - ai)}{9 - a^2} = \frac{13a - i(5a^2 + 6)}{9 - a^2}$ M0 M1 A0 No working so unsure if denominator multiplied by $3 - ai$ M1 M1 A0
	Use $i^2 = -1$ at least once and separate real and imaginary parts	M1	
	Obtain $\frac{13a - i(5a^2 + 6)}{9 + a^2}$ or $\frac{13a - 5a^2i - 6i}{9 + a^2}$	A1	OE If $15a - 2a = 13a$ seen later award this A1.
	Use $\arg z$ to form equation in a $-\frac{5a^2 + 6}{13a} = \pm \tan\left(\pm \frac{\pi}{4}\right) \quad \text{or} \quad -\frac{13a}{5a^2 + 6} = \pm \tan\left(\pm \frac{\pi}{4}\right)$ or $\tan^{-1}\left(-\frac{5a^2 + 6}{13a}\right) = \pm \frac{\pi}{4}$ or $\tan^{-1}\left(-\frac{13a}{5a^2 + 6}\right) = \pm \frac{\pi}{4}$	M1	Allow expression given in answer column or $5a^2 + 6 = \pm 13a$ or use $-(x \pm xi) = (13a - i(5a^2 + 6))/(9 + a^2)$ and eliminate x so $5a^2 + 6 = \pm 13a$ M1.
	Obtain $a = 2$	A1	Need to reject $a = \frac{3}{5}$ or ignore it in future work. May not see second root, but if present, must be $\frac{3}{5}$.
	Obtain $z = 2 - 2i$ only	A1	Allow $z = -2i + 2$.

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Question	Answer	Marks	Guidance
11(a)	Alternative Method 1 for the first four marks		
	$\arg z = \arg (5a - 2i) - \arg (3 + ai)$	M1	
	$= \tan^{-1}\left(\frac{-2}{5a}\right) - \tan^{-1}\left(\frac{a}{3}\right)$ $= \tan^{-1}\left(\frac{-2 - \frac{a}{3}}{5a}\right) / \left(1 + \left(\frac{-2}{5a}\right)\left(\frac{a}{3}\right)\right)$	M1	Allow one sign error in second M1.
	$= \tan^{-1}\left(-\frac{5a^2 + 6}{13a}\right) \text{ or } \tan^{-1}\left(-\frac{13a}{5a^2 + 6}\right)$	A1	
	$\pm \frac{\pi}{4} = \tan^{-1}\left(-\frac{5a^2 + 6}{13a}\right) \text{ or } \tan^{-1}\left(-\frac{13a}{5a^2 + 6}\right)$	M1	Equate <i>their</i> $\tan^{-1}\left(-\frac{5a^2 + 6}{13a}\right)$ to $\pm \frac{\pi}{4}$. Then as original scheme for final 2 marks.
	Alternative Method 2 for the first four marks		
	$(x + iy)(3 + ai) = 5a - 2i$ $3x - ay = 5a$ and $ax + 3y = -2$	M1 A1	
	$x = \pm y$ Find x or y in terms of a , e.g. $x = \frac{2}{3-a}$ or $x = \frac{5a}{3+a}$	M1	
	Substitute in other equation, for example $3\left(\frac{2}{3-a}\right) + a\left(\frac{2}{3-a}\right) = 5a$	M1	Then as original scheme for final 2 marks.
		6	

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Question	Answer	Marks	Guidance
11(b)	State $\arg(z^3) = -\frac{3}{4}\pi$ or evaluate from $z = b - bi$ or from $-2b^3(1 + i)$	B1	If 2 different values given award B0. Do not ISW.
	Complete method to obtain r from <i>their</i> z	M1	$ z^3 = \left(\sqrt{x^2 + y^2}\right)^3$. If z correct, may see $ z^3 = \left(\sqrt{2^2 + (-2)^2}\right)^3$ or $ z^3 = \sqrt{(-16)^2 + (-16)^2}$.
	$r = 16\sqrt{2}$	A1	CAO A1 if $z = 2 - 2i$ obtained correctly. or $z =$ used with $a = 2$ found correctly, otherwise A0XP. May see $\arg$ and r given in a final answer i.e. $16\sqrt{2}e^{-\frac{3}{4}\pi i}$. Allow this form for $\arg$ and r to collect full marks, even if i missing. Ignore answers outside the given interval. If 2 different values given award A0.
		3	