



Cambridge International AS & A Level

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MATHEMATICS

9709/31

Paper 3 Pure Mathematics 3

May/June 2021

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
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- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.



1 Solve the inequality $2|3x - 1| < |x + 1|$.

[4]

[illegible]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- 3 (a) Given that $\cos(x - 30^\circ) = 2 \sin(x + 30^\circ)$, show that $\tan x = \frac{2 - \sqrt{3}}{1 - 2\sqrt{3}}$. [4]

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- (b) Hence solve the equation

$$\cos(x - 30^\circ) = 2 \sin(x + 30^\circ),$$

for $0^\circ < x < 360^\circ$. [2]

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- 4 (a) Prove that $\frac{1 - \cos 2\theta}{1 + \cos 2\theta} \equiv \tan^2 \theta$. [2]

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- (b) Hence find the exact value of $\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} \frac{1 - \cos 2\theta}{1 + \cos 2\theta} d\theta$. [4]

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- 5 (a) Solve the equation $z^2 - 2piz - q = 0$, where p and q are real constants. [2]

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In an Argand diagram with origin O , the roots of this equation are represented by the distinct points A and B .

- (b) Given that A and B lie on the imaginary axis, find a relation between p and q . [2]

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6 The parametric equations of a curve are

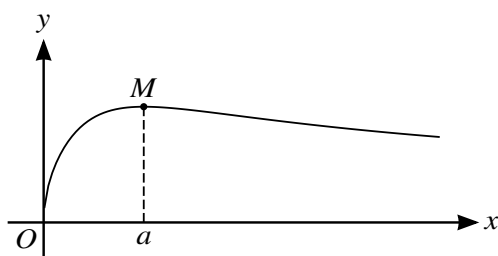
$$x = \ln(2 + 3t), \quad y = \frac{t}{2 + 3t}.$$

(a) Show that the gradient of the curve is always positive.

[5]

[illegible]

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(a) Show that a satisfies the equation

$$a = \tan\left(\frac{2a}{1+a^2}\right). \quad [4]$$

[illegible]

- (b) Verify by calculation that a lies between 1.3 and 1.5.

[2]

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- (c) Use an iterative formula based on the equation in part (a) to determine a correct to 2 decimal places. Give the result of each iteration to 4 decimal places.

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- 8** With respect to the origin O , the points A and B have position vectors given by $\overrightarrow{OA} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}$. The line l has equation $\mathbf{r} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$.

- (a) Find the acute angle between the directions of AB and l . [4]

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- 9** The equation of a curve is $y = x^{-\frac{2}{3}} \ln x$ for $x > 0$. The curve has one stationary point.

(a) Find the exact coordinates of the stationary point.

[5]

This image shows a full page of a worksheet designed for handwriting practice. It consists of approximately 20 horizontal rows. Each row is defined by two parallel dotted lines, creating a series of uniform gaps for writing. The entire page is otherwise blank, with no margins, text, or other markings.

(b) Show that $\int_1^8 y \, dx = 18 \ln 2 - 9$.

[5]

This image shows a full page of white paper with horizontal dotted lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the entire width of the page. There are no margins, text, or other markings present.

- 10** The variables x and t satisfy the differential equation $\frac{dx}{dt} = x^2(1 + 2x)$, and $x = 1$ when $t = 0$.

Using partial fractions, solve the differential equation, obtaining an expression for t in terms of x .

[11]

[illegible]

[illegible]

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MATHEMATICS

9709/32

Paper 3 Pure Mathematics 3

May/June 2021

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
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1 Solve the inequality $|2x - 1| < 3|x + 1|$.

[4]

[illegible]

- 2 On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z + 1 - i| \leq 1$ and $\arg(z - 1) \leq \frac{3}{4}\pi$. [4]

3 The variables x and y satisfy the equation $x = A(3^{-y})$, where A is a constant.

- (a) Explain why the graph of y against $\ln x$ is a straight line and state the exact value of the gradient of the line. [3]

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It is given that the line intersects the y -axis at the point where $y = 1.3$.

- (b) Calculate the value of A , giving your answer correct to 2 decimal places. [2]

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[illegible]

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting or typing. There are no margins, text, or other markings on the page.

- 6 (a) Prove that $\operatorname{cosec} 2\theta - \cot 2\theta \equiv \tan \theta$. [3]

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- (b) Hence show that $\int_{\frac{1}{4}\pi}^{\frac{1}{3}\pi} (\operatorname{cosec} 2\theta - \cot 2\theta) \, d\theta = \frac{1}{2} \ln 2$. [4]

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- 7** A curve is such that the gradient at a general point with coordinates (x, y) is proportional to $\frac{y}{\sqrt{x+1}}$. The curve passes through the points with coordinates $(0, 1)$ and $(3, e)$.

By setting up and solving a differential equation, find the equation of the curve, expressing y in terms of x . [7]

This image shows a single page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- 8** The equation of a curve is $y = e^{-5x} \tan^2 x$ for $-\frac{1}{2}\pi < x < \frac{1}{2}\pi$.

Find the x -coordinates of the stationary points of the curve. Give your answers correct to 3 decimal places where appropriate. [8]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

9 Let $f(x) = \frac{14 - 3x + 2x^2}{(2 + x)(3 + x^2)}$.

(a) Express $f(x)$ in partial fractions.

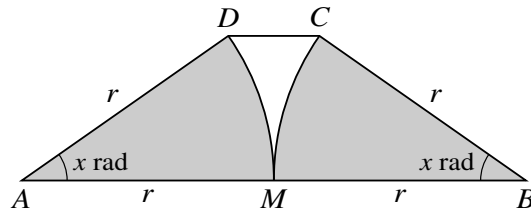
[5]

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- (b)** Hence obtain the expansion of $f(x)$ in ascending powers of x , up to and including the term in x^2 . [5]

This image shows a full page of white paper with horizontal dotted lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

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The diagram shows a trapezium $ABCD$ in which $AD = BC = r$ and $AB = 2r$. The acute angles BAD and ABC are both equal to x radians. Circular arcs of radius r with centres A and B meet at M , the midpoint of AB .

- (a) Given that the sum of the areas of the shaded sectors is 90% of the area of the trapezium, show that x satisfies the equation $x = 0.9(2 - \cos x) \sin x$. [3]

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- (b) Verify by calculation that x lies between 0.5 and 0.7. [2]

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- (c) Show that if a sequence of values in the interval $0 < x < \frac{1}{2}\pi$ given by the iterative formula

$$x_{n+1} = \cos^{-1} \left(2 - \frac{x_n}{0.9 \sin x_n} \right)$$

converges, then it converges to the root of the equation in part (a). [2]

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- (d) Use this iterative formula to determine x correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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- 11** With respect to the origin O , the points A and B have position vectors given by $\overrightarrow{OA} = 2\mathbf{i} - \mathbf{j}$ and $\overrightarrow{OB} = \mathbf{j} - 2\mathbf{k}$.

(a) Show that $OA = OB$ and use a scalar product to calculate angle AOB in degrees. [4]

This image shows a full page of white paper designed for handwriting practice. It features approximately 20 evenly spaced horizontal dotted lines running from left to right across the entire width of the page. There are no margins, text, or other markings present.

[illegible]

[illegible]

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MATHEMATICS

9709/33

Paper 3 Pure Mathematics 3

May/June 2021

1 hour 50 minutes

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- 1** Expand $(1 + 3x)^{\frac{2}{3}}$ in ascending powers of x , up to and including the term in x^3 , simplifying the coefficients. [4]

[illegible]

- 2** Solve the equation $4^x = 3 + 4^{-x}$. Give your answer correct to 3 decimal places. [5]

This image shows a full page of white paper with horizontal dotted lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the entire width of the page. There are no margins, text, or other markings present.

- 3 The parametric equations of a curve are

$$x = t + \ln(t + 2), \quad y = (t - 1)e^{-2t},$$

where $t > -2$.

- (a) Express $\frac{dy}{dx}$ in terms of t , simplifying your answer. [5]

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- (b) Find the exact y-coordinate of the stationary point of the curve. [2]

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4 Let $f(x) = \frac{15 - 6x}{(1 + 2x)(4 - x)}$.

(a) Express $f(x)$ in partial fractions.

[3]

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(b) Hence find $\int_1^2 f(x) \, dx$, giving your answer in the form $\ln\left(\frac{a}{b}\right)$, where a and b are integers. [4]

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- 5 (a)** By first expanding $\tan(2\theta + 2\theta)$, show that the equation $\tan 4\theta = \frac{1}{2} \tan \theta$ may be expressed as $\tan^4 \theta + 2 \tan^2 \theta - 7 = 0$. [4]

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- 6 (a) By sketching a suitable pair of graphs, show that the equation $\cot \frac{1}{2}x = 1 + e^{-x}$ has exactly one root in the interval $0 < x \leq \pi$. [2]

- (b) Verify by calculation that this root lies between 1 and 1.5. [2]

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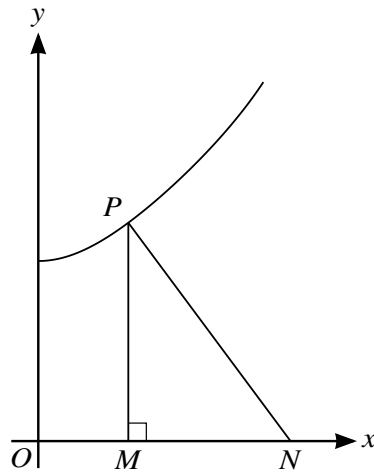
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For the curve shown in the diagram, the normal to the curve at the point P with coordinates (x, y) meets the x -axis at N . The point M is the foot of the perpendicular from P to the x -axis.

The curve is such that for all values of x in the interval $0 \leq x < \frac{1}{2}\pi$, the area of triangle PMN is equal to $\tan x$.

- (a) (i) Show that $\frac{MN}{y} = \frac{dy}{dx}$. [1]

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- (ii) Hence show that x and y satisfy the differential equation $\frac{1}{2}y^2 \frac{dy}{dx} = \tan x$. [2]

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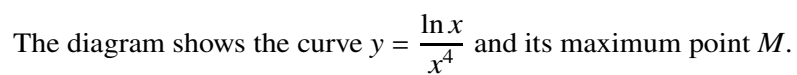
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- (b) Given that $y = 1$ when $x = 0$, solve this differential equation to find the equation of the curve, expressing y in terms of x . [6]

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(b) By using integration by parts, show that for all $a > 1$, $\int_1^a \frac{\ln x}{x^4} dx < \frac{1}{9}$. [6]

[illegible]

- 9 The quadrilateral $ABCD$ is a trapezium in which AB and DC are parallel. With respect to the origin O , the position vectors of A , B and C are given by $\vec{OA} = -\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$, $\vec{OB} = \mathbf{i} + 3\mathbf{j} + \mathbf{k}$ and $\vec{OC} = 2\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$.

- (a) Given that $\overrightarrow{DC} = 3\overrightarrow{AB}$, find the position vector of D . [3]

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- (b) State a vector equation for the line through A and B . [1]

[illegible]

This image shows a full page of white paper with horizontal dotted lines, typical of primary school writing paper. The lines are evenly spaced and run across the entire width of the page. There are no margins, text, or other markings present.

- 10 (a) Verify that $-1 + \sqrt{2}i$ is a root of the equation $z^4 + 3z^2 + 2z + 12 = 0$. [3]

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- (b) Find the other roots of this equation. [7]

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