
MATHEMATICS

9709/31

Paper 3

May/June 2019

MARK SCHEME

Maximum Mark: 75

Published

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This document consists of **17** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

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Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

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Mark Scheme Notes

Marks are of the following three types:

- M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
 - A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
 - B Mark for a correct result or statement independent of method marks.
- When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
 - The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously “correct” answers or results obtained from incorrect working.
 - Note: B2 or A2 means that the candidate can earn 2 or 0.
B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.
- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

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The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE Any Equivalent Form (of answer is equally acceptable) / Or Equivalent

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

CAO Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)

CWO Correct Working Only – often written by a ‘fortuitous’ answer

ISW Ignore Subsequent Working

SOI Seen or implied

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks	Guidance
1	State or imply ordinates 3, 2, 0, 4	B1	These and no more Accept in unsimplified form $ 2^0 - 4 $ etc.
	Use correct formula, or equivalent, with $h = 1$ and four ordinates	M1	
	Obtain answer 5.5	A1	
		3	

Question	Answer	Marks	Guidance
2	Use law for the logarithm of a product, quotient or power	M1	Condone $\ln \frac{x}{x-1}$ for M1
	Obtain a correct equation free of logarithms	A1	e.g. $(2x-3)(x-1) = x^2$ or $x^2 - 5x + 3 = 0$
	Solve a 3-term quadratic obtaining at least one root	M1	Must see working if using an incorrect quadratic $\left(\frac{5 \pm \sqrt{13}}{2} \right)$
	Obtain answer $x = 4.30$ only	A1	Q asks for 2 dp. Do not ISW. Overspecified answers score A0 Overspecified and no working can score M1A0
		4	

Question	Answer	Marks	Guidance
3	State or imply $3y^2 + 6xy \frac{dy}{dx}$ as derivative of $3xy^2$	B1	
	State or imply $3y^2 \frac{dy}{dx}$ as derivative of y^3	B1	
	Equate derivative of LHS to zero, substitute (1, 3) and find the gradient	M1	$\left(\frac{dy}{dx} = \frac{x^2 + y^2}{y^2 - 2xy} \right)$ For incorrect derivative need to see the substitution
	Obtain final answer $\frac{10}{3}$ or equivalent	A1	3.33 or better. Allow $\frac{30}{9}$ ISW after correct answer seen
		4	

Question	Answer	Marks	Guidance
4	Use correct trig formula and obtain an equation in $\tan \theta$	M1	Allow with 45° e.g. $\frac{1}{\tan \theta} - \frac{1}{\frac{\tan \theta + \tan 45^\circ}{1 - \tan \theta \tan 45^\circ}} = 3$
	Obtain a correct horizontal equation in any form	A1	e.g. $1 + \tan \theta - \tan \theta(1 - \tan \theta) = 3 \tan \theta(1 + \tan \theta)$
	Reduce to $2\tan^2 \theta + 3 \tan \theta - 1 = 0$	A1	or 3-term equivalent
	Solve 3-term quadratic and find a value of θ	M1	Must see working if using an incorrect quadratic
	Obtain answer 15.7°	A1	One correct solution (degrees to at least 3 sf)
	Obtain answer $119.(3)^\circ$	A1	Second correct solution and no others in range (degrees to at least 3 sf) Mark 0.274, 2.082 as MR: A0A1
		6	

Question	Answer	Marks	Guidance
5(i)	Use chain rule	M1	$k \cos \theta \sin^{-3} \theta (= -k \operatorname{cosec}^2 \theta \cot \theta)$ Allow M1 for $-2 \cos \theta \sin^{-1} \theta$
	Obtain correct answer in any form	A1	e.g. $-2 \operatorname{cosec}^2 \theta \cot \theta$, $\frac{-2 \cos \theta}{\sin^3 \theta}$ Accept $\frac{-2 \sin \theta \cos \theta}{\sin^4 \theta}$
		2	
5(ii)	Separate variables correctly and integrate at least one side	B1	$\int x \, dx = \int -\operatorname{cosec}^2 \theta \cot \theta \, d\theta$
	Obtain term $\frac{1}{2}x^2$	B1	
	Obtain term of the form $\frac{k}{\sin^2 \theta}$	M1*	or equivalent
	Obtain term $\frac{1}{2\sin^2 \theta}$	A1	or equivalent
	Use $x = 4$, $\theta = \frac{1}{6}\pi$ to evaluate a constant, or as limits, in a solution with terms ax^2 and $\frac{b}{\sin^2 \theta}$, where $ab \neq 0$	DM1	Dependent on the preceding M1
	Obtain solution $x = \sqrt{(\operatorname{cosec}^2 \theta + 12)}$	A1	or equivalent
		6	

Question	Answer	Marks	Guidance
6(i)	State correct expansion of $\sin(2x + x)$	B1	
	Use trig formulae and Pythagoras to express $\sin 3x$ in terms of $\sin x$	M1	
	Obtain a correct expression in any form	A1	e.g. $2\sin x(1 - \sin^2 x) + \sin x(1 - 2\sin^2 x)$
	Obtain $\sin 3x \equiv 3\sin x - 4\sin^3 x$ correctly AG	A1	Accept = for $\equiv$
		4	
6(ii)	Use identity, integrate and obtain $-\frac{3}{4}\cos x + \frac{1}{12}\cos 3x$	B1 B1	One mark for each term correct
	Use limits correctly in an integral of the form $a \cos x + b \cos 3x$, where $ab \neq 0$	M1	$\left(-\frac{3}{8} - \frac{1}{12} + \frac{3}{4} - \frac{1}{12} = -\frac{11}{24} + \frac{2}{3}\right)$
	Obtain answer $\frac{5}{24}$	A1	Must be exact. Accept simplified equivalent e.g. $\frac{15}{72}$ Answer only with no working is 0/4
		4	

Question	Answer	Marks	Guidance
7(i)	State at least one correct derivative	B1	$-2\sin\frac{1}{2}x, \frac{1}{(4-x)^2}$
	Equate product of derivatives to -1	M1	or equivalent
	Obtain a correct equation, e.g. $2\sin\frac{1}{2}x = (4-x)^2$	A1	
	Rearrange correctly to obtain $a = 4 - \sqrt{2\sin\frac{a}{2}}$ AG	A1	
		4	
7(ii)	Calculate values of a relevant expression or pair of expressions at $a = 2$ and $a = 3$	M1	e.g. $a = 2 \quad 2 < 2.7027.. \quad \begin{pmatrix} 0.703 \\ -0.412 \end{pmatrix} \quad \begin{pmatrix} 2.317 \\ -0.995 \end{pmatrix}$ Values correct to at least 2 dp
	Complete the argument correctly with correct calculated values	A1	
		2	
7(iii)	Use the iterative formula $a_{n+1} = 4 - \sqrt{2\sin\frac{1}{2}a_n}$ correctly at least once	M1	
	Obtain final answer 2.611	A1	
	Show sufficient iterations to 5 d.p. to justify 2.611 to 3 d.p., or show there is a sign change in the interval (2.6105, 2.6115)	A1	2, 2.70272, 2.60285, 2.61152, 2.61070, 2.61077 2.5, 2.62233, 2.60969, 2.61087, 2.61076 3, 2.58756, 2.61301, 2.61056, 2.61079 Condone truncation. Accept more than 5 dp
		3	

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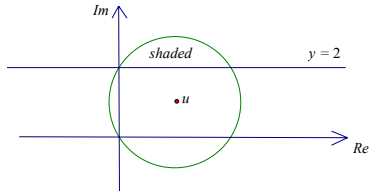
Question	Answer	Marks	Guidance
8(i)	State or imply the form $\frac{A}{2+x} + \frac{B}{3-x} + \frac{C}{(3-x)^2}$	B1	
	Use a correct method to obtain a constant	M1	
	Obtain one of $A = 2, B = 2, C = -7$	A1	
	Obtain a second value	A1	
	Obtain the third value	A1	[Mark the form $\frac{A}{2+x} + \frac{Dx+E}{(3-x)^2}$, where $A = 2, D = -2$ and $E = -1$, B1M1A1A1A1.]
		5	
8(ii)	Use a correct method to find the first two terms of the expansion of $(2+x)^{-1}, (3-x)^{-1}$ or $(3-x)^{-2}$, or equivalent, e.g. $\left(1 + \frac{1}{2}x\right)^{-1}$	M1	
	Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction	A1 A1 A1	FT on A, B and C $1 - \frac{x}{2} + \frac{x^2}{4} - \frac{2}{3}\left(1 + \frac{x}{3} + \frac{x^2}{9}\right) - \frac{7}{9}\left(1 + \frac{2x}{3} + \frac{3x^2}{9}\right)$
	Obtain final answer $\frac{8}{9} - \frac{43}{54}x + \frac{7}{108}x^2$	A1	
			For the A, D, E form of fractions give M1A1ftA1ft for the expanded partial fractions, then, if $D \neq 0$, M1 for multiplying out fully, and A1 for the final answer.
		5	

Question	Answer	Marks	Guidance
9(i)	Obtain a vector parallel to the plane, e.g. $\overrightarrow{CB} = 2\mathbf{i} + \mathbf{j}$	B1	
	Use scalar product to obtain an equation in a, b, c ,	M1	e.g. $2a + b = 0, a + 5c = 0, a + b - 5c = 0$
	Obtain two correct equations in a, b, c	A1	
	Solve to obtain $a : b : c$,	M1	or equivalent
	Obtain $a : b : c = 5 : -10 : -1$,	A1	or equivalent
	Obtain equation $5x - 10y - z = -25$,	A1	or equivalent
	Alternative method 1		
	Obtain a vector parallel to the plane, e.g. $\overrightarrow{CD} = \mathbf{i} + 5\mathbf{k}$	B1	$\overrightarrow{BD} = -\mathbf{i} - \mathbf{j} + 5\mathbf{k}$
	Obtain a second such vector and calculate their vector product, e.g. $(2\mathbf{i} + \mathbf{j}) \times (\mathbf{i} + 5\mathbf{k})$	M1	
	Obtain two correct components	A1	
	Obtain correct answer, e.g. $5\mathbf{i} - 10\mathbf{j} - \mathbf{k}$	A1	
	Substitute to find d	M1	
	Obtain equation $5x - 10y - z = -25$,	A1	or equivalent

Question	Answer	Marks	Guidance
9(i)	Alternative method 2		
	Obtain a vector parallel to the plane, e.g. $\overrightarrow{DB} = \mathbf{i} + \mathbf{j} - 5\mathbf{k}$	B1	
	Obtain a second such vector and form correctly a 2-parameter equation for the plane	M1	
	State a correct equation, e.g. $\mathbf{r} = 3\mathbf{i} + 4\mathbf{j} + \lambda(\mathbf{i} + 5\mathbf{k}) + \mu(\mathbf{i} + \mathbf{j} - 5\mathbf{k})$	A1	
	State three equations in x, y, z, λ and μ	A1	
	Eliminate λ and μ	M1	
	Obtain equation $5x - 10y - z = -25$	A1	or equivalent
	Alternative method 3		
	Substitute for B and C and obtain $3a + 4b = d$ and $a + 3b = d$	B1	
	Substitute for D to obtain a third equation and eliminate one unknown (a, b , or d) entirely	M1	
	Obtain two correct equations in two unknowns, e.g. a, b, c	A1	
	Solve to obtain their ratio, e.g. $a : b : c$	M1	
	Obtain $a : b : c = 5 : -10 : -1$, $a : c : d = 5 : -1 : -25$, or $b : c : d = 10 : 1 : 25$	A1	or equivalent
	Obtain equation $5x - 10y - z = -25$	A1	or equivalent

Question	Answer	Marks	Guidance
9(i)	Alternative method 4		
	Substitute for B and C and obtain $3a + 4b = d$ and $a + 3b = d$	B1	
	Solve to obtain $a : b : d$	M2	or equivalent
	Obtain $a : b : d = 1 : -2 : -5$	A1	or equivalent
	Substitute for C to obtain c	M1	
	Obtain equation $5x - 10y - z = -25$	A1	or equivalent
		6	
9(ii)	State or imply a normal vector for the plane $OABC$ is $\mathbf{k}$	B1	
	Carry out correct process for evaluating a scalar product of two relevant vectors, e.g. $(5\mathbf{i} - 10\mathbf{j} - \mathbf{k}) \cdot (\mathbf{k})$	M1	i.e. correct process using $\mathbf{k}$ and their normal
	Using the correct process for calculating the moduli, divide the scalar product by the product of the moduli and evaluate the inverse cosine of the result	M1	Allow M1M1 for clear use of an incorrect vector that has been stated to be the normal to $OABC$
	Obtain answer 84.9° or 1.48 radians	A1	
		4	

Question	Answer	Marks	Guidance
10(i)	State or imply $r = 2$	B1	Accept $\sqrt{4}$
	State or imply $\theta = \frac{1}{6}\pi$	B1	
	Use a correct method for finding the modulus or the argument of u^4	M1	Allow correct answers from correct u with minimal working shown
	Obtain modulus 16	A1	
	Obtain argument $\frac{2}{3}\pi$	A1	Accept $16e^{i\frac{2\pi}{3}}$
		5	
10(ii)	Substitute u and carry out a correct method for finding u^3	M1	$(u^3 = 8i)$ Follow <i>their</i> u^3 if found in part (i)
	Verify u is a root of the given equation	A1	
	State that the other root is $\sqrt{3} - i$	B1	
	Alternative method		
	State that the other root is $\sqrt{3} - i$	B1	
	Form quadratic factor and divide cubic by quadratic	M1	$(z - \sqrt{3} - i)(z - \sqrt{3} + i)(= z^2 - 2\sqrt{3}z + 4)$
	Verify that remainder is zero and hence that u is a root of the given equation	A1	
		3	

Question	Answer	Marks	Guidance
10(iii)	Show the point representing u in a relatively correct position	B1	
	Show a circle with centre u and radius 2	B1	FT on the point representing u . Condone near miss of origin
	Show the line $y = 2$	B1	
	Shade the correct region	B1	
	Show that the line and circle intersect on $x = 0$	B1	Condone near miss
		5	

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1	State unsimplified term in x^2 , or its coefficient in the expansion of $(1+3x)^{\frac{1}{3}} \left(\frac{\frac{1}{3} \times \frac{-2}{3}}{2} (3x)^2 \right)$	B1	Symbolic binomial coefficients are not sufficient for the B marks
	State unsimplified term in x^3 , or its coefficient in the expansion of $(1+3x)^{\frac{1}{3}} \left(\frac{\frac{1}{3} \times \frac{-2}{3} \times \frac{-5}{3}}{6} (3x)^3 \right)$	B1	
	Multiply by $(3-x)$ to give 2 terms in x^3 , or their coefficients	M1	$\left(3 \times \frac{10}{6} + 1\right)$ Ignore errors in terms other than x^3 $3 \times x^3 \text{coeff} - x^2 \text{coeff}$ and no other term in x^3
	Obtain answer 6	A1	Not $6x^3$
		4	

Question	Answer	Marks	Guidance
2	State or imply $u^2 - u - 12 (= 0)$, or equivalent in 3^x	B1	Need to be convinced they know $3^{2x} = (3^x)^2$
	Solve for u , or for 3^x , and obtain root 4	B1	
	Use a correct method to solve an equation of the form $3^x = a$ where $a > 0$	M1	Need to see evidence of method. Do not penalise an attempt to use the negative root as well. e.g. $x \ln 3 = \ln a$, $x = \log_3 a$ If seen, accept solution of straight forward cases such as $3^x = 3$, $x = 1$ without working
	Obtain final answer $x = 1.26$ only	A1	The Q asks for 2 dp
		4	

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Question	Answer	Marks	Guidance
3	Use correct trig formulae to obtain an equation in $\tan \theta$ or equivalent (e.g all in $\sin \theta$ or all in $\cos \theta$)	*M1	$\frac{1 - \tan^2 \theta}{2 \tan \theta} = 2 \tan \theta$. Allow $\frac{\cot^2 \theta - 1}{2 \cot \theta} = \frac{2}{\cot \theta}$
	Obtain a correct simplified equation	A1	$5 \tan^2 \theta = 1$ or $\sin^2 \theta = \frac{1}{6}$ or $\cos^2 \theta = \frac{5}{6}$
	Solve for θ	DM1	Dependent on the first M1
	Obtain answer 24.1° (or 155.9°)	A1	One correct in range to at least 3 sf
	Obtain second answer	A1	FT $180^\circ - \text{their } 24.1^\circ$ and no others in range. Correct to at least 3 sf. Accept 156° but not 156.0 Ignore values outside range If working in $\tan \theta$ or $\cos \theta$ need to be considering both square roots to score the second A1 Mark 0.421, 2.72 as a MR, so A0A1
		5	

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Question	Answer	Marks	Guidance
4	Use correct quotient rule	M1	Allow use of correct product rule on $x \times (1 + \ln x)^{-1}$
	Obtain correct derivative in any form	A1	$\frac{dy}{dx} = \frac{(1 + \ln x) - x \times \frac{1}{x}}{(1 + \ln x)^2} = \left(\frac{1}{1 + \ln x} - \frac{1}{(1 + \ln x)^2} \right)$
	Equate derivative to $\frac{1}{4}$ and obtain a quadratic in $\ln x$ or $(1 + \ln x)$	M1	Horizontal form. Accept $\ln x = \frac{1}{4}(1 + \ln x)^2$
	Reduce to $(\ln x)^2 - 2 \ln x + 1 = 0$	A1	or 3-term equivalent. Condone $\ln x^2$ if later used correctly
	Solve a 3-term quadratic in $\ln x$ for x	M1	Must see working if solving incorrect quadratic
	Obtain answer $x = e$	A1	Accept e^1
	Obtain answer $y = \frac{1}{2} e$	A1	Exact only with no decimals seen before the exact value. Accept $\frac{e^1}{2}$ but not $\frac{e}{1 + \ln e}$
		7	

Question	Answer	Marks	Guidance
5(i)	State answer $-1 - \sqrt{3}i$	B1	If $-\frac{1}{2}$ given as well at this point, still just B1
		1	

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Question	Answer	Marks	Guidance
5(ii)	Substitute $x = -1 + \sqrt{3}i$ in the equation and attempt expansions of x^2 and x^3	M1	Need to see sufficient working to be convinced that a calculator has not been used.
	Use $i^2 = -1$ correctly at least once	M1	Allow for relevant use at any point in the solution
	Obtain $k = 2$	A1	
	Carry out a complete method for finding a quadratic factor with zeros $-1 + \sqrt{3}i$ and $-1 - \sqrt{3}i$	M1	Could use factor theorem from this point. Need to see working. M1 for correct testing of correct root or allow M1 for three unsuccessful valid attempts.
	Obtain $x^2 + 2x + 4$	A1	Using factor theorem, obtain $f\left(-\frac{1}{2}\right) = 0$
	Obtain root $x = -\frac{1}{2}$, or equivalent, <i>via</i> division or inspection	A1	Final answer

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Question	Answer	Marks	Guidance
5(ii)	Alternative method 1		
	Carry out a complete method for finding a quadratic factor with zeros $-1 + \sqrt{3}i$ and $-1 - \sqrt{3}i$ (multiplying two linear factors or using sum and product of roots)	M1	Need to see sufficient working to be convinced that a calculator has not been used.
	Use $i^2 = -1$ correctly at least once	M1	Allow for relevant use at any point in the solution
	Obtain $x^2 + 2x + 4$	A1	Allow M1A0 for $x^2 + 2x + 3$
	Obtain linear factor $kx + 1$ and compare coefficients of x or x^2 and solve for k	M1	Can find the factor by inspection or by long division Must get to zero remainder
	Obtain $k = 2$	A1	
	Obtain root $x = -\frac{1}{2}$	A1	Final answer
			Note: Verification that $x = -\frac{1}{2}$ is a root is worth no marks without a clear demonstration of how the root was obtained

Question	Answer	Marks	Guidance
5(ii)	Alternative method 2		
	Use equation for sum of roots of cubic and use equation for product of roots of cubic	M1	
	Use $i^2 = -1$ correctly at least once	M1	Allow for relevant use at any point in the solution
	Obtain $-\frac{5}{k} = -2 + \gamma$, $-\frac{4}{k} = 4\gamma$	A1	
	Solve simultaneous equations for k and γ	M1	
	Obtain $k = 2$	A1	
	Obtain root $\gamma = -\frac{1}{2}$	A1	Final answer
		6	

Question	Answer	Marks	Guidance
6(i)	Correct use of trigonometry to obtain $AB = 2r \cos x$	B1	AG
		1	

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Question	Answer	Marks	Guidance
6(ii)	Use correct method for finding the area of the sector and the semicircle and form an equation in x	M1	$\frac{1}{2} \times \frac{1}{2} \pi r^2 = \frac{1}{2} (2r \cos x)^2 2x$
	Obtain $x = \cos^{-1} \sqrt{\frac{\pi}{16x}}$ correctly AG	A1	Via correct simplification e.g. from $\cos^2 x = \frac{\pi}{16x}$
		2	
6(iii)	Calculate values of a relevant expression or pair of expressions at $x = 1$ and $x = 1.5$ Must be working in radians	M1	e.g. $x = 1 \quad 1 \rightarrow 1.11$ $x = 1.5 \quad 1.5 \rightarrow 1.20$ Accept $f(1) = 1.11$ $f(1.5) = 1.20$ $f(x) = x - \cos^{-1} \sqrt{\frac{\pi}{16x}} : f(1) = -0.111., f(1.5) = 0.3..$ $f(x) = \cos x - \sqrt{\frac{\pi}{16x}} : f(1) = 0.097., f(1.5) = -0.291.$ For $16x \cos^2 x - \pi$ $f(1) = 1.529.., f(1.5) = -3.02..$ Must find values. M1 if at least one value correct
	Correct values and complete the argument correctly	A1	
		2	

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Question	Answer	Marks	Guidance
6(iv)	Use $x_{n+1} = \cos^{-1} \sqrt{\left(\frac{\pi}{16x_n}\right)}$ correctly at least twice Must be working in radians	M1	1, 1.11173, 1.13707, 1.14225, 1.14329, 1.14349, 1.14354, 1.14354 1.25, 1.16328, 1.14742, 1.14432, 1.14370 1.5, 1.20060, 1.15447, 1.14570, 1.14397, 1.14363
	Obtain final answer 1.144	A1	
	Show sufficient iterations to at least 5 d.p. to justify 1.144 to 3 d.p. or show there is a sign change in the interval (1.1435, 1.1445)	A1	
		3	

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Question	Answer	Marks	Guidance
7(i)	Separate variables correctly and attempt integration of at least one side	B1	$\int e^{-y} dy = \int xe^x dx$
	Obtain term $-e^{-y}$	B1	B0B1 is possible
	Commence integration by parts and reach $xe^x \pm \int e^x dx$	M1	B0B0M1A1 is possible
	Obtain $xe^x - e^x$	A1	or equivalent
			B1B1M1A1 is available if there is no constant of integration
	Use $x = 0, y = 0$ to evaluate a constant, or as limits in a definite integral, in a solution with terms ae^{-y} , bxe^x and ce^x , where $abc \neq 0$	M1	Must see this step
	Obtain correct solution in any form	A1	e.g. $e^{-y} = e^x - xe^x$
	Rearrange as $y = -\ln(1-x) - x$	A1	or equivalent e.g. $y = \ln \frac{1}{e^x(1-x)}$ ISW
		7	
7(ii)	Justify the given statement	B1	e.g. require $1-x > 0$ for the $\ln$ term to exist, hence $x < 1$ Must be considering the range of values of x , and must be relevant to <i>their</i> y involving $\ln(1-x)$
		1	

Question	Answer	Marks	Guidance
8(i)	State or imply the form $\frac{A}{2x+1} + \frac{B}{2x+3} + \frac{C}{(2x+3)^2}$	B1	
	Use a correct method to find a constant	M1	
	Obtain the values $A = 1, B = -1, C = 3$	A1 A1 A1	
	[Mark the form $\frac{A}{2x+1} + \frac{Dx+E}{(2x+3)^2}$, where $A = 1, D = -2$ and $E = 0$, B1M1A1A1A1 as above.]		Full marks for the three correct constants – do not actually need to see the partial fractions
		5	
8(ii)	Integrate and obtain terms $\frac{1}{2} \ln(2x+1) - \frac{1}{2} \ln(2x+3) - \frac{3}{2(2x+3)}$ [Correct integration of the A, D, E form of fractions gives $\frac{1}{2} \ln(2x+1) + \frac{x}{2x+3} - \frac{1}{2} \ln(2x+3)$ if integration by parts is used for the second partial fraction.]	B1 B1 B1	FT on A, B and C .
	Substitute limits correctly in an integral with terms $a \ln(2x+1)$, $b \ln(2x+3)$ and $c/(2x+3)$, where $abc \neq 0$ If using alternative form: $cx/(2x+3)$	M1	value for upper limit – value for lower limit 1 slip in substituting can still score M1 Condone omission of $\ln(1)$
	Obtain the given answer following full and correct working	A1	Need to see at least one interim step of valid log work. AG
		5	

Question	Answer	Marks	Guidance																															
9(i)	Carry out correct method for finding a vector equation for AB	M1																																
	Obtain $(\mathbf{r} =)\mathbf{i} + 2\mathbf{j} - \mathbf{k} + \lambda(2\mathbf{i} - \mathbf{j} + 2\mathbf{k})$, or equivalent	A1																																
	Equate two pairs of components of general points on <i>their</i> AB and l and solve for λ or for μ	M1	$\begin{pmatrix} 1+2\lambda \\ 2-\lambda \\ -1+2\lambda \end{pmatrix} = \begin{pmatrix} 2+\mu \\ 1+\mu \\ 1+2\mu \end{pmatrix}$																															
	Obtain correct answer for λ or μ , e.g. $\lambda = 0, \mu = -1$	A1																																
	Verify that all three equations are not satisfied and the lines fail to intersect ($\neq$ is sufficient justification e.g. $2 \neq 0$) Conclusion needs to follow correct values	A1	Alternatives <table><tr><th>A</th><th>λ</th><th>μ</th><th></th><th>B</th><th>λ</th><th>μ</th><th></th></tr><tr><td>ij</td><td>$\frac{2}{3}$</td><td>$\frac{1}{3}$</td><td>$\frac{1}{3} \neq \frac{5}{3}$</td><td></td><td>$-\frac{1}{3}$</td><td>$\frac{1}{3}$</td><td>$\frac{1}{3} \neq \frac{5}{3}$</td></tr><tr><td>ik</td><td>0</td><td>-1</td><td>$2 \neq 0$</td><td></td><td>-1</td><td>-1</td><td>$2 \neq 0$</td></tr><tr><td>jk</td><td>1</td><td>0</td><td>$3 \neq 2$</td><td></td><td>0</td><td>0</td><td>$3 \neq 2$</td></tr></table>	A	λ	μ		B	λ	μ		ij	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{3} \neq \frac{5}{3}$		$-\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3} \neq \frac{5}{3}$	ik	0	-1	$2 \neq 0$		-1	-1	$2 \neq 0$	jk	1	0	$3 \neq 2$		0	0
A	λ	μ		B	λ	μ																												
ij	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{3} \neq \frac{5}{3}$		$-\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3} \neq \frac{5}{3}$																											
ik	0	-1	$2 \neq 0$		-1	-1	$2 \neq 0$																											
jk	1	0	$3 \neq 2$		0	0	$3 \neq 2$																											
		5																																

Question	Answer	Marks	Guidance
9(ii)	State or imply midpoint has position vector $2\mathbf{i} + \frac{3}{2}\mathbf{j}$	B1	
	Substitute in $2x - y + 2z = d$ and find d	M1	Correct use of <i>their</i> direction for AB and <i>their</i> midpoint
	Obtain plane equation $4x - 2y + 4z = 5$	A1	or equivalent e.g. $\mathbf{r} \cdot \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} = \frac{5}{2}$
	Substitute components of l in plane equation and solve for μ	M1	Correct use of their plane equation.
	Obtain $\mu = -\frac{1}{2}$ and position vector $\frac{3}{2}\mathbf{i} + \frac{1}{2}\mathbf{j}$ for the point P	A1	Final answer Accept coordinates in place of position vector
		5	

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Question	Answer	Marks	Guidance
10(i)	State correct expansion of $\sin(3x+x)$ or $\sin(3x-x)$	B1	B0 If their formula retains $\pm$ in the middle
	Substitute expansions in $\frac{1}{2}(\sin 4x + \sin 2x)$	M1	
	Obtain $\sin 3x \cos x = \frac{1}{2}(\sin 4x + \sin 2x)$ correctly	A1	Must see the $\sin 4x$ and $\sin 2x$ or reference to LHS and RHS for A1 AG
		3	
10(ii)	Integrate and obtain $-\frac{1}{8}\cos 4x - \frac{1}{4}\cos 2x$	B1 B1	
	Substitute limits $x = 0$ and $x = \frac{1}{3}\pi$ correctly	M1	In their expression
	Obtain answer $\frac{9}{16}$	A1	From correct working seen.
		4	

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Question	Answer	Marks	Guidance
10(iii)	State correct derivative $2\cos 4x + \cos 2x$	B1	
	Using correct double angle formula, express derivative in terms of $\cos 2x$ and equate the result to zero	M1	
	Obtain $4\cos^2 2x + \cos 2x - 2 = 0$	A1	
	Solve for x or $2x$ (could be labelled x) $\left(\cos 2x = \frac{-1 \pm \sqrt{33}}{8} \right)$	M1	Must see working if solving an incorrect quadratic The roots of the correct quadratic are -0.843 and 0.593 Need to get as far as $x = \dots$ The wrong value of x is 0.468 and can imply M1 if correct quadratic seen Could be working from a quartic in $\cos x$: $16\cos^4 x - 14\cos^2 x + 1 = 0$
	Obtain answer $x = 1.29$ only	A1	
		5	

MATHEMATICS

9709/33

Paper 3

May/June 2019

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2019 series for most Cambridge IGCSE™, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

This document consists of **15** printed pages.



Cambridge Assessment
International Education

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mark Scheme Notes

Marks are of the following three types:

- M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
 - A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
 - B Mark for a correct result or statement independent of method marks.
- When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
 - The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously “correct” answers or results obtained from incorrect working.
 - Note: B2 or A2 means that the candidate can earn 2 or 0.
B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.
- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

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The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE Any Equivalent Form (of answer is equally acceptable) / Or Equivalent

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

CAO Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)

CWO Correct Working Only – often written by a ‘fortuitous’ answer

ISW Ignore Subsequent Working

SOI Seen or implied

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks	Guidance
1	Use law of the logarithm of a product or quotient	M1	
	Use law of the logarithm of power twice	M1	
	Obtain a correct linear equation in x , e.g. $(3-2x)\ln 5 = \ln 4 + x\ln 7$	A1	
	Obtain answer $x = 0.666$	A1	
		4	

Question	Answer	Marks	Guidance
2	Commence integration and reach $ax^2 \sin 2x + b \int x \sin 2x \, dx$	M1*	
	Obtain $\frac{1}{2}x^2 \sin 2x - \int x \sin 2x \, dx$, or equivalent	A1	
	Complete the integration and obtain $\frac{1}{2}x^2 \sin 2x + \frac{1}{2}x \cos 2x - \frac{1}{4} \sin 2x$, or equivalent	A1	
	Use limits correctly, having integrated twice	DM1	
	Obtain given answer correctly	A1	
		5	

Question	Answer	Marks	Guidance
3(i)	Use double angle formulae and express entire fraction in terms of $\sin\theta$ and $\cos\theta$	M1	
	Obtain a correct expression	A1	
	Obtain the given answer	A1	
		3	
3(ii)	State integral of the form $\pm \ln \cos \theta$	M1*	
	Use correct limits correctly and insert exact values for the trig ratios	DM1	
	Obtain a correct expression, e.g. $-\ln \frac{1}{\sqrt{2}} + \ln \frac{\sqrt{3}}{2}$	A1	
	Obtain the given answer following full and exact working	A1	
		4	

Question	Answer	Marks	Guidance
4(i)	Use the quotient or product rule	M1	
	Obtain correct derivative in any form	A1	
	Reduce to $-\frac{2e^{-x}}{(1-e^{-x})^2}$, or equivalent, and explain why this is always negative	A1	
		3	

Question	Answer	Marks	Guidance
4(ii)	Equate derivative to -1 and obtain the given equation	B1	
	State or imply $u^2 - 4u + 1 = 0$, or equivalent in e^a	B1	
	Solve for a	M1	
	Obtain answer $a = \ln(2 + \sqrt{3})$ and no other	A1	
		4	

Question	Answer	Marks	Guidance
5	Separate variables correctly and integrate at least one side	B1	
	Obtain term $\ln(x+1)$	B1	
	Obtain term of the form $a \ln(y^2 + 5)$	M1	
	Obtain term $\frac{1}{2} \ln(y^2 + 5)$	A1	
	Use $y = 2, x = 0$ to determine a constant, or as limits, in a solution containing terms $a \ln(y^2 + 5)$ and $b \ln(x+1)$, where $ab \neq 0$	M1	
	Obtain correct solution in any form	A1	
	Obtain final answer $y^2 = 9(x+1)^2 - 5$	A1	
		7	

Question	Answer	Marks	Guidance
6(i)	State $b = 3$	B1	
		1	
6(ii)	Commence division by $x - b$ and reach partial quotient $x^3 + kx^2$	M1	
	Obtain quotient $x^3 + x^2 + 3x + 2$	A1	There being no remainder
	Equate quotient to zero and rearrange to make the subject a	M1	
	Obtain the given equation	A1	
		4	
6(iii)	Use the iterative formula $a_{n+1} = -\frac{1}{3}(2 + a_n^2 + a_n^3)$ correctly at least once	M1	
	Obtain final answer -0.715	A1	
	Show sufficient iterations to 5 d.p. to justify -0.715 to 3 d.p., or show there is a sign change in the interval $(-0.7145, -0.7155)$	A1	
		3	

Question	Answer	Marks	Guidance
7(i)	Use product rule	M1	
	Obtain correct derivative in any form	A1	
		2	
7(ii)	Equate derivative to zero and use correct $\cos(A + B)$ formula	M1	
	Obtain the given equation	A1	
		2	
7(iii)	Use correct method to solve for x	M1	
	Obtain answer, e.g. $x = \frac{1}{12}\pi$	A1	
	Obtain second answer, e.g. $\frac{7}{12}\pi$, and no other	A1	
		3	

Question	Answer	Marks	Guidance
8(i)	Multiply numerator and denominator by $1 + \sqrt{3}i$, or equivalent	M1	
	$4i - 4\sqrt{3}$ and $3 + 1$	A1	
	Obtain final answer $-\sqrt{3} + i$	A1	
		3	

Question	Answer	Marks	Guidance
8(ii)	State that the modulus of u is 2	B1	
	State that the argument of u is $\frac{5}{6}\pi$ (or 150°)	B1	
		2	
8(iii)	Show a circle with centre the origin and radius 2	B1	
	Show u in a relatively correct position	B1	FT
	Show the perpendicular bisector of the line joining u and the origin	B1	FT
	Shade the correct region	B1	
		4	

Question	Answer	Marks	Guidance
9(i)	State or imply the form $\frac{A}{3+x} + \frac{B}{1-x} + \frac{C}{(1-x)^2}$	B1	
	Use a correct method for finding a constant	M1	
	Obtain one of $A = -3$, $B = -1$, $C = 2$	A1	
	Obtain a second value	A1	
	Obtain the third value	A1	Mark the form $\frac{A}{3+x} + \frac{Dx+E}{(1-x)^2}$, where $A = -3$, $D = 1$ and $E = 1$, B1M1A1A1A1 as above.
		5	
9(ii)	Use a correct method to find the first two terms of the expansion of $(3+x)^{-1}$, $(1+\frac{1}{3}x)^{-1}$, $(1-x)^{-1}$ or $(1-x)^{-2}$	M1	
	Obtain correct unsimplified expansions up to the term in x^3 of each partial fraction	A1	FT on A
		A1	FT on B
		A1	FT on C
	Obtain final answer $\frac{10}{3}x + \frac{44}{9}x^2 + \frac{190}{27}x^3$	A1	For the A, D, E form of fractions give M1A1ftA1ft for the expanded partial fractions, then, if $D \neq 0$, M1 for multiplying out fully, and A1 for the final answer.
		5	

Question	Answer	Marks	Guidance
10(i)	Find $\overrightarrow{PQ}$ for a general point Q on l , e.g. $-3\mathbf{i} + 6\mathbf{k} + \mu(2\mathbf{i} - \mathbf{j} - 2\mathbf{k})$	B1	
	Calculate scalar product of $\overrightarrow{PQ}$ and a direction vector for l and equate the result to zero	M1	
	Solve for μ and obtain $\mu = 2$	A1	
	Carry out a complete method for finding the length of $\overrightarrow{PQ}$	M1	
	Obtain answer 3	A1	
	Alternative method for question 10(i)		
	Calling the point $(1, 2, 3)$ A , state $\overrightarrow{AP}$ (or $\overrightarrow{PA}$) in component form, e.g. $3\mathbf{i} - 6\mathbf{k}$	B1	
	Use a scalar product with a direction vector for l to find the projection of $\overrightarrow{AP}$ (or $\overrightarrow{PA}$) on l	M1	
	Obtain correct answer in any form, e.g. $\frac{18}{\sqrt{9}}$	A1	
	Use Pythagoras to find the perpendicular	M1	
	Obtain answer 3	A1	

Question	Answer	Marks	Guidance
10(i)	Alternative method for question 10(i)		
	State $\overrightarrow{AP}$ (or $\overrightarrow{PA}$) in component form	B1	
	Calculate a vector product with a direction vector for l	M1	
	Obtain correct answer, e.g. $6\mathbf{i} - 6\mathbf{j} - 3\mathbf{k}$	A1	
	Divide modulus of the product by that of the direction vector	M1	
	Obtain answer 3	A1	
		5	

Question	Answer	Marks	Guidance
10(ii)	Substitute coordinates of a general point of l in the plane equation and equate constant terms	M1	
	Obtain a correct equation, e.g. $a + 2b + 6 = 13$	A1	
	Equate the coefficient of μ to zero	M1	
	Obtain a correct equation, e.g. $2a - b - 4 = 0$	A1	
	Substitute (1, 2, 3) in the plane equation	M1	
	Obtain a correct equation, e.g. $a + 2b + 6 = 13$	A1	
	Alternative method for question 10(ii)		
	Find a second point on l and obtain an equation in a and/or b	M1	
	Obtain a correct equation, e.g. $5a - 2 = 13$	A1	
	Equate scalar product of a direction vector for l and a vector normal for the plane to zero	M1	
	Obtain a correct equation, e.g. $2a - b - 4 = 0$	A1	
	Solve for a or for b	M1	
	Obtain $a = 3$ and $b = 2$	A1	
		6	