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MATHEMATICS

9709/71

Paper 7 Probability & Statistics 2 **(S2)**

May/June 2019

1 hour 15 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

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The total number of marks for this paper is 50.

This document consists of **12** printed pages.



- 1** At an internet café, the charge for using a computer is 5 cents per minute. The number of minutes for which people use a computer has mean 23 and standard deviation 8.

(i) Find, in cents, the mean and standard deviation of the amount people pay when using a computer. [2]

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(ii) Each day, 15 people use computers independently. Find, in cents, the mean and standard deviation of the total amount paid by 15 people. [3]

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- 2 The time, in minutes, that John takes to travel to work has a normal distribution. Last year the mean and standard deviation were 26.5 and 4.8 respectively. This year John uses a different route and he finds that the mean time for his first 150 journeys is 27.5 minutes.

- (i) Stating a necessary assumption, test at the 1% significance level whether the mean time for his journey to work has increased. [6]

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- (ii) State, with a reason, whether it was necessary to use the Central Limit theorem in your answer to part (i). [1]

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- 3** Sumitra has a six-sided die. She suspects that it is biased so that it shows a six less often than it would if it were fair. She decides to test the die by throwing it 30 times and noting the number of throws on which it shows a six.
- (i) It shows a six on exactly 2 throws. Use a binomial distribution to carry out the test at the 5% significance level. [5]

This image shows a full page of primary-ruled paper. It consists of multiple horizontal rows, each defined by two parallel dotted lines. The rows are evenly spaced and cover the entire area of the page, providing a guide for handwriting practice. There are no margins, text, or other markings present.

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- (ii) Later, Sumitra repeats the test at the 5% significance level by throwing the die 30 times again. Find the probability of a Type I error in this second test. [2]

4 (a)



The diagram shows the graph of the probability density function, f , of a random variable X , where a is a constant greater than 0.5. The graph between $x = 0$ and $x = a$ is a straight line parallel to the x -axis.

- (i) Find $P(X < 0.5)$ in terms of a . [2]

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- (ii) Find $E(X)$ in terms of a . [1]

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- (iii) Show that $\text{Var}(X) = \frac{1}{12}a^2$. [2]

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5 (a) The random variable X has the distribution $\text{Po}(2.3)$.

(i) Find $P(2 \leq X \leq 4)$. [2]

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(ii) Find the probability that the sum of two independent values of X is greater than 2. [3]

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(iii) The random variable S is the sum of 50 independent values of X . Use a suitable approximating distribution to find $P(S \leq 110)$. [4]

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- (b) The random variable Y has the distribution $\text{Po}(\lambda)$. Given that $P(Y = 3) = P(Y = 5)$, find λ . [3]

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- 6** Ramesh plans to carry out a survey in order to find out what adults in his town think about local sports facilities. He chooses a random sample from the adult members of a tennis club and gives each of them a questionnaire.

- (i)** Give a reason why this will not result in Ramesh having a random sample of adults who live in the town. [1]

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- (ii)** Describe briefly a valid method that Ramesh could use to choose a random sample of adults in the town. [2]

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Ramesh now uses a valid method to choose a random sample of 350 adults from the town. He finds that 47 adults think that the local sports facilities are good.

- (iii)** Calculate an approximate 90% confidence interval for the proportion of all adults in the town who think that the local sports facilities are good. [4]

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- (iv) Ramesh calculates a confidence interval whose width is 1.25 times the width of this 90% confidence interval. Ramesh's new interval is an $x\%$ confidence interval. Find the value of x . [3]

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[illegible]

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1 The random variable X has the distribution $\text{Po}(5)$.

(i) Find $P(X = 2)$. [1]

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It is given that $P(X = n) = P(X = n + 1)$.

(ii) Write down an equation in n . [1]

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(iii) Hence or otherwise find the value of n . [1]

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2 The random variable X has mean 372 and standard deviation 54.

- (i) Describe fully the distribution of the mean of a random sample of 36 values of X . [3]

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- (ii) The distribution in part (i) might be either exact or approximate. State a condition under which the distribution is exact. [1]

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- 3** It is claimed that, on average, a particular train journey takes less than 1.9 hours. The times, t hours, taken for this journey on a random sample of 50 days were recorded. The results are summarised below.

$$n = 50 \qquad \Sigma t = 92.5 \qquad \Sigma t^2 = 175.25$$

- (i) Calculate unbiased estimates of the population mean and variance. [3]

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page, providing a template for handwriting practice or general writing. There are no margins, text, or other markings on the page.

(ii) Test the claim at the 5% significance level.

[5]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the entire width of the page. There are no margins, text, or other markings present.

- 4 The heights of a certain variety of plant are normally distributed with mean 110 cm and variance 1050 cm^2 . Two plants of this variety are chosen at random. Find the probability that the height of one of these plants is at least 1.5 times the height of the other. [7]

[illegible]

- 5** The manufacturer of a certain type of biscuit claims that 10% of packets include a free offer printed on the packet. Jyothi suspects that the true proportion is less than 10%. He plans to test the claim by looking at 40 randomly selected packets and, if the number which include the offer is less than 2, he will reject the manufacturer's claim.

(i) State suitable hypotheses for the test.

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(ii) Find the probability of a Type I error.

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On another occasion Jyothi looks at 80 randomly selected packets and finds that exactly 6 include the free offer.

- (iii) Calculate an approximate 90% confidence interval for the proportion of packets that include the offer. [3]

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page. There are no margins, text, or other markings on the paper.

- (iv) Use your confidence interval to comment on the manufacturer's claim. [1]

[illegible]

- 6 X is a random variable with probability density function given by

$$f(x) = \begin{cases} \frac{a}{x^2} & 1 \leq x \leq b, \\ 0 & \text{otherwise,} \end{cases}$$

where a and b are constants.

- (i) Show that $b = \frac{a}{a-1}$. [3]

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- (ii) Given that the median of X is $\frac{3}{2}$, find the values of a and b . [3]

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(iii) Use your values of a and b from part **(ii)** to find $E(X)$. [3]

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- 7 All the seats on a certain daily flight are always sold. The number of passengers who have bought seats but fail to arrive for this flight on a particular day is modelled by the distribution $B(320, 0.005)$.

(i) Explain what the number 320 represents in this context. [1]

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(ii) The total number of passengers who have bought seats but fail to arrive for this flight on 2 randomly chosen days is denoted by X . Use a suitable approximating distribution to find $P(2 < X < 6)$. [3]

[illegible]

(iii) Justify the use of your approximating distribution. [2]

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After some changes, the airline wishes to test whether the mean number of passengers per day who fail to arrive for this flight has decreased.

- (iv) During 5 randomly chosen days, a total of 2 passengers failed to arrive. Carry out the test at the 2.5% significance level. [5]

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[illegible]

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- 1** A coin is thrown 100 times and it shows heads 60 times. Calculate an approximate 98% confidence interval for the probability, p , that the coin shows heads on any throw. [3]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- 2 The length of worms is denoted by X cm. The lengths of a random sample of 50 worms were measured. Some of the results were lost, but the following results are available.

- $\Sigma x^2 = 4361$
- An unbiased estimate of the population variance of X is 9.62.

Calculate the mean length of the 50 worms.

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- 3** Luis has to choose one person at random from four people, *A*, *B*, *C* and *D*. He throws a fair six-sided die. If the score is 1, he will choose *A*. If the score is 2 he will choose *B*. If the score is 3, he will choose *C*. If the score is 4 or more he will choose *D*.

(i) Explain why the choice made by this method is not random.

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(ii) Describe how Luis could use a single throw of the die to make a random choice.

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On another day, Luis has to choose two people at random from the same four people, *A*, *B*, *C* and *D*.

- (iii)** List the possible choices of two people and hence describe how Luis could use a single throw of the die to make this random choice.

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- This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- 5 The amount of money, in dollars, spent by a customer on one visit to a certain shop is modelled by the distribution $N(\mu, 1.94)$. In the past, the value of μ has been found to be 20.00, but following a rearrangement in the shop, the manager suspects that the value of μ has changed. He takes a random sample of 6 customers and notes how much they each spend, in dollars. The results are as follows.

17.60 23.50 17.30 22.00 31.00 15.50

The manager carries out a hypothesis test using a significance level of $\alpha\%$. The test does not support his suspicion. Find the largest possible value of α . [6]

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page, providing a template for handwriting practice or general writing. There are no margins, text, or other markings on the page.

- 6 A function f is defined by

$$f(x) = \begin{cases} \frac{3x^2}{a^3} & 0 \leq x \leq a, \\ 0 & \text{otherwise,} \end{cases}$$

where a is a constant.

- (i) Show that f is a probability density function for all positive values of a . [3]

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The random variable X has probability density function f and the median of X is 2.

- (ii) Show that $a = 2.52$, correct to 3 significant figures. [3]

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(iii) Find $E(X)$.

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- 7 Each day at a certain doctor's surgery there are 70 appointments available in the morning and 60 in the afternoon. All the appointments are filled every day. The probability that any patient misses a particular morning appointment is 0.04, and the probability that any patient misses a particular afternoon appointment is 0.05. All missed appointments are independent of each other.

Use suitable approximating distributions to answer the following.

- (i) Find the probability that on a randomly chosen morning there are at least 3 missed appointments. [3]

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- (ii) Find the probability that on a randomly chosen day there are a total of exactly 6 missed appointments. [3]

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- [illegible]

- 8 The four sides of a spinner are A, B, C, D . The spinner is supposed to be fair, but Sonam suspects that the spinner is biased so that the probability, p , that it will land on side A is greater than $\frac{1}{4}$. He spins the spinner 10 times and finds that it lands on side A 6 times.

(i) Test Sonam's suspicion using a 1% significance level.

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Later Sonam carries out a similar test at the 1% significance level, using another 10 spins of the spinner.

- (ii) Calculate the probability of a Type I error. [2]

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- (iii) Assuming that the value of p is actually $\frac{3}{5}$, calculate the probability of a Type II error. [3]

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