

Cambridge International AS & A Level

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MATHEMATICS

9709/61

Paper 6 Probability & Statistics 2

May/June 2020

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
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- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **12** pages. Blank pages are indicated.

- 1** The lengths, X centimetres, of a random sample of 7 leaves from a certain variety of tree are as follows.

5.2 4.8 5.5 6.1 4.8 3.9 4.4

- (a)** Calculate unbiased estimates of the population mean and variance of X . [3]

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It is now given that the true value of the population variance of X is 0.55, and that X has a normal distribution.

- (b)** Find a 95% confidence interval for the population mean of X . [3]

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- 2 In the past the yield of a certain crop, in tonnes per hectare, had mean 0.56 and standard deviation 0.08. Following the introduction of a new fertilizer, the farmer intends to test at the 2.5% significance level whether the mean yield has increased. He finds that the mean yield over 10 years is 0.61 tonnes per hectare.

(a) State two assumptions that are necessary for the test. [2]

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(b) Carry out the test. [5]

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- 3** The masses, in kilograms, of large sacks of flour and small sacks of flour have the independent distributions $N(40, 1.5^2)$ and $N(12, 0.7^2)$ respectively.
- (a)** Find the probability that the total mass of 6 randomly chosen large sacks of flour is more than 245 kg. [4]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- (b) Find the probability that the mass of a randomly chosen large sack of flour is less than 4 times the mass of a randomly chosen small sack of flour. [6]

[illegible]

- 4** A fair spinner has five sides numbered 1, 2, 3, 4, 5. The score on one spin is denoted by X .

(a) Show that $\text{Var}(X) = 2$.

[1]

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Fiona has another spinner, also with five sides numbered 1, 2, 3, 4, 5. She suspects that it is biased so that the expected score is less than 3. In order to test her suspicion, she plans to spin her spinner 40 times. If the mean score is less than 2.6 she will conclude that her spinner is biased in this way.

(b) Find the probability of a Type I error.

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(c) State what is meant by a Type II error in this context.

[1]

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- 5** Each week a sports team plays one home match and one away match. In their home matches they score goals at a constant average rate of 2.1 goals per match. In their away matches they score goals at a constant average rate of 0.8 goals per match. You may assume that goals are scored at random times and independently of one another.

(a) A week is chosen at random.

- (i)** Find the probability that the team scores a total of 4 goals in their two matches. [2]

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- (ii)** Find the probability that the team scores a total of 4 goals, with more goals scored in the home match than in the away match. [3]

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- (b) Use a suitable approximating distribution to find the probability that the team scores fewer than 25 goals in 10 randomly chosen weeks. [4]

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- (c) Justify the use of the approximating distribution used in part (b). [1]

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- 6** The length of time, T minutes, that a passenger has to wait for a bus at a certain bus stop is modelled by the probability density function given by

$$f(t) = \begin{cases} \frac{3}{4000}(20t - t^2) & 0 \leq t \leq 20, \\ 0 & \text{otherwise.} \end{cases}$$

- (a)** Sketch the graph of $y = f(t)$. [1]

- (b)** Hence explain, without calculation, why $E(T) = 10$. [1]

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- (c)** Find $\text{Var}(T)$. [3]

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- (d) It is given that $P(T < 10 + a) = p$, where $0 < a < 10$.

Find $P(10 - a < T < 10 + a)$ in terms of p . [2]

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- (e) Find $P(8 < T < 12)$. [3]

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- (f) Give one reason why this model may be unrealistic. [1]

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[illegible]

Cambridge Assessment International Education is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which itself is a department of the University of Cambridge.

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MATHEMATICS

9709/62

Paper 6 Probability & Statistics 2

May/June 2020

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- 1 The masses, in grams, of plums of a certain type have the distribution $N(40.4, 5.2^2)$. The plums are packed in bags, with each bag containing 6 randomly chosen plums. If the total weight of the plums in a bag is less than 220 g the bag is rejected.

Find the percentage of bags that are rejected.

[4]

This image shows a single page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- 2** A shop obtains apples from a certain farm. It has been found that 5% of apples from this farm are Grade A. Following a change in growing conditions at the farm, the shop management plan to carry out a hypothesis test to find out whether the proportion of Grade A apples has increased. They select 25 apples at random. If the number of Grade A apples is more than 3 they will conclude that the proportion has increased.

(a) State suitable null and alternative hypotheses for the test. [1]

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(b) Find the probability of a Type I error. [3]

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In fact 2 of the 25 apples were Grade A.

(c) Which of the errors, Type I or Type II, is possible? Justify your answer. [2]

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3 In the data-entry department of a certain firm, it is known that 0.12% of data items are entered incorrectly, and that these errors occur randomly and independently.

(a) A random sample of 3600 data items is chosen. The number of these data items that are incorrectly entered is denoted by X .

(i) State the distribution of X , including the values of any parameters. [1]

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(ii) State an appropriate approximating distribution for X , including the values of any parameters.

Justify your choice of approximating distribution. [3]

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(iii) Use your approximating distribution to find $P(X > 2)$. [2]

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- (b) Another large random sample of n data items is chosen. The probability that the sample contains no data items that are entered incorrectly is more than 0.1.

Use an approximating distribution to find the largest possible value of n . [3]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting or typing. There are no margins, text, or other markings on the page.

- 4 The score on one spin of a 5-sided spinner is denoted by the random variable X with probability distribution as shown in the table.

x	0	1	2	3	4
$P(X = x)$	0.1	0.2	0.4	0.2	0.1

- (a) Show that $\text{Var}(X) = 1.2$. [2]

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The spinner is spun 200 times. The score on each spin is noted and the mean, $\bar{X}$, of the 200 scores is found.

- (b) Given that $P(\bar{X} > a) = 0.1$, find the value of a . [4]

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- (c) Explain whether it was necessary to use the Central Limit theorem in your answer to part (b). [1]

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- (d) Johann has another, similar, spinner. He suspects that it is biased so that the mean score is less than 2. He spins his spinner 200 times and finds that the mean of the 200 scores is 1.86.

Given that the variance of the score on one spin of this spinner is also 1.2, test Johann's suspicion at the 5% significance level. [5]

[illegible]

- 5 (a) The random variable X has the distribution $\text{Po}(\lambda)$.

(i) State the values that X can take. [1]

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It is given that $P(X = 1) = 3 \times P(X = 0)$.

(ii) Find λ . [1]

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(iii) Find $P(4 \leq X \leq 6)$. [2]

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- (b) The random variable Y has the distribution $\text{Po}(\mu)$ where μ is large. Using a suitable approximating distribution, it is found that $P(Y < 46) = 0.0668$, correct to 4 decimal places.

Find μ .

[5]

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- 6 A random variable X has probability density function given by

$$f(x) = \begin{cases} \frac{k}{x^2} & 1 \leq x \leq a, \\ 0 & \text{otherwise,} \end{cases}$$

where k and a are positive constants.

- (a) Show that $k = \frac{a}{a-1}$. [3]

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- (b) Find $E(X)$ in terms of a . [3]

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- (c) Find the 60th percentile of X in terms of a . [4]

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[illegible]

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- 1** A random sample of 100 values of a variable X is taken. These values are summarised below.

$$n = 100 \quad \Sigma x = 1556 \quad \Sigma x^2 = 29\,004$$

Calculate unbiased estimates of the population mean and variance of X . [3]

[illegible]

- 2 Each day at the gym, Sarah completes three runs. The distances, in metres, that she completes in the three runs have the independent distributions $W \sim N(1520, 450)$, $X \sim N(2250, 720)$ and $Y \sim N(3860, 1050)$.

Find the probability that, on a particular day, Y is less than the total of W and X . [5]

This image shows a single page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- 3 The number of customers who visit a particular shop between 9.00 am and 10.00 am has the distribution $Po(\lambda)$. In the past the value of λ was 5.2. Following some new advertising, the manager wishes to test whether the value of λ has increased. He chooses a random sample of 20 days and finds that the total number of customers who visited the shop between 9.00 am and 10.00 am on those days is 125.

Use an approximating distribution to test at the 2.5% significance level whether the value of λ has increased. [6]

This image shows a single page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- 4 The random variable A has the distribution $\text{Po}(1.5)$. A_1 and A_2 are independent values of A .

(a) Find $P(A_1 + A_2 < 2)$. [3]

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(b) Given that $A_1 + A_2 < 2$, find $P(A_1 = 1)$. [4]

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- 5** Sunita has a six-sided die with faces marked 1, 2, 3, 4, 5, 6. The probability that the die shows a six on any throw is p . Sunita throws the die 500 times and finds that it shows a six 70 times.

(a) Calculate an approximate 99% confidence interval for p . [4]

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(b) Sunita believes that the die is fair. Use your answer to part **(a)** to comment on her belief. [1]

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- (c) Sunita uses the result of her 500 throws to calculate an $\alpha\%$ confidence interval for p . This interval has width 0.04.

Find the value of α .

[5]

[illegible]

- 6** The length, X centimetres, of worms of a certain type is modelled by the probability density function

$$f(x) = \begin{cases} \frac{6}{125}(10-x)(x-5) & 5 \leq x \leq 10, \\ 0 & \text{otherwise.} \end{cases}$$

- (a) State the value of $E(X)$. [1]

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- (b)** Find $\text{Var}(X)$. [3]

[illegible]

Find the probability that exactly one of them has length less than 6 cm.

[5]

[illegible]

- 7 A market researcher is investigating the length of time that customers spend at an information desk. He plans to choose a sample of 50 customers on a particular day.

(a) He considers choosing the first 50 customers who visit the information desk.

Explain why this method is unsuitable.

[1]

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The actual lengths of time, in minutes, that customers spend at the information desk may be assumed to have mean μ and variance 4.8. The researcher knows that in the past the value of μ was 6.0. He wishes to test, at the 2% significance level, whether this is still true. He chooses a random sample of 50 customers and notes how long they each spend at the information desk.

(b) State the probability of making a Type I error and explain what is meant by a Type I error in this context.

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- (c) Given that the mean time spent at the information desk by the 50 customers is 6.8 minutes, carry out the test. [5]

[illegible]

- (d)** Give a reason why it was necessary to use the Central Limit theorem in your answer to part (c). [1]

[illegible]

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