

Cambridge International AS & A Level

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MATHEMATICS

9709/31

Paper 3 Pure Mathematics 3

May/June 2020

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
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- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Blank pages are indicated.

- 1 Find the set of values of x for which $2(3^{1-2x}) < 5^x$. Give your answer in a simplified exact form. [4]

[illegible]

- 2 (a) Expand $(2 - 3x)^{-2}$ in ascending powers of x , up to and including the term in x^2 , simplifying the coefficients. [4]

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- (b) State the set of values of x for which the expansion is valid. [1]

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- 3 Express the equation $\tan(\theta + 60^\circ) = 2 + \tan(60^\circ - \theta)$ as a quadratic equation in $\tan \theta$, and hence solve the equation for $0^\circ \leq \theta \leq 180^\circ$. [6]

[illegible]

- 4** The curve with equation $y = e^{2x}(\sin x + 3 \cos x)$ has a stationary point in the interval $0 \leq x \leq \pi$.

(a) Find the x -coordinate of this point, giving your answer correct to 2 decimal places. [4]

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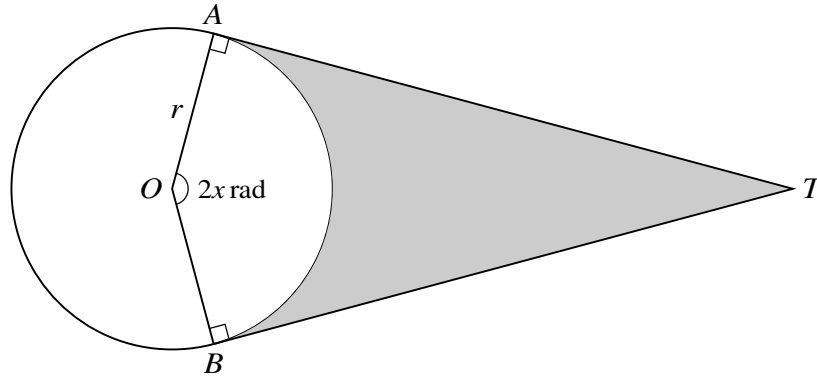
(b) Determine whether the stationary point is a maximum or a minimum. [2]

[illegible]

- 5** (a) Find the quotient and remainder when $2x^3 - x^2 + 6x + 3$ is divided by $x^2 + 3$. [3]

[illegible]

(b) Using your answer to part (a), find the exact value of $\int_1^3 \frac{2x^3 - x^2 + 6x + 3}{x^2 + 3} dx$. [5]



The diagram shows a circle with centre O and radius r . The tangents to the circle at the points A and B meet at T , and angle AOB is $2x$ radians. The shaded region is bounded by the tangents AT and BT , and by the minor arc AB . The area of the shaded region is equal to the area of the circle.

- (a) Show that x satisfies the equation $\tan x = \pi + x$. [3]

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- (b) This equation has one root in the interval $0 < x < \frac{1}{2}\pi$. Verify by calculation that this root lies between 1 and 1.4. [2]

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- (c) Use the iterative formula

$$x_{n+1} = \tan^{-1}(\pi + x_n)$$

to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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7 Let $f(x) = \frac{\cos x}{1 + \sin x}$.

(a) Show that $f'(x) < 0$ for all x in the interval $-\frac{1}{2}\pi < x < \frac{3}{2}\pi$. [4]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

(b) Find $\int_{\frac{1}{6}\pi}^{\frac{1}{2}\pi} f(x) \, dx$. Give your answer in a simplified exact form.

[4]

This image shows a full page of a handwriting practice worksheet. It consists of multiple rows of horizontal dotted lines spaced evenly down the page, providing a guide for letter height and placement. The background is plain white, and there are no margins or additional markings.

- 8** A certain curve is such that its gradient at a point (x, y) is proportional to $\frac{y}{x\sqrt{x}}$. The curve passes through the points with coordinates $(1, 1)$ and $(4, e)$.

- (a) By setting up and solving a differential equation, find the equation of the curve, expressing y in terms of x . [8]

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(b) Describe what happens to y as x tends to infinity.

[1]

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- 9 With respect to the origin O , the vertices of a triangle ABC have position vectors

$$\overrightarrow{OA} = 2\mathbf{i} + 5\mathbf{k}, \quad \overrightarrow{OB} = 3\mathbf{i} + 2\mathbf{j} + 3\mathbf{k} \quad \text{and} \quad \overrightarrow{OC} = \mathbf{i} + \mathbf{j} + \mathbf{k}.$$

- (a) Using a scalar product, show that angle ABC is a right angle. [3]

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- (b) Show that triangle ABC is isosceles. [2]

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[illegible]

10 (a) The complex number u is defined by $u = \frac{3i}{a + 2i}$, where a is real.

(i) Express u in the Cartesian form $x + iy$, where x and y are in terms of a . [3]

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(ii) Find the exact value of a for which $\arg u^* = \frac{1}{3}\pi$. [3]

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- (b) (i) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z - 2i| \leq |z - 1 - i|$ and $|z - 2 - i| \leq 2$. [4]

- (ii) Calculate the least value of $\arg z$ for points in this region. [2]

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MATHEMATICS

9709/32

Paper 3 Pure Mathematics 3

May/June 2020

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
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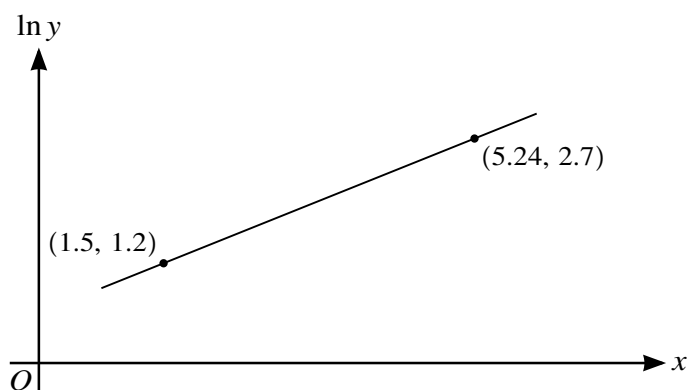
INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Blank pages are indicated.

- 1** Find the quotient and remainder when $6x^4 + x^3 - x^2 + 5x - 6$ is divided by $2x^2 - x + 1$. [3]

[illegible]



The variables x and y satisfy the equation $y^2 = Ae^{kx}$, where A and k are constants. The graph of $\ln y$ against x is a straight line passing through the points $(1.5, 1.2)$ and $(5.24, 2.7)$ as shown in the diagram.

Find the values of A and k correct to 2 decimal places.

[5]

[illegible]

3 Find the exact value of

$$\int_1^4 x^{\frac{3}{2}} \ln x \, dx.$$

[5]

This image shows a single page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

4 A curve has equation $y = \cos x \sin 2x$.

Find the x -coordinate of the stationary point in the interval $0 < x < \frac{1}{2}\pi$, giving your answer correct to 3 significant figures. [6]

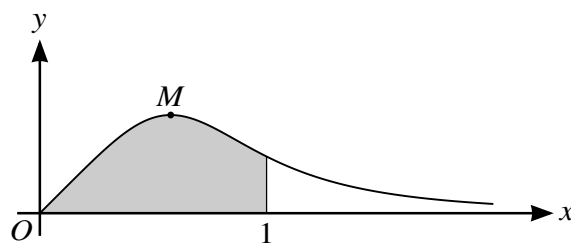
This image shows a single page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- 5 (a)** Express $\sqrt{2} \cos x - \sqrt{5} \sin x$ in the form $R \cos(x + \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. Give the exact value of R and the value of α correct to 3 decimal places. [3]

[illegible]

- (b)** Hence solve the equation $\sqrt{2} \cos 2\theta - \sqrt{5} \sin 2\theta = 1$, for $0^\circ < \theta < 180^\circ$. [4]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.



(a) Find the x -coordinate of M , giving your answer correct to 3 decimal places.

[4]

[illegible]

- (b) Using the substitution $u = \sqrt{3}x^2$, find by integration the exact area of the shaded region bounded by the curve, the x -axis and the line $x = 1$. [5]

This image shows a full page of white paper with horizontal dotted lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

7 The variables x and y satisfy the differential equation

$$\frac{dy}{dx} = \frac{y-1}{(x+1)(x+3)}.$$

It is given that $y = 2$ when $x = 0$.

Solve the differential equation, obtaining an expression for y in terms of x . [9]

[illegible]

- 8** (a) Solve the equation $(1 + 2i)w + iw^* = 3 + 5i$. Give your answer in the form $x + iy$, where x and y are real. [4]

[illegible]

- (b) (i) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z - 2 - 2i| \leq 1$ and $\arg(z - 4i) \geq -\frac{1}{4}\pi$. [4]

- (ii) Find the least value of $\text{Im } z$ for points in this region, giving your answer in an exact form. [2]

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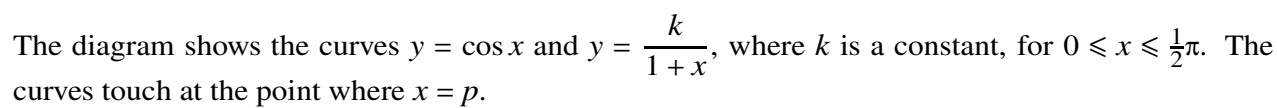
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- [illegible]

- (b) Use the iterative formula $p_{n+1} = \tan^{-1}\left(\frac{1}{1+p_n}\right)$ to determine the value of p correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

This image shows a full page of blank handwriting practice paper. It features ten sets of horizontal lines, each consisting of a solid top line, a dashed midline, and a solid bottom line. These lines are evenly spaced across the entire page to guide letter formation and alignment. The background is plain white, and there are no margins or additional markings.

- (c) Hence find the value of k correct to 2 decimal places. [2]

[illegible]

- 10** With respect to the origin O , the points A and B have position vectors given by $\vec{OA} = 6\mathbf{i} + 2\mathbf{j}$ and $\vec{OB} = 2\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$. The midpoint of OA is M . The point N lying on AB , between A and B , is such that $AN = 2NB$.

(a) Find a vector equation for the line through M and N .

[5]

[illegible]

The line through M and N intersects the line through O and B at the point P .

- (b) Find the position vector of P . [3]

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- (c) Calculate angle OPM , giving your answer in degrees. [3]

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[illegible]

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MATHEMATICS

9709/33

Paper 3 Pure Mathematics 3

May/June 2020

1 hour 50 minutes

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1 Solve the inequality $|2x - 1| > 3|x + 2|$.

[4]

[illegible]

2 Find the exact value of $\int_0^1 (2-x)e^{-2x} dx$.

[5]

This image shows a full page of a handwriting practice worksheet. It consists of multiple rows of horizontal dashed lines spaced evenly down the page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.

- 3 (a) Show that the equation

$$\ln(1 + e^{-x}) + 2x = 0$$

can be expressed as a quadratic equation in e^x .

[2]

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- (b) Hence solve the equation $\ln(1 + e^{-x}) + 2x = 0$, giving your answer correct to 3 decimal places.

[4]

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4 The equation of a curve is $y = x \tan^{-1}\left(\frac{1}{2}x\right)$.

(a) Find $\frac{dy}{dx}$. [3]

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(b) The tangent to the curve at the point where $x = 2$ meets the y -axis at the point with coordinates $(0, p)$.

Find p . [3]

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5 By first expressing the equation

$$\tan \theta \tan(\theta + 45^\circ) = 2 \cot 2\theta$$

as a quadratic equation in $\tan \theta$, solve the equation for $0^\circ < \theta < 90^\circ$.

[6]

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page, typical of notebook or legal stationery. There are no margins, text, or other markings on the page.

- 6 (a)** By sketching a suitable pair of graphs, show that the equation $x^5 = 2 + x$ has exactly one real root. [2]

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- (b)** Show that if a sequence of values given by the iterative formula

$$x_{n+1} = \frac{4x_n^5 + 2}{5x_n^4 - 1}$$

converges, then it converges to the root of the equation in part **(a)**. [2]

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- (c) Use the iterative formula with initial value $x_1 = 1.5$ to calculate the root correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

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7 Let $f(x) = \frac{2}{(2x-1)(2x+1)}$.

(a) Express $f(x)$ in partial fractions.

[2]

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(b) Using your answer to part (a), show that

$$(f(x))^2 = \frac{1}{(2x-1)^2} - \frac{1}{2x-1} + \frac{1}{2x+1} + \frac{1}{(2x+1)^2}. \quad [2]$$

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(c) Hence show that $\int_1^2 (f(x))^2 dx = \frac{2}{5} + \frac{1}{2} \ln\left(\frac{5}{9}\right)$. [5]

[illegible]

- 8** Relative to the origin O , the points A , B and D have position vectors given by

$$\overrightarrow{OA} = \mathbf{i} + 2\mathbf{j} + \mathbf{k}, \quad \overrightarrow{OB} = 2\mathbf{i} + 5\mathbf{j} + 3\mathbf{k} \quad \text{and} \quad \overrightarrow{OD} = 3\mathbf{i} + 2\mathbf{k}.$$

A fourth point C is such that $ABCD$ is a parallelogram.

- (a) Find the position vector of C and verify that the parallelogram is not a rhombus. [5]

[illegible]

(b) Find angle BAD , giving your answer in degrees.

[3]

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(c) Find the area of the parallelogram correct to 3 significant figures.

[2]

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- 9 (a) The complex numbers u and w are such that

$$u - w = 2i \quad \text{and} \quad uw = 6.$$

Find u and w , giving your answers in the form $x + iy$, where x and y are real and exact. [5]

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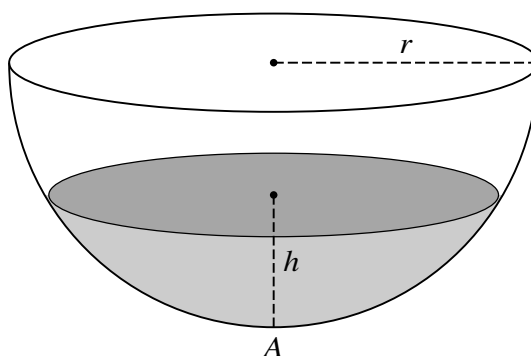
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- (b) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities

$$|z - 2 - 2i| \leq 2, \quad 0 \leq \arg z \leq \frac{1}{4}\pi \quad \text{and} \quad \operatorname{Re} z \leq 3. \quad [5]$$



The volume of water in the tank is V when the depth is h . It is given that $V = \frac{1}{3}\pi(3rh^2 - h^3)$.

$$\frac{dh}{dt} = -\frac{B}{2rh^{\frac{1}{2}} - h^{\frac{3}{2}}},$$

[4]

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- (b)** Solve the differential equation and obtain an expression for t in terms of h and r . [8]

[illegible]

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