

CANDIDATE
NAME

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NUMBER

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MATHEMATICS

9709/31

Paper 3 Pure Mathematics 3 **(P3)**

May/June 2018

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name in the spaces at the top of this page.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

DO NOT WRITE IN ANY BARCODES.

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an electronic calculator is expected, where appropriate.

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At the end of the examination, fasten all your work securely together.

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The total number of marks for this paper is 75.

This document consists of **18** printed pages and **2** blank pages.



- 1** Showing all necessary working, solve the equation $\ln(x^4 - 4) = 4 \ln x - \ln 4$, giving your answer correct to 2 decimal places. [4]

This image shows a full page of white paper with horizontal dotted lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

2 (i) Given that $\sin(x - 60^\circ) = 3 \cos(x - 45^\circ)$, find the exact value of $\tan x$. [4]

[illegible]

(ii) Hence solve the equation $\sin(x - 60^\circ) = 3 \cos(x - 45^\circ)$, for $0^\circ < x < 360^\circ$. [2]

[illegible]

- 3** A curve has equation $y = \frac{e^{3x}}{\tan \frac{1}{2}x}$. Find the x -coordinates of the stationary points of the curve in the interval $0 < x < \pi$. Give your answers correct to 3 decimal places. [6]

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page, typical of notebook or legal stationery. There are no margins, text, or other markings on the page.

- 4** The polynomial $x^4 + 2x^3 + ax + b$, where a and b are constants, is divisible by $x^2 - x + 1$. Find the values of a and b . [5]

[illegible]

5 Let $I = \int_{\frac{1}{4}}^{\frac{3}{4}} \sqrt{\left(\frac{x}{1-x}\right)} dx$.

(i) Using the substitution $x = \cos^2 \theta$, show that $I = \int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} 2 \cos^2 \theta \, d\theta$. [4]

[illegible]

[Turn over

- 6 In a certain chemical reaction the amount, x grams, of a substance is decreasing. The differential equation relating x and t , the time in seconds since the reaction started, is

$$\frac{dx}{dt} = -kx\sqrt{t},$$

where k is a positive constant. It is given that $x = 100$ at the start of the reaction.

- (i) Solve the differential equation, obtaining a relation between x , t and k . [5]

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- 7 (i) Showing all working and without using a calculator, solve the equation $z^2 + (2\sqrt{6})z + 8 = 0$, giving your answers in the form $x + iy$, where x and y are real and exact. [3]

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- (ii) Sketch an Argand diagram showing the points representing the roots. [1]

- (iii) The points representing the roots are A and B , and O is the origin. Find angle AOB . [3]

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- (iv) Prove that triangle AOB is equilateral. [1]

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This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

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- (ii) Verify by calculation that a lies between 3 and 3.5. [2]

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- (iii) Use an iteration based on the equation in part (i) to determine a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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9 Let $f(x) = \frac{12x^2 + 4x - 1}{(x - 1)(3x + 2)}$.

(i) Express $f(x)$ in partial fractions.

[5]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- (ii)** Hence obtain the expansion of $f(x)$ in ascending powers of x , up to and including the term in x^2 . [5]

[illegible]

- 10** The point P has position vector $3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$. The line l has equation $\mathbf{r} = 4\mathbf{i} + 2\mathbf{j} + 5\mathbf{k} + \mu(\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$.

- (i) Find the length of the perpendicular from P to l , giving your answer correct to 3 significant figures. [5]

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- (ii) Find the equation of the plane containing l and P , giving your answer in the form $ax + by + cz = d$.
[5]

This image shows a full page of a document template designed for handwriting practice or general note-taking. It consists of approximately 20 evenly spaced horizontal dotted lines across the entire width of the page. The background is plain white, and there are no margins, headers, footers, or other markings present.

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

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The total number of marks for this paper is 75.

This document consists of **20** printed pages.



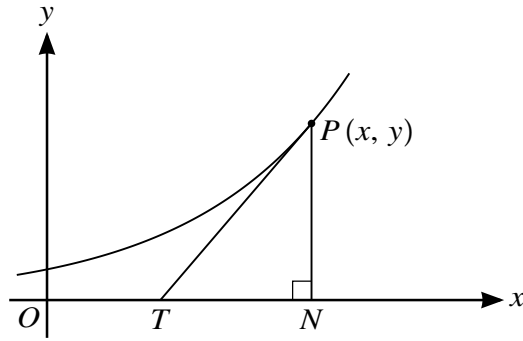
- 1 Showing all necessary working, solve the equation $3|2^x - 1| = 2^x$, giving your answers correct to 3 significant figures. [4]

This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- 2** Showing all necessary working, solve the equation $\cot \theta + \cot(\theta + 45^\circ) = 2$, for $0^\circ < \theta < 180^\circ$. [5]

[illegible]

3



In the diagram, the tangent to a curve at the point P with coordinates (x, y) meets the x -axis at T . The point N is the foot of the perpendicular from P to the x -axis. The curve is such that, for all values of x , the gradient of the curve is positive and $TN = 2$.

- (i) Show that the differential equation satisfied by x and y is $\frac{dy}{dx} = \frac{1}{2}y$. [1]

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The point with coordinates $(4, 3)$ lies on the curve.

- (ii) Solve the differential equation to obtain the equation of the curve, expressing y in terms of x . [5]

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4 (i) Show that $\frac{2 \sin x - \sin 2x}{1 - \cos 2x} \equiv \frac{\sin x}{1 + \cos x}$. [4]

[illegible]

(ii) Hence, showing all necessary working, find $\int_{\frac{1}{3}\pi}^{\frac{1}{2}\pi} \frac{2 \sin x - \sin 2x}{1 - \cos 2x} dx$, giving your answer in the form $\ln k$. [4]

This image shows a full page of primary-ruled paper. It features approximately 20 horizontal dotted lines spaced evenly down the page, providing a guide for handwriting practice. The paper is otherwise blank, with no margins, text, or other markings.

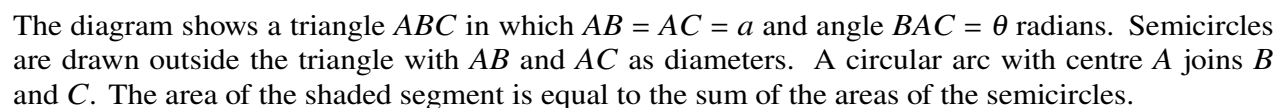
5 The equation of a curve is $x^2(x + 3y) - y^3 = 3$.

(i) Show that $\frac{dy}{dx} = \frac{x^2 + 2xy}{y^2 - x^2}$. [4]

[illegible]

- (ii) Hence find the exact coordinates of the two points on the curve at which the gradient of the normal is 1. [4]

[illegible]



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- This image shows a full page of a handwriting practice worksheet. It features 18 horizontal rows, each consisting of three parallel dashed lines. The top row is slightly larger than the others, providing extra space for a title or introduction. The remaining 17 rows are uniform in size and are designed to help children learn letter formation and alignment through repeated tracing and writing exercises. The entire page is white with no other markings or text.

- (ii) Verify by calculation that θ lies between 2.2 and 2.4.

[2]

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- (iii) Use an iterative formula based on the equation in part (i) to determine θ correct to 2 decimal places. Give the result of each iteration to 4 decimal places.

[3]

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(i) Find, in the form $x + iy$, where x and y are real and exact, the complex numbers uv and $\frac{u}{v}$. [5]

[illegible]

- (ii) On a sketch of an Argand diagram with origin O , show the points A and B representing the complex numbers u and v respectively. Prove that angle $AOB = \frac{2}{3}\pi$. [3]

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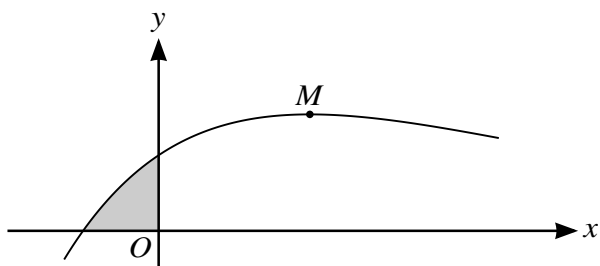
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(i) Find the x -coordinate of M .

[4]

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- (ii) Find the area of the shaded region enclosed by the curve and the axes, giving your answer in terms of e . [5]

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9 Let $f(x) = \frac{x - 4x^2}{(3 - x)(2 + x^2)}$.

(i) Express $f(x)$ in the form $\frac{A}{3-x} + \frac{Bx+C}{2+x^2}$. [4]

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- (ii)** Hence obtain the expansion of $f(x)$ in ascending powers of x , up to and including the term in x^3 .
[5]

[illegible]

- 10** Two lines l and m have equations $\mathbf{r} = 2\mathbf{i} - \mathbf{j} + \mathbf{k} + s(2\mathbf{i} + 3\mathbf{j} - \mathbf{k})$ and $\mathbf{r} = \mathbf{i} + 3\mathbf{j} + 4\mathbf{k} + t(\mathbf{i} + 2\mathbf{j} + \mathbf{k})$ respectively.

(i) Show that the lines are skew.

[4]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the entire width of the page. There are no margins, text, or other markings present.

A plane p is parallel to the lines l and m .

(ii) Find a vector that is normal to p .

[3]

[illegible]

- [illegible]

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- 1** Expand $\frac{4}{\sqrt{4-3x}}$ in ascending powers of x , up to and including the term in x^2 , simplifying the coefficients. [4]

[illegible]

- 2 Showing all necessary working, solve the equation $5^{2x} = 5^x + 5$. Give your answer correct to 3 decimal places. [5]

This image shows a full page of white paper with horizontal dotted lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- 3** Showing all necessary working, find the value of $\int_0^{\frac{1}{6}\pi} x \cos 3x \, dx$, giving your answer in terms of π . [5]

[illegible]

- 4** The curve with equation $y = \frac{\ln x}{3+x}$ has a stationary point at $x = p$.

(i) Show that p satisfies the equation $\ln x = 1 + \frac{3}{x}$. [3]

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

(iii) It is given that the equation in part (i) can be written in the form $x = \frac{3+x}{\ln x}$. Use an iterative formula based on this rearrangement to determine the value of p correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

5 (i) By first expanding $(\cos^2 x + \sin^2 x)^3$, or otherwise, show that

$$\cos^6 x + \sin^6 x = 1 - \frac{3}{4} \sin^2 2x. \quad [4]$$

This image shows a full page of white paper with horizontal dashed lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- 6 (i) Express $\frac{1}{4-y^2}$ in partial fractions. [2]

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- (ii) The variables x and y satisfy the differential equation

$$x \frac{dy}{dx} = 4 - y^2,$$

and $y = 1$ when $x = 1$. Solve the differential equation, obtaining an expression for y in terms of x . [6]

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7 Throughout this question the use of a calculator is not permitted.

- (i) Express $\cos \theta + 2 \sin \theta$ in the form $R \cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. Give the exact values of R and $\tan \alpha$. [3]

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- (ii) Hence, showing all necessary working, show that $\int_0^{\frac{1}{4}\pi} \frac{15}{(\cos \theta + 2 \sin \theta)^2} d\theta = 5$. [5]

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- 8** The equation of a curve is $2x^3 - y^3 - 3xy^2 = 2a^3$, where a is a non-zero constant.

(i) Show that $\frac{dy}{dx} = \frac{2x^2 - y^2}{y^2 + 2xy}$. [4]

[illegible]

- (ii) Find the coordinates of the two points on the curve at which the tangent is parallel to the y-axis. [5]

[illegible]

- 9 (a)** Find the complex number z satisfying the equation

$$3z - iz^* = 1 + 5i,$$

where z^* denotes the complex conjugate of z .

[4]

[illegible]

- [5]

[illegible]

- 10** The points A and B have position vectors $2\mathbf{i} + \mathbf{j} + 3\mathbf{k}$ and $4\mathbf{i} + \mathbf{j} + \mathbf{k}$ respectively. The line l has equation $\mathbf{r} = 4\mathbf{i} + 6\mathbf{j} + \mu(\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})$.

(i) Show that l does not intersect the line passing through A and B . [5]

[illegible]

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[illegible]

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