



Cambridge International A Level

MATHEMATICS

9709/61

Paper 6 Probability & Statistics 2

May/June 2021

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2021 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **13** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

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Mathematics Specific Marking Principles	
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2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

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The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
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CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1	$\lambda = (3.1 + 1.7) \times 2$	M1	Attempt combined mean. Allow 3.1 + 1.7 for M1
	$= 9.6$	A1	Correct mean
	$1 - e^{-9.6} (1 + 9.6 + \frac{9.6^2}{2} + \frac{9.6^3}{3!})$	M1	Allow incorrect mean. Allow one end error.
	$= 0.986$ (3 sf)	A1	SC If 9.6 seen and unsupported 0.986 M1A1B1. SC Unsupported correct answer of 0.986 only if 9.6 also not seen scores B2 only.
		4	

Question	Answer	Marks	Guidance
2(a)	$\pm \frac{123-125}{\frac{6}{\sqrt{40}}} [= -2.108...]$	M1	Must have $\sqrt{40}$ No standard deviation/variance mix. Ignore any continuity correction attempts for this mark.
	$P(z < '-2.108') = 1 - \Phi('2.108')$	M1	For correct probability area consistent with <i>their</i> working.
	$= 0.0175$ or 0.0176 (3 sf)	A1	
		3	
2(b)	No, population is normal	B1	Need both.
		1	

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Question	Answer	Marks	Guidance
3	$1 - \frac{20}{27}$ or $\frac{20}{27} - \frac{1}{2}$ $\frac{20}{27} - \left(1 - \frac{20}{27}\right)$ or $\left(\frac{20}{27} - \frac{1}{2}\right)$	M1	For either expression seen.
	$\frac{13}{27}$	A1	OE. Accept 0.481 or 0.482.
		2	

Question	Answer	Marks	Guidance
4	$\frac{3820}{100} [= 38.2]$	B1	
	$\frac{100}{99} \left(\frac{182200}{100} - '38.2'^2 \right)$ or $\frac{1}{99} \left(182200 - \frac{3820^2}{100} \right)$	M1	Use of biased (362.76) scores M0
	$= \frac{12092}{33}$ or 366.424 or 366 (3 sf)	A1	Accept SD=19.1422 or 19.1(3sf)
	$'38.2' \pm z \times \sqrt{\frac{'366.424'}{100}}$	M1	Expression of the correct form must be a z-value.
	$z = 1.881$ or 1.882	B1	Seen.
	34.6 to 41.8 (3 sf)	A1	Allow use of biased giving (34.6,41.8) Must be an interval.
		6	

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Question	Answer	Marks	Guidance
5(a)	$\text{Po}\left(\frac{2}{15}\right)$	M1	SOI. Allow $\text{Po}(0.133)$.
	$P(X \geq 1) = 1 - e^{-\frac{2}{15}}$	M1	Allow incorrect λ allow one end error
	$= 0.125$ (3 sf)	A1	SC Partially unsupported final answer: $\text{Po}\left(\frac{2}{15}\right)$ stated B1 then unsupported 0.125 B1 SC Use of Binomial (0.1248) B1 only Use of Normal scores M0
		3	

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Question	Answer	Marks	Guidance
5(b)	$\lambda = \frac{n}{75000}$	B1	
	$e^{-\frac{n}{75000}} > 0.9$	M1	Allow '=' Allow incorrect λ
	$-\frac{n}{75000} > \ln 0.9 \quad [n < 7902.04]$	M1	Attempt ln both sides
	Largest value of n is 7902	A1	CWO. Must be an integer.
	Alternative method for Question 5(b)		
	$e^{-\mu} > 0.9$	M1	Allow '='
	$-\mu > \ln 0.9 \quad [\mu < 0.10536]$	M1	Attempt ln both sides
	$n = \mu \times 75000$	B1	
	Largest value of n is 7902	A1	CWO. Must be an integer.
	Alternative method for Question 5(b)		
	$\frac{74999}{75000}$	B1	
	$\left(\frac{74999}{75000}\right)^n > 0.9$	M1	
	$n \ln \frac{74999}{75000} > \ln 0.9$	M1	Attempt ln or log both sides
	Largest value of n is 7901	A1	CWO Must be an integer
		4	

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Question	Answer	Marks	Guidance
6	$E(X) = 3$	B1	N.B. $E(X)=108k$ is B0 until correct k substituted in.
	$k \int_0^6 (6x - x^2) dx = 1$ $k \left[3x^2 - \frac{x^3}{3} \right]_0^6 [= 1]$	M1	Attempt integration of $f(x)$ and $=1$. Ignore limits at this stage.
	$k \left(108 - \frac{216}{3} \right) = 1$ $k = \frac{3}{108} \text{ or } \frac{1}{36}$	A1	
	$\frac{3}{108} \int_0^6 (6x^3 - x^4) dx$ $= \frac{3}{108} \left[\frac{3x^4}{2} - \frac{x^5}{5} \right]_0^6 = 10.8$	*M1	Attempt integration of <i>their</i> $k \times x^2 f(x)$. Ignore limits at this stage. Accept in terms of k .
	'10.8' – '3' ²	DM1	<i>Their</i> 10.8 (from use of limits 0 and 6) minus <i>their</i> $(E(X))^2$. Accept in terms of k : $388.8k - (108k)^2$
	$\frac{9}{5}$ or 1.8	A1	CWO. Must be convincingly obtained as AG.
		6	

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Question	Answer	Marks	Guidance
7(a)	$E(T) = 3 \times 55 + 6 \times 27 [= 327]$	B1	OE. Accept unsimplified.
	$\text{Var}(T) = 3 \times 3^2 + 6 \times 2.5^2 [= 64.5]$	B1	Accept unsimplified.
	$\frac{340 - '327'}{\sqrt{'64.5'}} [= 1.619]$	M1	Must have $\sqrt{}$
	$P(z < '1.619') = \Phi('1.619')$	M1	Correct probability area consistent with <i>their</i> working.
	0.947 (3 sf)	A1	
		5	
7(b)	$E(L - S_1 - S_2) = 55 - 2 \times 27 [= 1]$	B1	OE e.g. $E(S_1 + S_2 - L) = -1$. Accept unsimplified.
	$\text{Var}(L - S_1 - S_2) = 3^2 + 2 \times 2.5^2 [= 21.5]$	B1	Accept unsimplified.
	$\frac{0 - '1'}{\sqrt{'21.5'}} [= -0.216]$	M1	Standardising with <i>their</i> values. Must come from a combination attempt.
	$P(L - S_1 - S_2 > 0) = \Phi('0.216')$	M1	Correct probability area consistent with <i>their</i> working.
	0.586 or 0.585 (3 sf)	A1	
		5	

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Question	Answer	Marks	Guidance
8(a)	Not representative (of all students in the school)	B1	OE idea of ‘not being representative’ e.g. different grades in the school have different characteristics/proportions.... Don’t accept ‘not random’ or ‘biased’ without further explanation.
		1	
8(b)	$H_0: P(\text{not correct uniform}) = 0.15$ $H_1: P(\text{not correct uniform}) < 0.15$	B1	Allow " <i>p</i> "
		1	
8(c)	Any two probs attempted using B(50,0.15)	M1	
	$P(X \leq 3) = 0.85^{50} + 50 \times 0.85^{49} \times 0.15 + {}^{50}C_2 \times 0.85^{48} \times 0.15^2 + {}^{50}C_3 \times 0.85^{47} \times 0.15^3$	M1	Attempt the tail probability P(0,1,2,3) with B(50,0.15) must be added.
	$P(X \leq 4) = 0.04605 + {}^{50}C_4 \times 0.85^{46} \times 0.15^4$	M1	OE. <i>Their</i> P($X \leq 3$) + P($X = 4$) or P(0,1,2,3,4) with B(50,0.15) must be added.
	$P(X \leq 3) = 0.0460$ or 0.0461 [<0.05] $P(X \leq 4) = 0.112$ or [>0.05]	A1	Both correct. OR if P($X \leq 4$) not seen; P(4)=0.06606 and 0.06606>0.05 and P($X \leq 3$)=0.0460 scores M1 A1
	P(Type I) = 0.0460 or 0.0461 (3 sf)	A1	Dependent on second M1. SC If M1M1M1A0 scored allow A1FT for incorrect P($X \leq 3$) as long as <0.05
		5	

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Question	Answer	Marks	Guidance
8(d)	4 is outside critical region (≤ 3) OE or $P(X \leq 4) = 0.112$ which is > 0.05	M1	FT working from (c) .
	No evidence that proportion not wearing the correct uniform has decreased (Accept H_0)	A1	In context not definite, e.g. not 'Proportion has not decreased'. No contradiction.
		2	
8(e)	Not rejected H_0	*B1 FT	FT If Reject H_0 in (d)
	Type II	DB1 FT	FT Type I
		2	



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Question	Answer	Marks	Guidance
1(a)	$H_0: p = \frac{1}{4}$ $H_1: p \neq \frac{1}{4}$	B1	or $H_0: \mu = 25$ or $H_1: \mu \neq 25$
		1	
1(b)	$N\left(25, \frac{75}{4}\right)$	B1	SOI. Allow B1 for $N\left(25, \frac{75}{4}\right)$ or $N(0.25, 0.001875)$ SOI.
	$\pm \frac{15.5 - 25}{\sqrt{\frac{75}{4}}}$ or $\frac{\frac{15.5}{100} - 0.25}{\sqrt{\frac{0.25 \times 0.75}{100}}}$	M1	Standardise with <i>their</i> $N(25, \dots)$ Allow with no or wrong continuity correction.
	± -2.194 (2.19)	A1	
	$-2.326 < -2.194$ or $0.0141 > 0.01$ or $0.9859 < 0.99$	M1	For valid comparison (accept 2.326 to 2.329)
	No evidence to reject that the probability is $\frac{1}{4}$	A1 FT	OE must be in context and not definite, e.g. not ‘Claim untrue’. No contradictions. FT <i>their</i> z ; dependent on two-tailed test (one-tailed test can score B1 M1 A1 M1 A0) SC for use of Binomial $B(100, 0.25)$ $P = 0.0111$ for B1 and then comparison with 0.01 and correct conclusion for B1, maximum 2 out of 5 marks.
		5	

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Question	Answer	Marks	Guidance
2(a)	$\text{Var}(X) = 400 \times 0.01 \times 0.99 (= 3.96)$	M1	
	$\text{Var}(4X + 2) = 16 \times \text{Var}(X)$	M1	For $16 \times \text{their Var}(X)$
	63.36	A1	Accept 63.4
		3	
2(b)(i)	Po(4)	B1	
	$n = 400 > 50$ and either $np = 4 < 5$ or $p = 0.01 < 0.1$	B1	Must quote values 400 and 4 or clearly see $n=400$ and $np=4$ (or $p=0.01$) in working
		2	
2(b)(ii)	$e^{-4} \left(\frac{4^2}{2!} + \frac{4^3}{3!} + \frac{4^4}{4!} + \frac{4^5}{5!} \right)$	M1	FT <i>their</i> '4' Allow one end error FT from (b)(i) Use of Normal allow M1 for attempt at standardising (with correct continuity correction) using <i>their</i> $N(4, 3.96)$ and attempt at probability. FT from (b)(i) Use of Binomial allow M1 for attempt at $P(2, 3, 4, 5)$ Binomial terms clearly seen and added
	0.694 (3 sf)	A1	CWO SCB1 only for unsupported answer of 0.694
		2	

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Question	Answer	Marks	Guidance
3(a)	$\frac{1}{2}p(p-1) = 1$	M1	For area =1 For verification methods accept $\frac{1}{2} \times 2 \times 1 = 1$ or $\frac{1}{2} \times 2 \times (p-1) = 1$ or $\frac{1}{2} \times 1 \times p = 1$ as indication that area=1
	$p = 2$	A1	AG - Convincing method and answer. Must see quadratic rearranged to =0 and no errors seen. N.B. Accept convincing verification methods (e.g. statement such as ‘assume $p = 2$ ’ or ‘if $p = 2$ ’ or ‘using $p = 2$ ’ or showing by clear substitution that $p = 2$ fits $\frac{1}{2}p(p-1) = 1$ with clear conclusion)
		2	
3(b)	Gradient = 2 equation of line is $y = 2x + c$ line passes through (1, 0), hence $c = -2$	M1	Award for attempting equation of line $y = mx + c$ with $m = 2, -2, \frac{1}{2}$ or $-\frac{1}{2}$ and numerical c ($c \neq 0$)
	$y = 2x - 2$	A1	May be seen in (a) M1 can be implied by correct answer
	$2 \int_1^2 (x^2 - x) dx$	M1	For attempting $\int x f(x) dx$. Ignore limits, FT <i>their</i> equation.
	$2 \left[\frac{x^3}{3} - \frac{x^2}{2} \right]_1^2$	A1 FT	Correct integration FT <i>their</i> $f(x)$ and correct limits
	$\frac{5}{3}$ or 1.67 (3 sf)	A1	
		5	

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Question	Answer	Marks	Guidance
4(a)	Mean = $15.0+32.0+8.6$ [= 55.6]	B1	Allow unsimplified
	Var = $1.1^2+3.5^2+1.2^2$ [= 14.9]	B1	Allow unsimplified
		2	
4(b)	$\frac{60 - "55.6"}{\sqrt{"14.9"}}$ [= 1.140]	M1	FT <i>their</i> 55.6 and 14.9 Ignore continuity correction
	$1 - \Phi("1.140")$	M1	For correct probability area consistent with <i>their</i> working
	0.127 (3 sf)	A1	CWO
		3	
4(c)	$\frac{54.5 - "55.6"}{\sqrt{\frac{"14.9"}{15}}}$ or $\frac{817.5 - 834}{\sqrt{223.5}}$ [= -1.104]	M1	FT <i>their</i> 55.6 and 14.9 No mixed methods
	$1 - \Phi("1.104")$	M1	For correct probability area consistent with their working
	0.135 (3 sf)	A1	As final answer
		3	

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Question	Answer	Marks	Guidance
5(a)	Conclude that (population) mean time has changed (or is not 42.4) although μ has not changed (or is still 42.4)	B1	OE. In context.
		1	
5(b)	H ₀ : population mean (or μ) = 42.4 H ₁ : population mean (or μ) $\neq$ 42.4	B1	Not just ‘mean’. (could be seen in (a))
	$\pm \frac{45.6 - 42.4}{\sqrt{38.2 \div 20}}$	M1	For standardising (must have $\sqrt{20}$)
	± 2.315	A1	
	2.240 < ‘2.315’	M1	For valid comparison (accept 2.241) or $P(z > 2.315) = 0.0103 < 0.0125$ oe
	There is evidence that μ or mean time has changed	A1 FT	FT <i>their z</i> In context, not definite. No contradictions. Note: Accept correct alternative methods SC: One tail test no FT. Can score B0 M1 A1 M1 (comparison with 1.96) A0 (maximum 3 out of 5)
		5	

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Question	Answer	Marks	Guidance
6(a)	$\text{est}(\mu) = \frac{7570}{100} (= 75.7)$	B1	
	$\text{est}(\sigma^2) = \frac{100}{99} \left(\frac{\sum h^2}{100} - '75.7'^2 \right)$ or $\frac{1}{99} \left(588050 - \frac{7570^2}{100} \right)$ $= \frac{100}{99} \left(\frac{588050}{100} - '75.7'^2 \right) [= 151.525]$	M1	Attempted (Note: Biased variance (150.01) scores M0)
	= 152 (3 sf)	A1	Or $\frac{15001}{99}$
		3	
6(b)	$'75.7' \pm z \sqrt{\frac{151.525}{100}}$	M1	For expression of correct form. Must be a z value. Condone just + or just -.
	$z = 2.576$	B1	Accept 2.574 to 2.579
	72.5 to 78.9	A1 FT	FT biased variance only Must be an interval
		3	
6(c)	0.99^4	B1	
	0.961 (3 sf)	B1	
		2	

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Question	Answer	Marks	Guidance
7(a)	$e^{-4.2} \times \frac{4.2^4}{4!}$	M1	P(4), allow any λ
	0.194 (3 sf)	A1	As final answer. SC Unsupported correct answer scores B1 only.
		2	
7(b)	$1 - e^{-8.4} \left(1 + 8.4 + \frac{8.4^2}{2} + \frac{8.4^3}{3!} \right)$	M1	Allow M1 with incorrect λ . Accept one end error.
	0.968 (3 sf)	A1	As final answer. SC Unsupported correct answer scores B1 only.
		2	
7(c)	N(50.4, 50.4)	M1	SOI
	$\frac{39.5 - 50.4}{\sqrt{50.4}} \quad [= -1.535]$	M1	Allow wrong or no continuity correction. Must have $\sqrt{}$
	$\Phi(-1.535) = 1 - \Phi(1.535)$	M1	For correct probability area consistent with <i>their</i> working.
	0.0624 (3 sf) or 0.0623	A1	
		4	



Cambridge International A Level

MATHEMATICS

9709/63

Paper 6 Probability & Statistics 2

May/June 2021

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2021 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **13** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	$\lambda = 10 \times 1.36 [= 13.6]$	M1	
	$E(\text{amount}) = 5 \times 13.6 = [\text{\$}]68$	A1	
	$\text{Var}(\text{amount}) = 5^2 \times 13.6 [= 340]$	M1	$5^2 \times \dots$
		M1	$\dots \times \text{their } \lambda$
	Standard Deviation = $[\text{\$}]18.4(4)$ (3 s.f.)	A1	CAO condone $2\sqrt{85}$
		5	

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Question	Answer	Marks	Guidance
2(a)	Conclude (mean) (journey) time has not decreased when in fact it has.	B1	OE in context
		1	
2(b)	H ₀ : Pop mean (or μ) = 1.4 H ₁ : Pop mean (or μ) < 1.4	B1	May be seen in (a)
	$\frac{1.36 - 1.4}{\frac{0.12}{\sqrt{50}}}$	M1	Accept totals method $\frac{68 - 70}{\sqrt{50} \times 0.12}$ No mixed methods or no standard deviation/variance mixes
	–2.357 or – 2.36	A1	Correct z or correct area if used
	–2.357 < –1.96 or 0.0092 < 0.025 or 0.9908 > 0.975 Or CV method 1.36 < 1.367	M1	valid comparison
	There is evidence that (mean) (journey) times have decreased	A1 FT	in context not definite no contradictions NB use of two tail test scores max B0M1A1M1A0 no ft for two tail test
		5	
2(c)	H ₀ was rejected OE	*B1 FT	FT H ₀ was accepted OE
	Type I	DB1 FT	FT Type II
		2	

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Question	Answer	Marks	Guidance
3(a)(i)	$H_0: \lambda = 2.4$ $H_1: \lambda > 2.4$	B1	Accept λ or μ Accept 2.4 or 0.8 (per year)
		1	
3(a)(ii)	$1 - e^{-2.4}(1 + 2.4 + \frac{2.4^2}{2} + \frac{2.4^3}{3!} + \frac{2.4^4}{4!})$	M1	Any λ ; allow one end error
	0.0959 (3 sf)	A1	SC unsupported answer 0.0959 scores B1 only not M1A1
	$0.0959 > 0.05$	M1	Valid comparison Use of $0.9041 < 0.95$ can recover either M1A1 or B1
	There is evidence that Jane's claim not justified or There is insufficient evidence to support Jane's claim	A1 FT	OE. In context, not definite, e.g. not 'Jane is wrong', no contradictions. Condone omission of Jane.
		4	
3(b)	Mean not constant so Poisson model not valid	B1	
		1	

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Question	Answer	Marks	Guidance
4(a)	$\frac{4509}{90}$ [= 50.1]	B1	
	$\frac{90}{89} \left(\frac{225950}{90} - '50.1'^2 \right)$ or $\frac{1}{89} \left(225950 - \frac{4509^2}{90} \right)$	M1	Attempted. Use of biased = 0.5455 scores M0A0
	$\frac{491}{890}$ or 0.552 (3 sf)	A1	
		3	
4(b)	$'50.1' \pm z \sqrt{\frac{491}{890}}$	M1	Expression of the correct form, allow any z-value but must be a z-value
	$z = 2.326$	B1	Accept 2.326 to 2.329
	49.9 to 50.3 (3 sf)	A1	FT from biased variance. Must be an interval.
		3	
4(c)	Population of masses is unknown	B1	Accept population of masses is not normal
		1	
4(d)	$1 - 0.98$	M1	0.02 seen
	$0.02 \div 2 = 0.01$	A1	As final answer
		2	

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Question	Answer	Marks	Guidance
5(a)	Po(2.5)	B1	Accept Poisson with mean = 2.5 not just $np = 2.5$
	$n = 25\,000 > 50$ and np (or λ) = 2.5 which is < 5 or $n = 25\,000 > 50$ and $p = 0.0001 < 0.1$	B1	Must see 2.5 (or 0.0001) and 25000 OE, not just $np < 5$ (or $p < 0.1$) and $n > 50$
		2	
5(b)	$e^{-2.5}(1 + 2.5 + \frac{2.5^2}{2} + \frac{2.5^3}{3!})$	M1	Any λ , accept one end error. FT binomial from part (a) scores M1 only for equivalent binomial expressions FT normal from part (a) must use correct continuity correction and tables scores M1 only for complete method
	0.758 (3 sf)	A1	Unsupported answer of 0.758 scores B1 instead of M1A1
		2	
5(c)	$e^{-2.5} \times \frac{2.5^k}{(k)!} = 2e^{-2.5} \times \frac{2.5^{k+1}}{(k+1)!}$	M1	Any λ FT binomial from (a) scores M1 only for equivalent binomial expression FT from (a) normal for equivalent expressions continuity correction must be included
	$k = 4$	A1	No errors seen SC $k = 4$ unsupported scores B1 only, but see full Poisson expressions for P(4) and P(5) and 0.134 scores M1A1
		2	

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Question	Answer	Marks	Guidance
5(d)	$1 - e^{-\lambda} = 0.963$	M1	Accept <i>their</i> attempt at λ
	$\lambda = -\ln 0.037 (= 3.2968 \text{ or } 3.30 \text{ or } 3.3)$	M1	Correct use of lns
	$n = 33\,000$ (3 sf)	A1	Allow $n = 32\,950$ to $33\,050$ (must be an integer) SC use of binomial leading to 32 967 scores B1 for $(0.9999)^n = 0.037$ B1 for 33 000 to 3sf (32 967)
		3	
6(a)	$P(X > 10) = \int_{10}^{20} \frac{3}{8000} (x - 20)^2 dx$	M1	Attempt integration of $f(x)$, ignore limits.
	$= \left[\frac{3}{8000} \times \frac{(x-20)^3}{3} \right]_{10}^{20} \text{ or } \frac{3}{8000} \left[\frac{x^3}{3} - \frac{40x^2}{2} + 400x \right]_{10}^{20}$ $= \frac{1}{8000} [0 - (-10)^3]$	M1	Substitute correct limits 10 to 20 or 1 – ... limits 0 to 10 in <i>their</i> integral
	$\frac{1}{8}$ or 0.125	A1	SC Unsupported answer of $\frac{1}{8}$ scores B1 only
	$(\frac{1}{8})^2 = \frac{1}{64}$ or 0.0156 (3 sf)	B1 FT	FT <i>their</i> $P(X > 10)$ dependent on first M1 gained
		4	

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Question	Answer	Marks	Guidance
6(b)	$\int_0^{20} \frac{3}{8000} (x^3 - 40x^2 + 400x) dx$	M1	Attempt integration of $xf(x)$. Ignore limits.
	$\frac{3}{8000} \left[\frac{x^4}{4} - \frac{40x^3}{3} + \frac{400x^2}{2} \right]_0^{20}$ or $\left(\frac{3x}{8000} \times \frac{(x-20)^3}{3} \right) - \frac{1}{8000} \left(\frac{(x-20)^4}{4} \right)$	A1	Correct integral (by expanding or by parts)
	$\frac{3}{8000} \left[\frac{160000}{4} - \frac{40 \times 8000}{3} + 200 \times 400 \right]$	M1	Subst correct limits in their (4th degree) integral
	5	A1	
		4	
6(c)	$\int_0^m \frac{3}{8000} (x-20)^2 dx = 0.5$	M1	Attempt to integrate $f(x)$ and equate to 0.5. Ignore limits.
	$\left[\frac{3}{8000} \times \frac{(x-20)^3}{3} \right]_0^m = 0.5$ or $\frac{3}{8000} \left[\frac{x^3}{3} - \frac{40x^2}{2} + 400x \right]_0^m = 0.5$ $\frac{1}{8000} [(m-20)^3 - (-20)^3] = 0.5$	M1	Attempt integral and substitute limits 0 and m or m and 20 and = 0.5
	$(m-20)^3 = -4000$	A1	AG. Found convincingly.
	$(m = 20 + \sqrt[3]{-4000})$ $m = 4.13$ (3 sf)	B1	
		4	

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Question	Answer	Marks	Guidance
6(d)	Doesn't allow for trains > 20 mins late or Doesn't allow for trains being early	B1	or any relevant comment e.g. trains on Sun may be different to trains on Mon
		1	