

# Cambridge International AS & A Level

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## MATHEMATICS

**9709/31**

Paper 3 Pure Mathematics 3

May/June 2022

**1 hour 50 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

## INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
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- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

## INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

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- 1** Solve the equation  $2(3^{2x-1}) = 4^{x+1}$ , giving your answer correct to 2 decimal places. [4]

This image shows a full page of a handwriting practice worksheet. It consists of multiple sets of three horizontal dotted lines spaced evenly down the page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.

- 2 (a) Expand  $(2 - x^2)^{-2}$  in ascending powers of  $x$ , up to and including the term in  $x^4$ , simplifying the coefficients. [4]

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- (b) State the set of values of  $x$  for which the expansion is valid. [1]

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- 3** Solve the equation  $2 \cot 2x + 3 \cot x = 5$ , for  $0^\circ < x < 180^\circ$ .

[6]

[illegible]

- 4** The variables  $x$  and  $y$  satisfy the differential equation

$$\frac{dy}{dx} = \frac{xy}{1+x^2},$$

and  $y = 2$  when  $x = 0$ .

Solve the differential equation, obtaining a simplified expression for  $y$  in terms of  $x$ . [7]

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



- 5** The polynomial  $ax^3 - 10x^2 + bx + 8$ , where  $a$  and  $b$  are constants, is denoted by  $p(x)$ . It is given that  $(x - 2)$  is a factor of both  $p(x)$  and  $p'(x)$ .

(a) Find the values of  $a$  and  $b$ .

[5]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

**(b)** When  $a$  and  $b$  have these values, factorise  $p(x)$  completely.

[3]

[illegible]

**6** Let  $I = \int_0^3 \frac{27}{(9+x^2)^2} dx$ .

(a) Using the substitution  $x = 3 \tan \theta$ , show that  $I = \int_0^{\frac{1}{4}\pi} \cos^2 \theta \, d\theta$ . [4]

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**(b)** Hence find the exact value of  $I$ .

[4]

[illegible]

- 7 The complex number  $u$  is defined by  $u = \frac{\sqrt{2} - a\sqrt{2}i}{1 + 2i}$ , where  $a$  is a positive integer.

**(a)** Express  $u$  in terms of  $a$ , in the form  $x + iy$ , where  $x$  and  $y$  are real and exact.

[3]

[illegible]

It is now given that  $a = 3$ .

- (b) Express  $u$  in the form  $re^{i\theta}$ , where  $r > 0$  and  $-\pi < \theta \leq \pi$ , giving the exact values of  $r$  and  $\theta$ . [2]

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- (c) Using your answer to part (b), find the two square roots of  $u$ . Give your answers in the form  $re^{i\theta}$ , where  $r > 0$  and  $-\pi < \theta \leq \pi$ , giving the exact values of  $r$  and  $\theta$ . [3]

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**8** The equation of a curve is  $x^3 + y^3 + 2xy + 8 = 0$ .

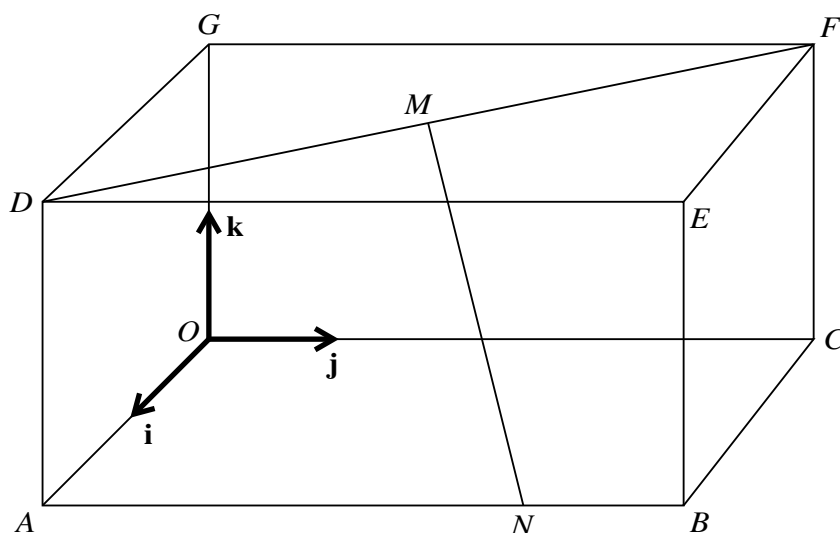
(a) Express  $\frac{dy}{dx}$  in terms of  $x$  and  $y$ . [4]

[illegible]

The tangent to the curve at the point where  $x = 0$  and the tangent at the point where  $y = 0$  intersect at the acute angle  $\alpha$ .

- (b) Find the exact value of  $\tan \alpha$ . [5]

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In the diagram,  $OABCDEFG$  is a cuboid in which  $OA = 2$  units,  $OC = 4$  units and  $OG = 2$  units. Unit vectors  $\mathbf{i}$ ,  $\mathbf{j}$  and  $\mathbf{k}$  are parallel to  $OA$ ,  $OC$  and  $OG$  respectively. The point  $M$  is the midpoint of  $DF$ . The point  $N$  on  $AB$  is such that  $AN = 3NB$ .

- (a) Express the vectors  $\overrightarrow{OM}$  and  $\overrightarrow{MN}$  in terms of  $\mathbf{i}$ ,  $\mathbf{j}$  and  $\mathbf{k}$ . [3]

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- (b) Find a vector equation for the line through  $M$  and  $N$ . [2]

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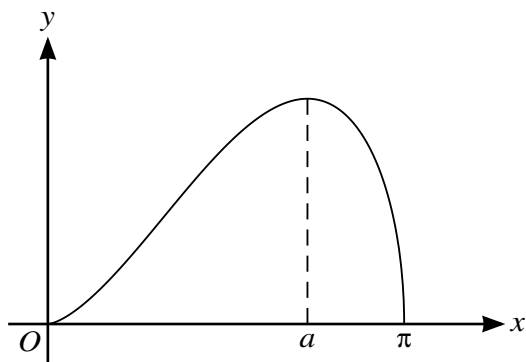
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- (c) Show that the length of the perpendicular from  $O$  to the line through  $M$  and  $N$  is  $\sqrt{\frac{53}{6}}$ . [4]

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(a) Show that  $\tan a = -\frac{1}{2}a$ . [4]

[illegible]

- (b) Verify by calculation that  $a$  lies between 2 and 2.5. [2]

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- (c) Show that if a sequence of values in the interval  $0 < x < \pi$  given by the iterative formula  $x_{n+1} = \pi - \tan^{-1}\left(\frac{1}{2}x_n\right)$  converges, then it converges to  $a$ , the root of the equation in part (a). [2]

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- (d) Use the iterative formula given in part (c) to determine  $a$  correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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[illegible]

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## MATHEMATICS

9709/32

Paper 3 Pure Mathematics 3

May/June 2022

1 hour 50 minutes

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## INFORMATION

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**BLANK PAGE**

- 1** Solve the equation  $\ln(e^{2x} + 3) = 2x + \ln 3$ , giving your answer correct to 3 decimal places. [4]

[illegible]

**2** Solve the equation  $3 \cos 2\theta = 3 \cos \theta + 2$ , for  $0^\circ \leq \theta \leq 360^\circ$ .

[5]

[illegible]

- 3** The polynomial  $ax^3 + x^2 + bx + 3$  is denoted by  $p(x)$ . It is given that  $p(x)$  is divisible by  $(2x - 1)$  and that when  $p(x)$  is divided by  $(x + 2)$  the remainder is 5.

Find the values of  $a$  and  $b$ .

[5]

This image shows a single page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- 4 The equation of a curve is  $y = \cos^3 x \sqrt{\sin x}$ . It is given that the curve has one stationary point in the interval  $0 < x < \frac{1}{2}\pi$ .

Find the  $x$ -coordinate of this stationary point, giving your answer correct to 3 significant figures. [6]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the entire width of the page. There are no margins, text, or other markings present.

- 5 (a) By sketching a suitable pair of graphs, show that the equation  $\ln x = 3x - x^2$  has one real root. [2]

- (b) Verify by calculation that the root lies between 2 and 2.8. [2]

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- (c) Use the iterative formula  $x_{n+1} = \sqrt{3x_n - \ln x_n}$  to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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**6** The variables  $x$  and  $y$  satisfy the differential equation

$$\frac{dy}{dx} = xe^{y-x},$$

and  $y = 0$  when  $x = 0$ .

(a) Solve the differential equation, obtaining an expression for  $y$  in terms of  $x$ .

[7]

[illegible]

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- (b) Find the value of  $y$  when  $x = 1$ , giving your answer in the form  $a - \ln b$ , where  $a$  and  $b$  are integers. [1]

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**7** The equation of a curve is  $x^3 + 3x^2y - y^3 = 3$ .

(a) Show that  $\frac{dy}{dx} = \frac{x^2 + 2xy}{y^2 - x^2}$ . [4]

[illegible]

- (b)** Find the coordinates of the points on the curve where the tangent is parallel to the  $x$ -axis. [5]

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**8** Let  $f(x) = \frac{x^2 + 9x}{(3x - 1)(x^2 + 3)}$ .

(a) Express  $f(x)$  in partial fractions.

[5]

[illegible]

- (b)** Hence find  $\int_1^3 f(x) \, dx$ , giving your answer in a simplified exact form. [5]

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**9** The lines  $l$  and  $m$  have vector equations

$$\mathbf{r} = -\mathbf{i} + 3\mathbf{j} + 4\mathbf{k} + \lambda(2\mathbf{i} - \mathbf{j} - \mathbf{k}) \quad \text{and} \quad \mathbf{r} = 5\mathbf{i} + 4\mathbf{j} + 3\mathbf{k} + \mu(a\mathbf{i} + b\mathbf{j} + \mathbf{k})$$

respectively, where  $a$  and  $b$  are constants.

(a) Given that  $l$  and  $m$  intersect, show that  $2b - a = 4$ .

[4]

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- (b) Given also that  $l$  and  $m$  are perpendicular, find the values of  $a$  and  $b$ . [4]

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- (c) When  $a$  and  $b$  have these values, find the position vector of the point of intersection of  $l$  and  $m$ . [2]

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- 10** The complex number  $-1 + \sqrt{7}i$  is denoted by  $u$ . It is given that  $u$  is a root of the equation

$$2x^3 + 3x^2 + 14x + k = 0,$$

where  $k$  is a real constant.

- (a)** Find the value of  $k$ .

[3]

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- (b)** Find the other two roots of the equation.

[4]

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- (c) On an Argand diagram, sketch the locus of points representing complex numbers  $z$  satisfying the equation  $|z - u| = 2$ . [2]

- (d) Determine the greatest value of  $\arg z$  for points on this locus, giving your answer in radians. [2]

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Paper 3 Pure Mathematics 3

May/June 2022

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- 1** Find, in terms of  $a$ , the set of values of  $x$  satisfying the inequality

$$2|3x + a| < |2x + 3a|,$$

where  $a$  is a positive constant.

[4]

[illegible]

- 2** Solve the equation  $\cos(\theta - 60^\circ) = 3 \sin \theta$ , for  $0^\circ \leq \theta \leq 360^\circ$ . [5]

[illegible]

- 3 (a)** Show that the equation  $\log_3(2x + 1) = 1 + 2\log_3(x - 1)$  can be written as a quadratic equation in  $x$ . [3]

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- (b) Hence solve the equation  $\log_3(4y + 1) = 1 + 2 \log_3(2y - 1)$ , giving your answer correct to 2 decimal places. [2]

[illegible]

- 4** The curve  $y = e^{-4x} \tan x$  has two stationary points in the interval  $0 \leq x < \frac{1}{2}\pi$ .

- (a) Obtain an expression for  $\frac{dy}{dx}$  and show it can be written in the form  $\sec^2 x(a + b \sin 2x)e^{-4x}$ , where  $a$  and  $b$  are constants. [4]

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**(b)** Hence find the exact  $x$ -coordinates of the two stationary points.

[3]

[illegible]

5 The complex number  $3 - i$  is denoted by  $u$ .

- (a) Show, on an Argand diagram with origin  $O$ , the points  $A$ ,  $B$  and  $C$  representing the complex numbers  $u$ ,  $u^*$  and  $u^* - u$  respectively.

State the type of quadrilateral formed by the points  $O$ ,  $A$ ,  $B$  and  $C$ .

[3]

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- (b) Express  $\frac{u^*}{u}$  in the form  $x + iy$ , where  $x$  and  $y$  are real.

[3]

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**[Turn over**

- 6** The parametric equations of a curve are  $x = \frac{1}{\cos t}$ ,  $y = \ln \tan t$ , where  $0 < t < \frac{1}{2}\pi$ .

(a) Show that  $\frac{dy}{dx} = \frac{\cos t}{\sin^2 t}$ . [5]

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- (b)** Find the equation of the tangent to the curve at the point where  $y = 0$ . [3]

[illegible]

7 Let  $f(x) = \frac{5x^2 + 8x - 3}{(x - 2)(2x^2 + 3)}$ .

(a) Express  $f(x)$  in partial fractions.

[5]

[illegible]

- (b)** Hence obtain the expansion of  $f(x)$  in ascending powers of  $x$ , up to and including the term in  $x^2$ . [5]

[illegible]

- 8** At time  $t$  days after the start of observations, the number of insects in a population is  $N$ . The variation in the number of insects is modelled by a differential equation of the form  $\frac{dN}{dt} = kN^{\frac{3}{2}} \cos 0.02t$ , where  $k$  is a constant and  $N$  is a continuous variable. It is given that when  $t = 0$ ,  $N = 100$ .

(a) Solve the differential equation, obtaining a relation between  $N$ ,  $k$  and  $t$ . [5]

[illegible]

- (b) Given also that  $N = 625$  when  $t = 50$ , find the value of  $k$ . [2]

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- (c) Obtain an expression for  $N$  in terms of  $t$ , and find the greatest value of  $N$  predicted by this model. [2]

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- 9** With respect to the origin  $O$ , the point  $A$  has position vector given by  $\overrightarrow{OA} = \mathbf{i} + 5\mathbf{j} + 6\mathbf{k}$ . The line  $l$  has vector equation  $\mathbf{r} = 4\mathbf{i} + \mathbf{k} + \lambda(-\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$ .

- (a) Find in degrees the acute angle between the directions of  $OA$  and  $l$ . [3]

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- (b) Find the position vector of the foot of the perpendicular from  $A$  to  $l$ . [4]

[illegible]

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- (c) Hence find the position vector of the reflection of  $A$  in  $l$ . [2]

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**10** The constant  $a$  is such that  $\int_1^a x^2 \ln x \, dx = 4$ .

(a) Show that  $a = \left( \frac{35}{3 \ln a - 1} \right)^{\frac{1}{3}}$ . [5]

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- (b) Verify by calculation that  $a$  lies between 2.4 and 2.8.

[2]

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- (c) Use an iterative formula based on the equation in part (a) to determine  $a$  correct to 2 decimal places. Give the result of each iteration to 4 decimal places.

[3]

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