



Cambridge International A Level

MATHEMATICS

9709/31

Paper 3 Pure Mathematics 3

May/June 2022

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

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This document consists of **17** printed pages.

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These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
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- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

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3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (ISW).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

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The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1	Use law of the logarithm of a product or a quotient or a power	*M1	
	Obtain a correct linear equation in any form	A1	e.g. $\ln 2 + (2x-1)\ln 3 = (x+1)\ln 4$ or $\log_2 2 + (2x-1)\log_2 3 = (2x+2)\log_2 2$
	Solve for x	DM1	Allow for unsimplified expression $x = \dots$ Allow M1 M1 for $x=1.45$ from $6^{2x-1} = 4^{x+1}$.
	Obtain answer $x = 2.21$	A1	The question asks for 2 dp.
	Alternative method for question 1		
	Correct use of indices to obtain $2.25^x = 6$ or $1.5^{2x} = 6$	M1 A1	
	Correct use of logarithms to solve for x	M1	Allow solution of $2.25^x = 6$ by trial and improvement as far as 2.2...
	Obtain answer $x = 2.21$	A1	Need to see an intermediate step / sequence of iterations.
		4	

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Question	Answer	Marks	Guidance
2(a)	State a correct unsimplified version of the x^2 or the x^4 term of the expansion of $(2 - x^2)^{-2}$ or $\left(1 - \frac{1}{2}x^2\right)^{-2}$	M1	$\frac{1}{4}\left(1 + 2\frac{x^2}{2} + \frac{-2 \cdot -3}{2}\left(\frac{x^2}{2}\right)^2 \dots\right)$ Symbolic binomial coefficients are not sufficient for the M1.
	State correct first term $\frac{1}{4}$	B1	Accept 2^{-2} .
	Obtain the next two terms $\frac{1}{4}x^2 + \frac{3}{16}x^4$	A1 A1	A1 for each one correct ISW. Full marks for $\frac{1}{4}\left(1 + x^2 + \frac{3}{4}x^4\right)$ ISW.
			SC allow M1 A1 A1 for $\frac{1}{4}$ and $1 + x^2 + \frac{3}{4}x^4$ SOI. SC allow M1 A1 for $1 + x^2 + \frac{3}{4}x^4$
		4	
2(b)	State answer $ x < \sqrt{2}$	B1	Or $-\sqrt{2} < x < \sqrt{2}$.
		1	

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Question	Answer	Marks	Guidance
3	Use correct trigonometric formulae to form an equation in $\tan x$	*M1	e.g. $\frac{1 - \tan^2 x}{\tan x} + \frac{3}{\tan x} = 5$
	Obtain a correct linear equation in any form	A1	$1 - \tan^2 x + 3 = 5 \tan x$
	Reduce equation to a 3-term quadratic	A1	$\tan^2 x + 5 \tan x - 4 = 0$, or 3-term equivalent
	Solve a 3-term quadratic in $\tan x$ and obtain a value of x	DM1	
	Obtain answer, e.g. $x = 35.1^\circ$	A1	
	Obtain second answer, e.g. $x = 99.9^\circ$, and no other in $(0^\circ, 180^\circ)$	A1	Ignore answers outside $(0^\circ, 180^\circ)$. Treat answers in radians $(0.612, 1.74)$ as a misread.
	Alternative method for question 3		
	Use correct formulae for $\sin 2x$ and $\cos 2x$ to form an equation in $\sin x$ and $\cos x$	*M1	
	Obtain $4 \frac{\cos x}{\sin x} - \frac{\sin x}{\cos x} = 5$	A1	
	Reduce equation to a 3-term quadratic	A1	$\tan^2 x + 5 \tan x - 4 = 0$, or 3-term equivalent
	Solve a 3-term quadratic in $\tan x$ and obtain a value of x	DM1	
	Obtain answer, e.g. $x = 35.1^\circ$	A1	
	Obtain second answer, e.g. $x = 99.9^\circ$, and no other in $(0^\circ, 180^\circ)$	A1	Ignore answers outside $(0^\circ, 180^\circ)$. Treat answers in radians $(0.612, 1.74)$ as a misread.
		6	

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Question	Answer	Marks	Guidance
4	Separate variables correctly	B1	$\int \frac{x}{1+x^2} dx = \int \frac{1}{y} dy$ Accept without integral signs.
	Obtain term $\ln y$	B1	
	State term of the form $k \ln(1+x^2)$	M1	
	State correct term $\frac{1}{2} \ln(1+x^2)$	A1	OE
	Evaluate a constant, or use limits $x = 0, y = 2$ in a solution containing terms $a \ln y$ and $b \ln(1+x^2)$ where $ab \neq 0$	M1	If they remove logs first the constant must be of the correct form.
	Obtain correct solution in any form	A1	e.g. $\ln y + \ln \frac{1}{2} = \frac{1}{2} \ln(1+x^2)$
	Simplify and obtain $y = 2\sqrt{1+x^2}$	A1	OE The question asks for simplification, so need to deal with $\exp(\ln(...))$.
		7	

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Question	Answer	Marks	Guidance
5(a)	Substitute $x = 2$, equate to zero	M1	Or divide by $x - 2$ and equate constant remainder to zero.
	Obtain a correct equation, e.g. $8a - 40 + 2b + 8 = 0$	A1	Seen or implied in subsequent work.
	Differentiate $p(x)$, substitute $x = 2$ and equate result to zero	M1	Or divide by $x - 2$ and equate constant remainder to zero.
	Obtain $12a - 40 + b = 0$, or equivalent	A1	SOI in subsequent work.
	Obtain $a = 3$ and $b = 4$	A1	
	Alternative method for question 5(a)		
	State or imply $(x - 2)^2$ is a factor	M1	
	$p(x) = (x - 2)^2(ax + 2)$	A1	
	Obtain an equation in b	M1	
	e.g. by comparing coefficients of x : $b = 4a - 8$	A1	
	Obtain $a = 3$ and $b = 4$	A1	
			SC If uses $x = -2$ in both equations allow M1 and allow A1 for $a = -3$, $b = -4$.
		5	

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Question	Answer	Marks	Guidance
5(b)	Attempt division by $(x - 2)$	M1	The M1 is earned if division reaches a partial quotient of $ax^2 + kx$, or if inspection has an unknown factor $ax^2 + ex + f$ and an equation in e and/or f . Where a has the value found in part 5(a).
	Obtain quadratic factor $3x^2 - 4x - 4$	A1	
	Obtain factorisation $(3x+2)(x-2)(x-2)$	A1	
	Alternative method for question 5(b)		
	State or imply $(x-2)^2$ is a factor	B1	
	Attempt division by $(x-2)^2$, reaching a quotient $ax + k$ or use inspection with unknown factor $cx + d$ reaching a value for c or for d	M1	
	Obtain factorisation $(3x+2)(x-2)^2$	A1	Accept $3\left(x + \frac{2}{3}\right)(x-2)^2$.
		3	

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Question	Answer	Marks	Guidance
6(a)	State or imply $dx = 3\sec^2\theta d\theta$	B1	
	Substitute throughout for x and dx	M1	
	Obtain any correct form in terms of θ	A1	e.g. $\int \frac{81\sec^2\theta}{(9+9\tan^2\theta)^2} d\theta$
	Justify change of limits and obtain $\int_0^{\frac{\pi}{4}} \cos^2\theta d\theta$ correctly	A1	AG
		4	
6(b)	Obtain indefinite integral of the form $\int a + b\cos 2\theta d\theta$, where $ab \neq 0$	*M1	
	Obtain $\frac{1}{2}\theta + \frac{1}{4}\sin 2\theta$	A1	
	Use correct limits correctly in an expression containing $p\theta$ and $q\sin 2\theta$ where $pq \neq 0$	DM1	$\frac{\pi}{8} + \frac{1}{4}(-0)$
	Obtain answer $\frac{1}{8}(\pi + 2)$	A1	Or exact equivalent e.g. $\frac{1}{8}\pi + \frac{1}{4}$.
		4	

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Question	Answer	Marks	Guidance
7(a)	Multiply numerator and denominator by $1 - 2i$, or equivalent	M1	At least one multiplication completed.
	Obtain correct numerator $(1 - 2a)\sqrt{2} - (2 + a)\sqrt{2}i$	A1	OE
	Obtain final answer $\frac{1 - 2a}{5}\sqrt{2} - \frac{2 + a}{5}\sqrt{2}i$	A1	OE
	Alternative method for question 7(a)		
	Multiply $x + iy$ by $1 + 2i$ and compare real and imaginary parts	M1	
	Obtain $x - 2y = \sqrt{2}$ and $2x + y = a\sqrt{2}$	A1	
	Obtain final answer $\frac{1 - 2a}{5}\sqrt{2} - \frac{2 + a}{5}\sqrt{2}i$	A1	OE
		3	
7(b)	Obtain $r = 2$	B1 FT	
	Obtain $\theta = -\frac{3}{4}\pi$	B1	
		2	
7(c)	Use correct method to find r or θ	M1	
	State answer $\sqrt{2}e^{-\frac{3}{8}\pi i}$	A1 FT	
	State answer $\sqrt{2}e^{\frac{5}{8}\pi i}$	A1 FT	
		3	

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Question	Answer	Marks	Guidance
8(a)	State or imply $3y^2 \frac{dy}{dx}$ as derivative of y^3	B1	
	State or imply $2y + 2x \frac{dy}{dx}$ as derivative of $2xy$	B1	
	Complete differentiation and equate attempted derivative to zero and solve for $\frac{dy}{dx}$	M1	
	Obtain answer $-\frac{3x^2 + 2y}{3y^2 + 2x}$	A1	
		4	
8(b)	Find gradient at either $(0, -2)$ or $(-2, 0)$	M1	
	Obtain answers $\frac{1}{3}$ and 3	A1 A1	
	Use $\tan(A \pm B)$ formula to find $\tan \alpha$	M1	
	Obtain answer $\tan \alpha = \frac{4}{3}$	A1	
		5	

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Question	Answer	Marks	Guidance
9(a)	Obtain $\overrightarrow{OM} = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$	B1	
	Use a correct method to find $\overrightarrow{MN}$	M1	e.g. $\overrightarrow{MO} + \overrightarrow{OA} + \overrightarrow{AN}$ or $\overrightarrow{MO} + \overrightarrow{ON}$
	Obtain $\overrightarrow{MN} = \mathbf{i} + \mathbf{j} - 2\mathbf{k}$	A1	Accept any notation.
		3	
9(b)	Use a correct method to form an equation for MN	M1	Allow without $\mathbf{r} = \dots$
	Obtain $\mathbf{r} = 2\mathbf{i} + 3\mathbf{j} + \lambda(\mathbf{i} + \mathbf{j} - 2\mathbf{k})$	A1 FT	OE e.g. $\mathbf{r} = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k} + \mu(\mathbf{i} + \mathbf{j} - 2\mathbf{k})$ Must have $\mathbf{r} = \dots$ Follow <i>their</i> answers to part 9(a).
		2	
9(c)	State $\overrightarrow{OP}$ for a general point P on MN in component form, e.g. $(2 + \lambda, 3 + \lambda, -2\lambda)$	B1	
	Equate scalar product of $\overrightarrow{OP}$ and a direction vector for MN to zero and solve for λ	M1	
	Obtain $\lambda = -\frac{5}{6}$	A1	OE e.g. $\mu = \frac{1}{6}$
	Obtain $\sqrt{\frac{53}{6}}$ correctly	A1	AG e.g. from $\sqrt{\left(\frac{7}{6}\right)^2 + \left(\frac{13}{6}\right)^2 + \left(\frac{5}{3}\right)^2}$
		4	

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Question	Answer	Marks	Guidance
10(a)	Use correct product rule	M1	Condone incorrect / missing chain rule
	Obtain correct derivative in any form	A1	e.g. $\frac{dy}{dx} = \sqrt{\sin x} + \frac{x \cos x}{2\sqrt{\sin x}}$ or $2y \frac{dy}{dx} = 2x \sin x + x^2 \cos x$
	Equate derivative to zero and obtain an equation in $\tan x$ or $\tan a$	M1	
	Obtain $\tan a = -\frac{1}{2}a$ correctly	A1	AG
		4	
10(b)	Calculate the value of a relevant expression or pair of expressions at $a = 2$ and $a = 2.5$	M1	Must be working in radians At least one correct
	Complete the argument correctly with correct calculated values	A1	e.g. $-1 > -2.18$ and $-1.25 < -0.747$
		2	
10(c)	State a suitable equation, e.g. $x = \pi - \tan^{-1}\left(\frac{1}{2}x\right)$	B1	A correct equation without subscripts or quote $\tan \theta = -\tan(\pi - \theta)$
	Using $\tan(A \pm B)$ formula, or otherwise, rearrange this as $\tan x = -\frac{1}{2}x$	B1	Complete argument correctly
		2	

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Question	Answer	Marks	Guidance
10(d)	Use the iterative process correctly at least once	M1	Must be working in radians
	Obtain answer $a = 2.29$	A1	
	Show sufficient iterations to 4 dp to justify 2.29 to 2 dp or show there is a sign change in the interval (2.285, 2.295)	A1	e.g. 2.25, 2.2974, 2.2871, 2.2893, 2.2888, ...
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Question	Answer	Marks	Guidance
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	State a correct equation without logs (in any form)	A1	e.g. $3 + e^{2x} = 3e^{2x}$
	Carry out correct method to solve an equation of the form $e^{2x} = a$, where $a > 0$, or for solving $e^x = b$ ($b > 0$) if they have already taken the square root	M1	Allow for $x = \frac{1}{2} \ln \frac{3}{2}$. M1 can be implied by correct answer.
	Obtain answer $x = 0.203$	A1	CAO. The question requires 3 d.p. Answer only with no working shown is 0/4.
		4	

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Question	Answer	Marks	Guidance
2	Use correct double-angle formula to obtain an equation in $\cos \theta$	M1	e.g. $3(2\cos^2 \theta - 1) = 3\cos \theta + 2$
	Obtain $6\cos^2 \theta - 3\cos \theta - 5 = 0$, or 3-term equivalent	A1	M1 A0 is scored if they use any correct formula for $\cos 2\theta$ and make a subsequent error.
	Solve a 3-term quadratic in $\cos \theta$ for θ	M1	As far as $\theta = \cos^{-1}\left(\frac{3-\sqrt{129}}{12}\right)$ if quadratic correct.
	Obtain a correct answer, e.g. 134.1°	A1	Accept greater accuracy e.g. 134.1456, 225.8544.
	Obtain a second answer, e.g. 225.9° and no other in $[0^\circ, 360^\circ]$	A1 FT	Treat answers in radians (2.34 and 3.94) as a misread. Ignore answers outside $[0^\circ, 360^\circ]$. The FT is for 360° minus the first answer.
			Special Ruling: If they have an incorrect quadratic that leads legitimately to 4 solutions for θ , allow FT for 360° minus an answer in $(0^\circ, 180^\circ)$. More than 4 solutions is maximum M1 A0 M1 A0 A0. If <i>their</i> equation should have 4 solutions and the candidate only gives 3 solutions then M1 A0 M1 A0 A0. Mis-read leading to a quadratic with 4 solutions could score maximum M1 A0 M1 A1 A1 or M1 A0 M1 A1 A0 if extra/missing solution.
		5	

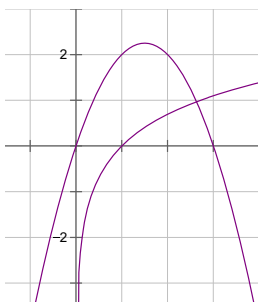
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Question	Answer	Marks	Guidance
3	Substitute $x = \frac{1}{2}$, equate result to zero	M1	Or divide by $2x-1$ and equate constant remainder to zero.
	Obtain a correct simplified equation	A1	e.g. $\frac{1}{8}a + \frac{1}{4} + \frac{1}{2}b + 3 = 0$ or $a + 4b = -26$
	Substitute $x = -2$, equate result to 5	M1	Or divide by $x+2$ and equate constant remainder to 5.
	Obtain a correct simplified equation	A1	e.g. $-8a + 4 - 2b + 3 = 5$ or $8a + 2b = 2$
	Obtain $a = 2$ and $b = -7$	A1	WWW
		5	

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Question	Answer	Marks	Guidance
4	Use the correct product rule and then the chain rule to differentiate either $\cos^3 x$ or $\sqrt{\sin x}$	M1	e.g. two terms with one part of $\frac{dy}{dx} = p \cos^2 x \sin x \sqrt{\sin x} + q \frac{\cos^3 x \cos x}{\sqrt{\sin x}}$.
	Obtain correct derivative in any form e.g. $\frac{dy}{dx} = -3 \cos^2 x \sin x \sqrt{\sin x} + \frac{\cos^3 x \cos x}{2\sqrt{\sin x}}$	A1 A1	A1 for each correct term substituted in the complete derivative.
	Equate their derivative to zero and obtain a horizontal equation with positive integer powers of $\sin x$ and/or $\cos x$ from an equation including $\sqrt{\sin x}$ or $\frac{1}{\sqrt{\sin x}}$ using sensible algebra.	M1	e.g. $-3 \cos^2 x \sin^2 x + \frac{1}{2} \cos^4 x = 0$
	Use correct formula(s) to express <i>their</i> equation/derivative in terms of one trigonometric function	M1	Can be awarded before the previous M1. May involve more than one trigonometric term.
	Obtain $7 \cos^2 x = 6$, $7 \sin^2 x = 1$, or $6 \tan^2 x = 1$, or equivalent, and obtain answer $x = 0.388$	A1	CAO. The question asks for 3 sf. Ignore additional answers outside $(0, \frac{\pi}{2})$. 22.2° is A0.
		6	

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Question	Answer	Marks	Guidance
5(a)	Sketch a relevant graph, e.g. $y = \ln x$	B1	 $\ln(x)$: sketch should imply y-axis is an asymptote. Through (1, 0) if marked. Correct shape. $3x - x^2$: Symmetrical. Through (0, 0) and (3, 0) if marked. If $\ln(x)$ correct accept parabola for +ve y only. If $\ln(x)$ incorrect then need parabola in 3 quadrants.
	Sketch a second relevant graph, e.g. $y = 3x - x^2$, and justify the given statement by marking the root on the sketch or by use of a suitable comment	B1	
		2	
5(b)	Calculate the values of a relevant expression or pair of expressions at $x = 2$ and $x = 2.8$	M1	Allow for a smaller interval. At least one value correct if comparing with 0. If using pairs then the pairing must be clear.
	Complete the argument correctly with correct calculated values	A1	e.g. $0.693 < 2$ and $1.03 > 0.56$ or $1.307 > 0, -0.47 < 0$ using $\sqrt{3x - \ln x}$ $0.304 > 0, -0.085 < 0$. Need to have calculated values to at least 2 sf.
		2	

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Question	Answer	Marks	Guidance
5(c)	Use the iterative process correctly at least once	M1	
	Obtain final answer 2.63	A1	
	Show sufficient iterations to at least 4 dp to justify 2.63 to 2 dp or show there is a sign change in the interval (2.625, 2.635)	A1	SC Allow M1 A1 A0 to a candidate who starts at a point in the interval and reaches a premature conclusion
		3	

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Question	Answer	Marks	Guidance
6(a)	Correct separation of variables	B1	$\int e^{-y} dy = \int xe^{-x} dx$ Condone missing integral signs.
	Obtain term $-e^{-y}$	B1	
	Commence integration by parts and reach $\pm xe^{-x} \pm \int e^{-x} dx$	*M1	M0 if clearly using differentiation of a product.
	Complete integration and obtain $-xe^{-x} - e^{-x}$	A1	
	Use $x = 0$ and $y = 0$ to evaluate a constant or as limits in a solution containing or derived from terms ae^{-y} , bxe^{-x} and ce^{-x} , where $abc \neq 0$	DM1	Must see working for this. In a correct solution they should have $-e^{-y} + C = -xe^{-x} - e^{-x}$ or equivalent. If they take logarithms before finding the constant, the constant must be of the right form.
	Correct solution in any form Must follow from correct working	A1	e.g. $-e^{-y} = -xe^{-x} - e^{-x}$ A0 if constant of integration ignored or assumed to be zero.
	Obtain final answer $y = -\ln((x+1)e^{-x})$ from correct working	A1	OE e.g. $y = x - \ln(x+1)$, $y = \ln\left(\frac{e^x}{x+1}\right)$. A0 if constant of integration ignored or assumed to be zero.
		7	
6(b)	Obtain answer $(y=)1 - \ln 2$	B1	Must follow from at least 6 or 7 obtained in part 6(a) .
		1	

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Question	Answer	Marks	Guidance
7(a)	State or imply $6xy + 3x^2 \frac{dy}{dx}$ as derivative of $3x^2y$	B1	
	State or imply $3y^2 \frac{dy}{dx}$ as derivative of y^3	B1	Allow B1 B1 for $(3x^2 dx +) 6xy dx + 3x^2 dy - 3y^2 dy [= 0]$
	Equate attempted derivative of left-hand side to zero and solve to obtain an equation with $\frac{dy}{dx}$ as subject	M1	Allow if zero implied by subsequent working. Allow if recover from an extra $\frac{dy}{dx} = \dots$ at the beginning of the left-hand side.
	Obtain $\frac{dy}{dx} = \frac{x^2 + 2xy}{y^2 - x^2}$ correctly	A1	AG Accept y' for $\frac{dy}{dx}$.
		4	
7(b)	Equate numerator to zero	*M1	Must be using the given derivative.
	Obtain $x = -2y$, or equivalent	A1	An equation with x or y as the subject SOI.
	Use $x^3 + 3x^2y - y^3 = 3$ to obtain an equation in x or y	DM1	$-8y^3 + 12y^3 - y^3 = 3$ or $x^3 - \frac{3}{2}x^3 + \frac{1}{8}x^3 = 3$ or any equivalent form (do not need to evaluate powers).
	Obtain the point $(-2, 1)$ and no others from solving their cubic equation	A1	Allow if each component stated separately. ISW.
	State the point $(0, -\sqrt[3]{3})$, or equivalent from correct work	B1	Accept $(0, \sqrt[3]{-3})$, or $(0, -1.44)$ (-1.44225) . Allow if each component stated separately. ISW.
		5	

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Question	Answer	Marks	Guidance
8(a)	State or imply the form $\frac{A}{3x-1} + \frac{Bx+C}{x^2+3}$	B1	
	Use a correct method for finding a constant	M1	
	Obtain one of $A = 1$, $B = 0$ and $C = 3$ from correct working	A1	A maximum of M1 A1 is available after B0.
	Obtain a second value from correct working	A1	
	Obtain the third value from correct working	A1	
		5	
8(b)	Integrate and obtain term $\frac{1}{3}\ln(3x-1)$	B1 FT	OE e.g. $\frac{1}{3}\ln(x-\frac{1}{3})$. The FT is on the value of A .
	Obtain term of the form $k\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)$	M1	
	Obtain term $\sqrt{3}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)$	A1 FT	OE. The FT is on the value of C .
	Substitute correct limits in an integral of the form $a\ln(3x-1) + k\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)$, where $ak \neq 0$, and evaluate trigonometry	M1	Must be subtracted the right way round. $\left(\frac{1}{3}\ln 8 - \frac{1}{3}\ln 2 + \sqrt{3} \times \frac{\pi}{3} - \sqrt{3} \times \frac{\pi}{6}\right)$ Angles should be in radians. Condone angles as decimals.
	Obtain answer $\frac{2}{3}\ln 2 + \frac{\sqrt{3}\pi}{6}$ from correct working in part 8(b)	A1	Or exact 2-term equivalent e.g. $\frac{1}{3}\ln 4 + \frac{\pi}{2\sqrt{3}}$ ISW.
		5	

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Question	Answer	Marks	Guidance
9(a)	Express general point of l or m in component form, i.e. $(-1+2\lambda, 3-\lambda, 4-\lambda)$ or $(5+a\mu, 4+b\mu, 3+\mu)$	B1	
	Equate components and eliminate either λ or μ	M1	e.g. $\mu = \frac{2}{1-b}$, $\lambda = \frac{-1-b}{1-b}$, $\mu = \frac{-4}{2+a}$, $\lambda = \frac{a+6}{a+2}$
	Eliminate the other parameter or obtain a second expression in the first	M1	λ and μ are not required to be the subject of the equations.
	Show intermediate steps to obtain $2b-a=4$	A1	AG
	Alternative method for question 9(a)		
	Express general point of l or m in component form, i.e. $(-1+2\lambda, 3-\lambda, 4-\lambda)$ or $(5+a\mu, 4+b\mu, 3+\mu)$	B1	
	Express a or b in terms of λ and μ	M1	$a = \frac{2\lambda-6}{\mu}$, $b = \frac{-1-\lambda}{\mu}$
	Use $\lambda=1-\mu$	M1	
	Obtain $2b-a=4$	A1	AG
		4	
9(b)	Using the correct process equate the scalar product of the direction vectors to zero	*M1	$(2\mathbf{i} - \mathbf{j} - \mathbf{k}) \cdot (a\mathbf{i} + b\mathbf{j} + \mathbf{k}) = 0$ SOL.
	Obtain $2a-b-1=0$	A1	OE e.g. $2(2b-4)-b-1=0$
	Solve simultaneous equations for a or for b	DM1	
	Obtain $a=2, b=3$	A1	
		4	

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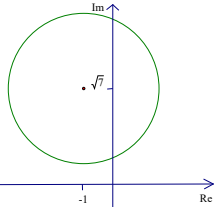
Question	Answer	Marks	Guidance
9(c)	Substitute found values in component equations and solve for λ or for μ	M1	
	Obtain answer $3\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ from either $\lambda = 2$ or $\mu = -1$	A1	Accept as coordinates or equivalent.
		2	

Question	Answer	Marks	Guidance
10(a)	Substitute $x = -1 + \sqrt{7}\mathbf{i}$ in the equation and attempt expansions of x^2 and x^3	*M1	
	Use $\mathbf{i}^2 = -1$ correctly at least once and solve for k	DM1	$2(20 - 4\sqrt{7}\mathbf{i}) + 3(-6 - 2\sqrt{7}\mathbf{i}) + 14(-1 + \sqrt{7}\mathbf{i}) + k = 0$
	Obtain answer $k = -8$	A1	
			SC B1 only for those who show no working for the cube and square and obtain answer $k = -8$.
	Alternative method for question 10(a)		
	Attempt division by $(x + 1 - \sqrt{7}\mathbf{i})$ as far as $2x^2 + z_1x + \dots$	*M1	See division on next page.
	Use $\mathbf{i}^2 = -1$ correctly at least once and obtain $2x^2 + z_1x + z_2 + \text{remainder}$	DM1	
	Obtain answer $k = -8$	A1	
		3	

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Question	Answer	Marks	Guidance
10(b)	State answer $-1 - \sqrt{7}i$	B1	Can be seen simply stated on its own, or in a list of roots. Allow if stated clearly in part 10(a) .
	Carry out a method for finding a quadratic factor with zeros $-1 + \sqrt{7}i$ and $-1 - \sqrt{7}i$	M1	Or state $\left(x - (-1 + \sqrt{7}i)\right)\left(x - (-1 - \sqrt{7}i)\right)(2x - p)$
	Obtain $x^2 + 2x + 8$	A1	Or obtain $(-1 + \sqrt{7}i)(-1 - \sqrt{7}i)p = -8$ Or obtain $(-1 + \sqrt{7}i) + (-1 - \sqrt{7}i) + \frac{p}{2} = -\frac{3}{2}$
	Obtain root $x = \frac{1}{2}$, or equivalent, via division or inspection	A1	Needs to follow from the working.
		4	

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Question	Answer	Marks	Guidance
10(c)	Show a circle with centre $-1 + \sqrt{7}i$	B1	 If the scales are very different from each other then B1 for centre in the correct position and B1 for an ellipse. If there is more than one circle the max score is B1.
	Show circle with radius 2 and centre not at the origin	B1	
	There needs to be some evidence of scale e.g. radius marked or a scale on the axes		
		2	
10(d)	Carry out a complete method for calculating the maximum value of $\arg z$ for correct circle	M1	e.g. $\frac{\pi}{2} + \tan^{-1} \frac{1}{\sqrt{7}} + \frac{\pi}{4}$ Can be implied by 155.7° .
	Obtain answer 2.72 radians	A1	CAO. The question requires radians.
		2	



Cambridge International A Level

MATHEMATICS

9709/33

Paper 3 Pure Mathematics 3

May/June 2022

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2022 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

This document consists of **23** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (ISW).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1	State or imply non-modular inequality $2^2(3x+a)^2 < (2x+3a)^2$, or corresponding quadratic equation, or pair of linear equations	B1	e.g. $(6x+2a)^2 = (2x+3a)^2$ or $32x^2 + 12xa - 5a^2 = 0$ $2(3x+a) = (2x+3a)$ and $-2(3x+a) = (2x+3a)$
	Solve 3-term quadratic, or solve two linear equations for x	M1	Apply general rules for solving quadratic equation by formula or by factors. Instead of $x = \{\text{formula}\}$, have $\{\text{formula}\} = 0$ and try to solve for a then M0
	Obtain critical values $x = \frac{1}{4}a$ and $x = -\frac{5}{8}a$	A1	
	State final answer $-\frac{5}{8}a < x < \frac{1}{4}a$ or $-0.625a < x < 0.25a$ or $x > -\frac{5}{8}a$ and $x < \frac{1}{4}a$ or $x > -\frac{5}{8}a \cap x < \frac{1}{4}a$	A1	Do not condone $\leq$ for $<$ in the final answer. Do not ISW. SC Set a to value, (say $a = 1$), after initial B1 gained, then $-\frac{5}{8} < x < \frac{1}{4}$ B1 maximum 2 out of 4.
	Alternative method for question 1		
	Obtain critical value $x = \frac{1}{4}a$ from a graphical method, or by solving a linear equation or linear inequality	B1	
	Obtain critical value $x = -\frac{5}{8}a$ similarly	B2	
	State final answer $-\frac{5}{8}a < x < \frac{1}{4}a$ or $-0.625a < x < 0.25a$ or $x > -\frac{5}{8}a$ and $x < \frac{1}{4}a$ or $x > -\frac{5}{8}a \cap x < \frac{1}{4}a$	B1	Do not condone $\leq$ for $<$ in the final answer. Do not ISW. SC Set a to value, (say $a = 1$), after initial B1 gained, then $-\frac{5}{8} < x < \frac{1}{4}$ B1 maximum 2 out of 4.
		4	

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Question	Answer	Marks	Guidance
2	Use correct $\cos(A - B)$ formula to obtain an equation in $\cos \theta$ and $\sin \theta$	B1	$\cos \theta \cos 60 + \sin \theta \sin 60 = 3 \sin \theta$
	Use trigonometric formula and substitute values for $\cos 60$ and $\sin 60$ to obtain an equation in $\tan \theta$ (or $\cos \theta$ or $\sin \theta$)	M1	Allow $\frac{1}{2}$ and $\frac{\sqrt{3}}{2}$ interchanged. $\frac{1}{2} + \frac{\sqrt{3}}{2} \tan \theta = 3 \tan \theta$ $\frac{1}{4} \cos^2 \theta = \left(3 - \frac{\sqrt{3}}{2}\right) \left(3 - \frac{\sqrt{3}}{2}\right) (1 - \cos^2 \theta)$ $\frac{1}{4} (1 - \sin^2 \theta) = \left(3 - \frac{\sqrt{3}}{2}\right) \left(3 - \frac{\sqrt{3}}{2}\right) \sin^2 \theta$
	Obtain $\tan \theta = \frac{1}{6 - \sqrt{3}}$ or $\tan \theta = \frac{6 + \sqrt{3}}{33}$ or 0.2343, $\cos \theta = \frac{3 \frac{\sqrt{3}}{2}}{\sqrt{10 - 3\sqrt{3}}}$ or 0.9736 or $\sin \theta = \frac{\frac{1}{2}}{\sqrt{10 - 3\sqrt{3}}}$ or 0.2281	A1	OE
	Obtain answer, e.g. $\theta = 13.2^\circ$	A1	May be more accurate, allow value rounding to 13.2° . $\theta = 13.1867^\circ$.
	Obtain second answer, e.g. $\theta = 193.2^\circ$ and no others in the given interval	A1 FT	May be more accurate. Allow value rounding to 193.2° . FT is on previous value of θ , must have scored M1. Note if θ is negative (e.g. -13.2): $-13.2 + 180 = 166.8$ A0 but $-13.2 + 360 = 346.8$ A1 FT. Ignore answers outside the given interval. Treat answers in radians as a misread. 0.23015, 3.3717.

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Question	Answer	Marks	Guidance
2	Alternative method for question 2 – using $R\cos(\theta \pm \alpha)$ or equivalent		
	Use correct $\cos(A - B)$ formula to obtain an equation in $\cos \theta$ and $\sin \theta$	B1	$\cos \theta \cos 60 + \sin \theta \sin 60 = 3 \sin \theta$
	Correct method for finding $\tan \alpha$ from $p\cos \theta + q\sin \theta = 0$	M1	$\tan \alpha = \pm \frac{q}{p}$
	Correct value of α	A1	76.8° or 1.34 radians (may be more accurate).
	Obtain answer, e.g. $\theta = 13.2^\circ$	A1	May be more accurate, allow value rounding to 13.2° . $\theta = 13.1867^\circ$.
	Obtain second answer, e.g. $\theta = 193.2^\circ$ and no others in the given interval	A1 FT	May be more accurate. Allow value rounding to 193.2° . FT is on previous value of θ , must have scored M1. Note if θ is negative (e.g. -13.2): $-13.2 + 180 = 166.8$ A0 but $-13.2 + 360 = 346.8$ A1 FT. Ignore answers outside the given interval. Treat answers in radians as a misread. 0.23015, 3.3717.
		5	

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Question	Answer	Marks	Guidance
3(a)	Use law of logarithm of a power	M1	$\log_3(2x + 1) = 1 + \log_3(x - 1)^2$
	Use $\log_3 3 = 1$	B1	$\log_3(2x + 1) = \log_3 3 + 2\log_3(x - 1)$ $\left[\log_3 \left(\frac{2x + 1}{(x - 1)^2} \right) = \log_3 3 \quad \text{or} \quad \left(\frac{2x + 1}{(x - 1)^2} \right) = 3 \right]$ SC For candidates scoring M0 B0 due to combining logs before dealing with coefficient 2, and confusing coefficients, allow $\log_3(\dots) = c$ leading to $(\dots) = 3^c$ B1 .
	Obtain $3x^2 - 8x + 2 = 0$ or $1.5x^2 - 4x + 1 = 0$	A1	OE 3 terms only and = 0 required.
		3	
3(b)	Solve 3-term quadratic equation from part 3(a) or restart to find y	M1	$y = \frac{4 \pm \sqrt{10}}{6}$ or $y = 1.1937\dots$ or $y = 0.1396\dots$ $(x = 2.3874 \text{ or } x = 0.2792)$ May solve for x but must find $y = \frac{x}{2}$ to gain M1.
	Obtain answer 1.19	A1	CAO. 2 dp required.
		2	

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Question	Answer	Marks	Guidance
4(a)	Use correct product rule or quotient rule, and attempt at chain rule	M1	$ke^{-4x} \tan x + e^{-4x} \sec^2 x$ or $\frac{e^{4x} \sec^2 x - \tan x(ke^{4x})}{(e^{4x})^2}$ Need to see $d(\tan x)/dx = \sec^2 x$ (formula sheet) and attempt at ke^{-4x} , where $k \neq 1$.
	Obtain correct derivative in any form	A1	$-4e^{-4x} \tan x + e^{-4x} \sec^2 x$ or $\frac{e^{4x} \sec^2 x - \tan x(4e^{4x})}{(e^{4x})^2}$
	Use trigonometric formulae to express derivative in the form $ke^{-4x} \sin x \cos x \sec^2 x + ae^{-4x} \sec^2 x$ or $ke^{-4x} \frac{\sin x \cos x}{\cos x \cos x} + ae^{-4x} \sec^2 x$ or $\sec^2 x(ke^{-4x} \sin x \cos x + ae^{-4x})$ Allow $\frac{1}{\cos^2 x}$ instead of $\sec^2 x$	M1	Need to use $\frac{\tan x}{\sec^2 x} = \sin x \cos x$ or $\tan x = \frac{\sin x}{\cos x} \cdot \frac{\cos x}{\cos x}$ OE. M1 is independent of previous M1, but expression must be of appropriate form.
	Obtain correct answer with $a = 1$ and $b = -2$	A1	At least one line of trigonometric working is required from $-4e^{-4x} \tan x + e^{-4x} \sec^2 x$ to given answer $\sec^2 x(1 - 2 \sin 2x) e^{-4x}$ with elements in any order. If only error: $4 \sin x \cos x = 4 \sin 2x$ M1 A1 M1 A0.
		4	

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Question	Answer	Marks	Guidance
4(b)	Equate derivative to zero and use correct method to solve for x	M1	$\sin 2x = \frac{1}{2}$, hence $x = \frac{1}{2} \sin^{-1} \frac{1}{2}$ or $x = \tan^{-1}(2 \pm \sqrt{3})$ Allow M1 for correct method for non-exact value.
	Obtain answer, e.g. $x = \frac{1}{12}\pi$	A1	[0.262 M1 A0]
	Obtain second answer, e.g. $\frac{5}{12}\pi$ and no other in the given interval	A1 FT	FT $\frac{\pi - \text{their } 2x}{2}$ if exact values; x must be $< \frac{\pi}{2}$. Ignore answers outside the given interval. Treat answers in degrees as a misread. 15° , 75° . SC No values found for a and b in 4(a) but chooses values in 4(b) : max M1 for x .
		3	

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Question	Answer	Marks	Guidance
5(a)	Show u and u^* in relatively correct positions. Must have sense of scale on axes	B1	$u = 3 - i$, $u^* = 3 + i$ Ignore labels.
	Show $u^* - u$ in a relatively correct position. Must have sense of scale on axes	B1	2i. Scale only on Imaginary axis is sufficient for this mark.
	State that $OABC$ is a parallelogram [independent of previous marks]	B1	Ignore ‘quadrilateral’. Allow ‘trapezium’ from correct work.
		3	

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Question	Answer	Marks	Guidance
5(b)	Multiply <i>their</i> numerator and the given denominator by $3 + i$ and attempt to evaluate either	M1	Can have missing term and arithmetic errors but need $i^2 = -1$ once, seen or implied.
	Obtain numerator $8 + 6i$ or denominator 10	A1	
	State final answer $\frac{4}{5} + \frac{3}{5}i$ or $\frac{8}{10} + \frac{6}{10}i$ or $0.8 + 0.6i$	A1	Correct answer with no working scores 0/3.
	Alternative method for question 5(b)		
	Obtain two equations in x and y , and attempt to solve for x or for y	M1	$3 = 3x + y$ and $1 = -x + 3y$
	Obtain $x = \frac{4}{5}$ or $\frac{8}{10}$ or 0.8 $y = \frac{3}{5}$ or $\frac{6}{10}$ or 0.6	A1	
	State final answer $\frac{4}{5} + \frac{3}{5}i$ or $\frac{8}{10} + \frac{6}{10}i$ or $0.8 + 0.6i$	A1	Correct answer with no working scores 0/3.
		3	

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Question	Answer	Marks	Guidance
5(c)	State or imply $\arg \frac{u^*}{u} = \arg u^* - \arg u$ or $2\arg u^*$	M1	
	Justify the given statement correctly	A1	AG $\arg \frac{u^*}{u} = \tan^{-1} \frac{3}{4}$, $\arg u^* = \tan^{-1} \frac{1}{3}$ and $\arg u = \tan^{-1} -\frac{1}{3}$ (or $\arg u = -\tan^{-1} \frac{1}{3}$), needed if use first expression in M1; or $\arg \frac{u^*}{u} = \tan^{-1} \frac{3}{4}$ and $\arg u^* = \tan^{-1} \frac{1}{3}$, needed if use second expression in M1.
	Alternative method for question 5(c)		
	Use $\tan 2A$ formula with $\tan A = \frac{1}{3}$	M1	$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$, $\tan A = \frac{1}{3}$, hence $\tan 2A = \frac{3}{4}$.
	Justify the given statement correctly	A1	AG So $2A = \tan^{-1} \frac{3}{4} = \arg \frac{u^*}{u}$ and $A = \tan^{-1} \frac{1}{3} = \arg u^*$ hence $\arg \frac{u^*}{u} = 2 \arg u^*$.
		2	

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Question	Answer	Marks	Guidance
6(a)	Use chain rule at least once	M1	Needs $\frac{dy}{dt} = \frac{1}{\tan t} \frac{d}{dt}(\tan t)$ or $\frac{dx}{dt} = (-1)(\cos^{-2} t) \frac{d}{dt}(\cos t)$. BOD if + and $(-1)(-1)$ not seen. $\frac{dx}{dt} = \sec t \tan t$ (from List of Formulae MF19) M1 A1. If $\frac{dx}{dt} = -\sec t \tan t$ M1 A0.
	Obtain $\frac{dx}{dt} = \sec t \tan t$	A1	OE e.g. $\sin t (\cos t)^{-2}$. If e.g. $\frac{dx}{dt} = \sec x \tan x$ or $\sec \theta \tan \theta$ or $\sec t \tan x$, condone recovery on next line.
	Obtain $\frac{dy}{dt} = \frac{\sec^2 t}{\tan t}$	A1	OE e.g. $\frac{1}{\sin t \cos t}$. If e.g. $\frac{dy}{dt} = \frac{\sec^2 x}{\tan x}$ or $\frac{\sec^2 \theta}{\tan \theta}$, condone recovery on next line. Only penalise notation errors once in $\frac{dx}{dt}$ and $\frac{dy}{dt}$ if no recovery.
	Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	M1	Allow even if previous M0 scored, but must be using derivatives.
	Obtain given answer $\frac{\cos t}{\sin^2 t}$	A1	AG After $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$ used, any notation error A0. Must cancel $\cos t$ correctly.
		5	

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Question	Answer	Marks	Guidance
6(b)	State or imply $t = \frac{1}{4}\pi$ when $y = 0$	B1	
	Form the equation of the tangent at $y = 0$ or find c	M1	$x = \sqrt{2}$, $\frac{dy}{dx} = \sqrt{2}$ and $y = 0$, <i>their</i> coordinates and gradient used in $y = mx + c$.
	Obtain answer $y = \sqrt{2}x - 2$	A1	OE e.g. $y = \sqrt{2}(x - \sqrt{2})$ ISW. Allow $y = 1.41x - 2[.00]$ or $1.41(x - 1.41)$.
		3	

Question	Answer	Marks	Guidance
7(a)	State or imply the form $\frac{A}{x-2} + \frac{Bx+C}{2x^2+3}$	B1	If $1 - \frac{A}{x-2} + \frac{Bx+C}{2x^2+3}$ or $\frac{A}{x-2} + \frac{C}{2x^2+3}$ B0 then M1 A1 (for $A = 3$) still possible.
	Use a correct method for finding a constant	M1	
	Obtain one of $A = 3$, $B = -1$ and $C = 6$	A1	Allow all A marks obtained even if method would give errors if equations solved in a different order.
	Obtain a second value	A1	
	Obtain the third value	A1	
		5	

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Question	Answer	Marks	Guidance
7(b)	Use correct method to find the first two terms of the expansion of $(x-2)^{-1}$, $\left(1-\frac{1}{2}x\right)^{-1}$, $(2x^2+3)^{-1}$ or $\left(1+\frac{2}{3}x^2\right)^{-1}$	M1	Symbolic binomial coefficients not sufficient for the M1.
	Obtain correct unsimplified expansions, up to the term in x^2 , of each partial fraction	A1 FT A1 FT	The FT is on A , B and C . $-\frac{A}{2}\left[1-\left(-\frac{x}{2}\right)+\frac{(-1)(-2)}{2}\left(-\frac{x}{2}\right)^2+\dots\right]$ $\frac{Bx+C}{3}\left[1-\frac{2x^2}{3}+\dots\right]$
	Extract the coefficient 3 correctly from $(2x^2+3)^{-1}$ with expansion to $1\pm\frac{2}{3}x^2$ then multiply by $Bx+C$ up to the terms in x^2 , where $BC \neq 0$	M1	$\frac{C}{3} + \frac{Bx}{3} \pm \frac{C}{3}\left(\frac{2}{3}\right)x^2$ or $\frac{1}{3}\left(C+Bx \pm C\left(\frac{2}{3}\right)x^2\right)$ Allow a slip in multiplication for M1. Allow miscopies in B and C from 7(a) .
	Obtain final answer $\frac{1}{2} - \frac{13}{12}x - \frac{41}{24}x^2$	A1	Do not ISW.
		5	

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Question	Answer	Marks	Guidance
8(a)	Separate variables correctly	B1	$\frac{dN}{N^{\frac{3}{2}}} = (k \cos 0.02t) dt$ Allow without integral signs.
	Obtain term $-\frac{2}{\sqrt{N}}$	B1	OE Ignore position of k .
	Obtain term $50 \sin 0.02t$	B1	OE Ignore position of k .
	Use $t = 0, N = 100$ to evaluate a constant, or as limits, in a solution containing terms $\frac{a}{\sqrt{N}}$ and $b \sin 0.02t$, where $ab \neq 0$	M1	$\left[\text{e.g. } c = -0.2 \text{ or } c = \frac{-0.2}{k} \right]$
	Obtain correct solution in any form, e.g. $-\frac{2}{\sqrt{N}} = 50k \sin 0.02t - 0.2$	A1	OE ISW e.g. $N = \frac{1}{(25k \sin 0.02t - 0.1)^2}$ $-2N^{-\frac{1}{2}} = \frac{k}{0.02} \sin 0.02t - \frac{1}{5}$ $50k \sin 0.02t = -\frac{2}{\sqrt{N}} + \frac{1}{5}$ $\frac{1}{\sqrt{N}} = -\frac{1}{2}k(50 \sin 0.02t) + \frac{1}{10}$ $50 \sin \left(\frac{1}{50}t \right) = -\frac{2\sqrt{N}}{kN} + \frac{20}{100k}$
		5	
8(b)	Use the substitution $N = 625$ and $t = 50$ in expression of appropriate form to evaluate k	M1	Expression must contain $a + b \sin 0.02t$, $(\sqrt{N})^{\pm n}$, where $n = -1, 1, 3$ or 5 and a and b are constants $ab \neq 0$ or $(a + b \sin 0.02t)^{\pm 2}$ and $(N)^{\pm n}$. Allow with k replaced by $\frac{1}{k}$, error due to $k(N^{-3/2})$ when separating variables in 8(a) . If invert term by term when 3 terms shown then M0.
	Obtain $k = 0.00285[2148]$	A1	Must evaluate $\sin 1$. Degrees $k = 0.138$ M1 A0.
		2	

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Question	Answer	Marks	Guidance
8(c)	Rearrange and obtain $N = 4(0.2 - 0.142(607)\sin 0.02t)^{-2}$ Substitution for k required	M1	Anything of the form $N = c(d - ek \sin 0.02t)^{-2}$, where c , d and e are constants $cde \neq 0$ and value of k substituted. Allow with k replaced by $1/k$, error due to $k(N^{-3/2})$ when separating variables in 8(a) . OE ISW e.g. $N = \left(-\frac{10}{0.7125\sin 0.02t - 1} \right)^2 \quad N = \frac{1}{(-0.0713\sin 0.02t + 0.1)^2}$ $N = \frac{100}{\left(\left(\frac{0.6}{\sin 1} \right) \sin 0.02t - 1 \right)^2} \quad N = \frac{1}{\left(\frac{3}{-50\sin 1} \times \sin 0.02t + \frac{1}{10} \right)^2}$ $N = \left(-\frac{0.06}{\sin 1} \sin 0.02t + 0.1 \right)^{-2} \quad N = \left(\frac{800}{80 - 57\sin 0.02t} \right)^2$ Do not need to substitute for $\sin(0.02t) = 1$, but must substitute for k .
	Accept answers between 1209 and 1215	A1	ISW Substitute $\sin 0.02t = 1$ or $t = 50 \sin^{-1} 1$ or 78.5 or 25π . Answer with no working (rubric) 0/2. SC $N = \dots$ not seen but correct numerical answer B1 1/2.
		2	

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Question	Answer	Marks	Guidance
9(a)	Using the correct process find the scalar product of direction vectors of l and OA	M1	$(1, 5, 6) \cdot (-1, 2, 3) = -1.1 + 5.2 + 6.3 = -1 + 10 + 18$
	Using the correct process for the moduli, divide the scalar product by the product of the moduli and find the inverse cosine of the result	M1	<i>Their</i> scalar product $\div [\sqrt{(1^2 + 5^2 + 6^2)}\sqrt{((-1)^2 + 2^2 + 3^2)}]$. Angle = $\cos^{-1} \frac{27}{\sqrt{62}\sqrt{14}}$.
	Obtain answer 23.6° .	A1	AWRT 23.6° . 23.5889° . Radians 0.412 scores A0 (0.4117...).
		3	
9(b)	Taking a general point P on l , state AP (or PA) in component form, e.g. $(3 - \lambda, -5 + 2\lambda, -5 + 3\lambda)$	B1	Note: $(4, 1, 0)$ or $(4, 1, 1)$, for $4\mathbf{i} + \mathbf{k}$ is not MR, but M1 possible.
	<i>Either</i> equate scalar product of AP and direction vector of l to zero and solve for λ <i>or</i> use Pythagoras in a relevant triangle and solve for λ	M1	$(3 - \lambda, -5 + 2\lambda, -5 + 3\lambda) \cdot (-1, 2, 3) = 0$ $-3 -10 -15 + \lambda + 4\lambda + 9\lambda = 0$ or let OQ = $(4, 0, 1)$ so AQ = $(3, -5, -5)$, QP = $(-\lambda, 2\lambda, 3\lambda)$, AP = $(3 - \lambda, -5 + 2\lambda, -5 + 3\lambda)$ hence $3^2 + (-5)^2 + (-5)^2 =$ $(3 - \lambda)^2 + (-5 + 2\lambda)^2 + (-5 + 3\lambda)^2 + (-\lambda)^2 + (2\lambda)^2 + (3\lambda)^2$ Other alternative approaches are possible, e.g. minimise AP or AP^2 , either by completing the square or by differentiating.
	Obtain $\lambda = 2$	A1	$\lambda = 2$
	State that the position vector OP* of the foot is $2\mathbf{i} + 4\mathbf{j} + 7\mathbf{k}$	A1	OE Condone coordinates.
		4	

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Question	Answer	Marks	Guidance
9(c)	Set up a correct method for finding the position vector of the reflection of A in l	M1	For all methods, allow a sign error in one component only: $\mathbf{OA}' = \mathbf{OP}^* + (\mathbf{OP}^* - \mathbf{OA})$ <i>their</i> $(2, 4, 7) + (\text{their } 2, 4, 7 - 1, 5, 6)$ or $\mathbf{OA}' = \mathbf{OP}^* - (\mathbf{OA} - \mathbf{OP}^*)$ <i>their</i> $(2, 4, 7) - (1, 5, 6 - \text{their } 2, 4, 7)$ or $\mathbf{OA}' = \mathbf{OA} + 2(\mathbf{OP}^* - \mathbf{OA})$ $\begin{pmatrix} 1 + 2(\text{their } 2 - 1) \\ 5 + 2(\text{their } 4 - 5) \\ 6 + 2(\text{their } 7 - 6) \end{pmatrix}$ or midpoint $\mathbf{OP}^* = (\mathbf{OA} + \mathbf{OA}')/2$ with <i>their</i> λ value substituted. $\frac{1+x}{2} = \text{their } 2$ $\frac{5+y}{2} = \text{their } 4$ $\frac{6+z}{2} = \text{their } 7$
	Obtain answer $3\mathbf{i} + 3\mathbf{j} + 8\mathbf{k}$ or $3\left(\mathbf{i} + \mathbf{j} + \frac{8}{3}\right)$	A1	OE Condone coordinates $x = 3, y = 3, z = 8$ A1. No method shown and correct answer 2/2.
		2	

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Question	Answer	Marks	Guidance
10(a)	Commence integration and reach $ax^3 \ln x + b \int x^3 \cdot \frac{1}{x} dx$	*M1	OE Allow omission of dx.
	Obtain $\frac{1}{3}x^3 \ln x - \frac{1}{3} \int x^3 \cdot \frac{1}{x} dx$	A1	OE Allow omission of dx.
	Complete integration and obtain $\frac{1}{3}x^3 \ln x - \frac{1}{9}x^3$	A1	Allow $-\frac{1}{3} \left(\frac{1}{3}x^3 \right)$.
	Use limits correctly and equate to 4, having integrated twice	DM1	$\frac{1}{3}a^3 \ln a - \frac{1}{9}a^3 - (0 - \frac{1}{9}) = 4$ allow one sign error OR one numerical error, but 0 may be absent or expressed as $\frac{a^3}{3} \ln 1$. Allow $-\frac{1}{3} \left(\frac{1}{3}ax^3 \right)$ and $-\frac{1}{3} \left(\frac{1}{3} \right)$.
	Obtain given result correctly	A1	$a = \left(\frac{35}{3 \ln a - 1} \right)^{\frac{1}{3}}$ AG After substitution, any errors even if corrected A0. Need to see at least one line of working between substitution and the given answer.
10(b)		5	
	Calculate the values of a relevant expression or pair of expressions at $a = 2.4$ and $a = 2.8$ All values must be correct for M1 (numerical question)	M1	
	Justify the given statement with correct calculated values	A1	$2.4 < 2.7(8)$ and $2.8 > 2.5(6)$ sign change here insufficient OR $-0.3(8)$ and $0.2(4) < 0, > 0$ or change of sign.
		2	

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Question	Answer	Marks	Guidance
10(c)	Use the iterative process $a_{n+1} = \left(\frac{35}{3 \ln a_n - 1} \right)^{\frac{1}{3}}$ correctly at least twice	M1	
	Obtain final answer $a = 2.64$	A1	Must be 2 dp.
	Show sufficient iterations to 4 dp to justify 2.64 to 2 dp, or show there is a sign change in (2.635, 2.645)	A1	$\begin{array}{ll} 2.635 & (35/(3 \ln a - 1))^{1/3} - a = 0.0029(4) > 0 \\ 2.645 & (35/(3 \ln a - 1))^{1/3} - a = -0.012 < 0 \end{array}$
		3	