

CANDIDATE
NAME

--

CENTRE
NUMBER

--	--	--	--	--

CANDIDATE
NUMBER

--	--	--	--

MATHEMATICS

9709/31

Paper 3 Pure Mathematics 3 **(P3)**

May/June 2019

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your centre number, candidate number and name in the spaces at the top of this page.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

DO NOT WRITE IN ANY BARCODES.

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 75.

This document consists of **18** printed pages and **2** blank pages.



- 1** Use the trapezium rule with 3 intervals to estimate the value of

$$\int_0^3 |2^x - 4| \, dx. \quad [3]$$

This image shows a single page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page, typical of notebook or legal stationery. There are no margins, text, or other markings on the page.

- 2 Showing all necessary working, solve the equation $\ln(2x - 3) = 2 \ln x - \ln(x - 1)$. Give your answer correct to 2 decimal places. [4]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the entire width of the page. There are no margins, text, or other markings present.

- 3** Find the gradient of the curve $x^3 + 3xy^2 - y^3 = 1$ at the point with coordinates $(1, 3)$. [4]

This image shows a full page of a handwriting practice worksheet. It consists of approximately 20 horizontal rows. Each row is defined by two parallel dotted lines, creating a series of uniform gaps for letter height. The entire page is otherwise blank, with no text or other markings.

- 4 By first expressing the equation $\cot \theta - \cot(\theta + 45^\circ) = 3$ as a quadratic equation in $\tan \theta$, solve the equation for $0^\circ < \theta < 180^\circ$. [6]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- 5 (i) Differentiate $\frac{1}{\sin^2 \theta}$ with respect to θ . [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (ii) The variables x and θ satisfy the differential equation

$$x \tan \theta \frac{dx}{d\theta} + \operatorname{cosec}^2 \theta = 0,$$

for $0 < \theta < \frac{1}{2}\pi$ and $x > 0$. It is given that $x = 4$ when $\theta = \frac{1}{6}\pi$. Solve the differential equation, obtaining an expression for x in terms of θ . [6]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

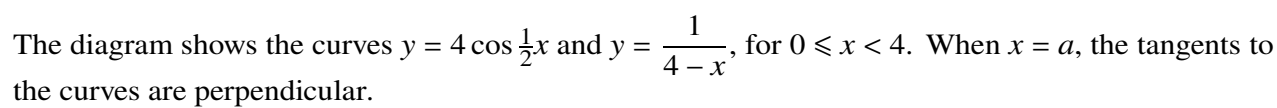
.....

.....

6 (i) By first expanding $\sin(2x + x)$, show that $\sin 3x \equiv 3 \sin x - 4 \sin^3 x$. [4]

This image shows a full page of a handwriting practice worksheet. It consists of approximately 20 horizontal rows. Each row is defined by two parallel dotted lines, creating a series of uniform gaps for letter height. The entire page is otherwise blank, with no margins, text, or other markings.

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.



[4]

This image shows a full page of a handwriting practice worksheet. It features 15 rows of horizontal dashed lines spaced evenly down the page, providing a guide for letter height and placement. The background is plain white, and there are no margins or additional markings.

- (ii) Verify by calculation that a lies between 2 and 3.

[2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (iii) Use an iterative formula based on the equation in part (i) to determine a correct to 3 decimal places. Give the result of each iteration to 5 decimal places.

[3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

8 Let $f(x) = \frac{16 - 17x}{(2 + x)(3 - x)^2}$.

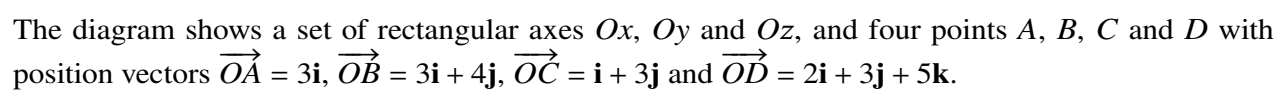
(i) Express $f(x)$ in partial fractions.

[5]

[illegible]

- (ii) Hence obtain the expansion of $f(x)$ in ascending powers of x , up to and including the term in x^2 . [5]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.



-
- This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

[Turn over

(i) Express u in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leq \pi$, giving the exact values of r and θ . Hence or otherwise state the exact values of the modulus and argument of u^4 . [5]

This image shows a full page of a worksheet designed for handwriting practice. It features 20 evenly spaced, horizontal dashed lines across the entire page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.

- (ii) Verify that u is a root of the equation $z^3 - 8z + 8\sqrt{3} = 0$ and state the other complex root of this equation. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (iii) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z - u| \leq 2$ and $\text{Im } z \geq 2$, where $\text{Im } z$ denotes the imaginary part of z . [5]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

BLANK PAGE

BLANK PAGE

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in the Cambridge Assessment International Education Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at www.cambridgeinternational.org after the live examination series.

Cambridge Assessment International Education is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which itself is a department of the University of Cambridge.

CANDIDATE
NAME

--

CENTRE
NUMBER

--	--	--	--	--

CANDIDATE
NUMBER

--	--	--	--

MATHEMATICS

9709/32

Paper 3 Pure Mathematics 3 **(P3)**

May/June 2019

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your centre number, candidate number and name in the spaces at the top of this page.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

DO NOT WRITE IN ANY BARCODES.

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 75.

This document consists of **18** printed pages and **2** blank pages.



- 1** Find the coefficient of x^3 in the expansion of $(3-x)(1+3x)^{\frac{1}{3}}$ in ascending powers of x . [4]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- 2 Showing all necessary working, solve the equation $9^x = 3^x + 12$. Give your answer correct to 2 decimal places. [4]

[illegible]

- 3** Showing all necessary working, solve the equation $\cot 2\theta = 2 \tan \theta$ for $0^\circ < \theta < 180^\circ$. [5]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- 4** Find the exact coordinates of the point on the curve $y = \frac{x}{1 + \ln x}$ at which the gradient of the tangent is equal to $\frac{1}{4}$. [7]

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

5 Throughout this question the use of a calculator is not permitted.

It is given that the complex number $-1 + (\sqrt{3})i$ is a root of the equation

$$kx^3 + 5x^2 + 10x + 4 = 0,$$

where k is a real constant.

(i) Write down another root of the equation.

[1]

.....

.....

.....

.....

.....

(ii) Find the value of k and the third root of the equation.

[6]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

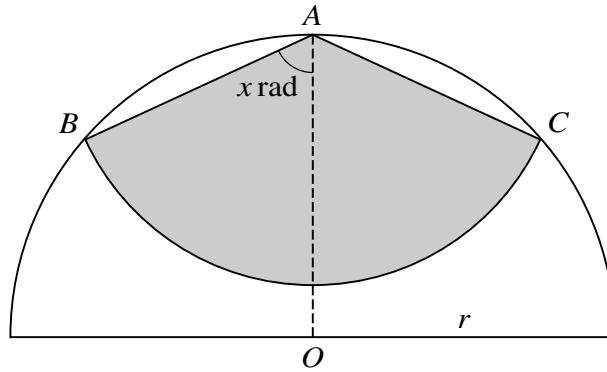
.....

.....

.....

.....

.....



In the diagram, A is the mid-point of the semicircle with centre O and radius r . A circular arc with centre A meets the semicircle at B and C . The angle OAB is equal to x radians. The area of the shaded region bounded by AB , AC and the arc with centre A is equal to half the area of the semicircle.

- (i) Use triangle OAB to show that $AB = 2r \cos x$. [1]

.....

.....

.....

.....

.....

- (ii) Hence show that $x = \cos^{-1} \sqrt{\left(\frac{\pi}{16x}\right)}$. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(iii) Verify by calculation that x lies between 1 and 1.5.

[2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

(iv) Use an iterative formula based on the equation in part (ii) to determine x correct to 3 decimal places. Give the result of each iteration to 5 decimal places.

[3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- 7 The variables x and y satisfy the differential equation $\frac{dy}{dx} = xe^{x+y}$. It is given that $y = 0$ when $x = 0$.

(i) Solve the differential equation, obtaining y in terms of x . [7]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(ii) Explain why x can only take values that are less than 1.

[1]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

8 Let $f(x) = \frac{10x + 9}{(2x + 1)(2x + 3)^2}$.

(i) Express $f(x)$ in partial fractions.

[5]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

(ii) Hence show that $\int_0^1 f(x) \, dx = \frac{1}{2} \ln \frac{9}{5} + \frac{1}{5}$.

[5]

[illegible]

- 9** The points A and B have position vectors $\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and $3\mathbf{i} + \mathbf{j} + \mathbf{k}$ respectively. The line l has equation $\mathbf{r} = 2\mathbf{i} + \mathbf{j} + \mathbf{k} + \mu(\mathbf{i} + \mathbf{j} + 2\mathbf{k})$.

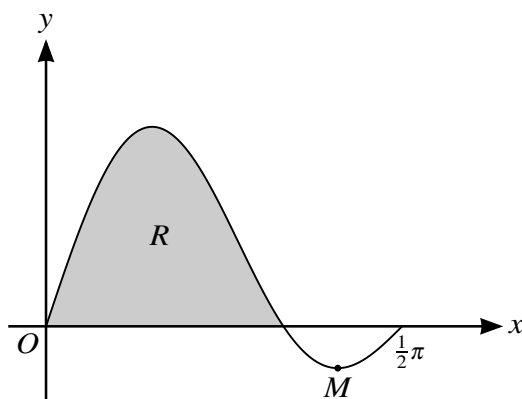
(i) Show that l does not intersect the line passing through A and B . [5]

[illegible]

- (ii) The plane m is perpendicular to AB and passes through the mid-point of AB . The plane m intersects the line l at the point P . Find the equation of m and the position vector of P . [5]

[illegible]

10



The diagram shows the curve $y = \sin 3x \cos x$ for $0 \leq x \leq \frac{1}{2}\pi$ and its minimum point M . The shaded region R is bounded by the curve and the x -axis.

- (i) By expanding $\sin(3x + x)$ and $\sin(3x - x)$ show that

$$\sin 3x \cos x = \frac{1}{2}(\sin 4x + \sin 2x). \quad [3]$$

.....

.....

.....

.....

.....

.....

.....

.....

- (ii) Using the result of part (i) and showing all necessary working, find the exact area of the region R . [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (iii) Using the result of part (i), express $\frac{dy}{dx}$ in terms of $\cos 2x$ and hence find the x -coordinate of M , giving your answer correct to 2 decimal places. [5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting or typing. There are no margins, text, or other markings on the page.

BLANK PAGE

BLANK PAGE

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in the Cambridge Assessment International Education Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at www.cambridgeinternational.org after the live examination series.

Cambridge Assessment International Education is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which itself is a department of the University of Cambridge.

CANDIDATE
NAME

--

CENTRE
NUMBER

--	--	--	--	--

CANDIDATE
NUMBER

--	--	--	--

MATHEMATICS

9709/33

Paper 3 Pure Mathematics 3 (P3)

May/June 2019

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your centre number, candidate number and name in the spaces at the top of this page.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

DO NOT WRITE IN ANY BARCODES.

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 75.

This document consists of **20** printed pages.



- 1** Use logarithms to solve the equation $5^{3-2x} = 4(7^x)$, giving your answer correct to 3 decimal places. [4]

[illegible]

2

[5]

[illegible]

3 Let $f(\theta) = \frac{1 - \cos 2\theta + \sin 2\theta}{1 + \cos 2\theta + \sin 2\theta}$.

(i) Show that $f(\theta) = \tan \theta$.

[3]

[illegible]

(ii) Hence show that $\int_{\frac{1}{6}\pi}^{\frac{1}{4}\pi} f(\theta) \, d\theta = \frac{1}{2} \ln \frac{3}{2}$.

[4]

[illegible]

- 4** The equation of a curve is $y = \frac{1 + e^{-x}}{1 - e^{-x}}$, for $x > 0$.

(i) Show that $\frac{dy}{dx}$ is always negative.

[3]

This image shows a single page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- (ii) The gradient of the curve is equal to -1 when $x = a$. Show that a satisfies the equation $e^{2a} - 4e^a + 1 = 0$. Hence find the exact value of a . [4]

[illegible]

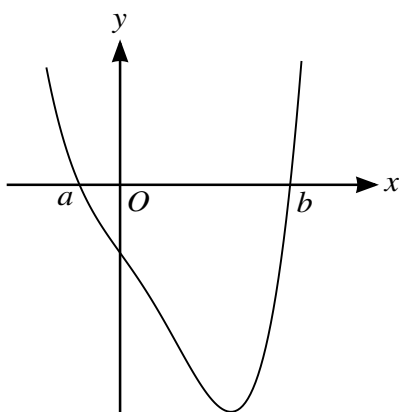
5 The variables x and y satisfy the differential equation

$$(x + 1)y \frac{dy}{dx} = y^2 + 5.$$

It is given that $y = 2$ when $x = 0$. Solve the differential equation obtaining an expression for y^2 in terms of x . [7]

This image shows a full page of a worksheet designed for handwriting practice. It features 20 evenly spaced, horizontal dashed lines across the entire width of the page. The background is plain white, and there are no margins, text, or other markings present.

6



The diagram shows the curve $y = x^4 - 2x^3 - 7x - 6$. The curve intersects the x -axis at the points $(a, 0)$ and $(b, 0)$, where $a < b$. It is given that b is an integer.

(i) Find the value of b .

[1]

.....

.....

.....

.....

.....

.....

.....

(ii) Hence show that a satisfies the equation $a = -\frac{1}{3}(2 + a^2 + a^3)$.

[4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (iii) Use an iterative formula based on the equation in part (ii) to determine a correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

7 The curve $y = \sin\left(x + \frac{1}{3}\pi\right) \cos x$ has two stationary points in the interval $0 \leq x \leq \pi$.

(i) Find $\frac{dy}{dx}$. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(ii) By considering the formula for $\cos(A + B)$, show that, at the stationary points on the curve, $\cos\left(2x + \frac{1}{3}\pi\right) = 0$. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

8 Throughout this question the use of a calculator is not permitted.

The complex number u is defined by

$$u = \frac{4i}{1 - (\sqrt{3})i}.$$

(i) Express u in the form $x + iy$, where x and y are real and exact. [3]

This image shows a full page of a document template designed for handwriting practice. It features 20 evenly spaced, horizontal dashed lines across the entire width of the page. The lines are thin and black, providing a guide for letter height and placement. There are no margins, text, or other markings present on the page.

(ii) Find the exact modulus and argument of u .

[2]

.....

.....

.....

.....

.....

.....

.....

.....

(iii) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z| < 2$ and $|z - u| < |z|$. [4]

9 Let $f(x) = \frac{2x(5-x)}{(3+x)(1-x)^2}$.

(i) Express $f(x)$ in partial fractions.

[5]

[illegible]

- (ii)** Hence obtain the expansion of $f(x)$ in ascending powers of x up to and including the term in x^3 . [5]

[illegible]

- 10** The line l has equation $\mathbf{r} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k} + \mu(2\mathbf{i} - \mathbf{j} - 2\mathbf{k})$.

- (i) The point P has position vector $4\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$. Find the length of the perpendicular from P to l . [5]

[illegible]

- (ii) It is given that l lies in the plane with equation $ax + by + 2z = 13$, where a and b are constants. Find the values of a and b . [6]

[illegible]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

Cambridge Assessment International Education is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which itself is a department of the University of Cambridge.