
MATHEMATICS

9709/31

Paper 3

May/June 2018

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2018 series for most Cambridge IGCSE™, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

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Generic Marking Principles

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GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously ‘correct’ answers or results obtained from incorrect working.
- Note: B2 or A2 means that the candidate can earn 2 or 0.
B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.
- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE Any Equivalent Form (of answer is equally acceptable) / Or Equivalent

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

CAO Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)

CWO Correct Working Only – often written by a ‘fortuitous’ answer

ISW Ignore Subsequent Working

SOI Seen or implied

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become ‘follow through’ marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks
1	Use law for the logarithm of a product, quotient or power	M1
	Obtain a correct equation free of logarithms, e.g. $4(x^4 - 4) = x^4$	A1
	Solve for x	M1
	Obtain answer $x = 1.52$ only	A1
		4

Question	Answer	Marks
2(i)	Use trig formulae and obtain an equation in $\sin x$ and $\cos x$	M1*
	Obtain a correct equation in any form	A1
	Substitute exact trig ratios and obtain an expression for $\tan x$	M1(dep*)
	Obtain answer $\tan x = \frac{-(6 + \sqrt{6})}{(6 - \sqrt{2})}$ or equivalent	A1
		4
2(ii)	State answer, e.g. 118.5°	B1
	State second answer, e.g. 298.5°	B1ft
		2

Question	Answer	Marks
3	Use quotient or product rule	M1
	Obtain correct derivative in any form	A1
	Equate derivative to zero and obtain a quadratic in $\tan \frac{1}{2}x$ or an equation of the form $a \sin x = b$	M1*
	Solve for x	M1(dep*)
	Obtain answer 0.340	A1
	Obtain second answer 2.802 and no other in the given interval	A1
		6

Question	Answer	Marks
4	<i>EITHER:</i> Commence division by $x^2 - x + 1$ and reach a partial quotient of the form $x^2 + kx$	M1
	Obtain quotient $x^2 + 3x + 2$	A1
	<i>Either</i> Set remainder identically equal to zero and solve for a or for b , <i>or</i> multiply given divisor and found quotient and obtain a or b	M1
	Obtain $a = 1$	A1
	Obtain $b = 2$	A1
	<i>OR:</i> Assume an unknown factor $x^2 + Bx + C$ and obtain an equation in B and/or C	M1
	Obtain $B = 3$ and $A = 2$	A1
	<i>Either</i> Use equations to obtain a or b <i>or</i> multiply given divisor and found factor to obtain a or b	M1
	Obtain $a = 1$	A1
	Obtain $b = 2$	A1
		5

Question	Answer	Marks
5(i)	State or imply $dx = -2\cos\theta \sin\theta d\theta$, or equivalent	B1
	Substitute for x and dx , and use Pythagoras	M1
	Obtain integrand $\pm 2\cos^2\theta$	A1
	Justify change of limits and obtain given answer correctly	A1
		4
5(ii)	Obtain indefinite integral of the form $a\theta + b\sin 2\theta$	M1*
	Obtain $\theta + \frac{1}{2}\sin 2\theta$	A1
	Use correct limits correctly	M1(dep*)
	Obtain answer $\frac{1}{6}\pi$ with no errors seen	A1
		4

Question	Answer	Marks
6(i)	Separate variables correctly and integrate at least one side	B1
	Obtain term $\ln x$	B1
	Obtain term $-\frac{2}{3}kt\sqrt{t}$, or equivalent	B1
	Evaluate a constant, or use limits $x = 100$ and $t = 0$, in a solution containing terms $a \ln x$ and $b t\sqrt{t}$	M1
	Obtain correct solution in any form, e.g. $\ln x = -\frac{2}{3}kt\sqrt{t} + \ln 100$	A1
		5
6(ii)	Substitute $x = 80$ and $t = 25$ to form equation in k	M1
	Substitute $x = 40$ and eliminate k	M1
	Obtain answer $t = 64.1$	A1
		3

Question	Answer	Marks
7(i)	Use quadratic formula, or completing the square, or the substitution $z = x + iy$ to find a root, using $i^2 = -1$	M1
	Obtain a root, e.g. $-\sqrt{6} - \sqrt{2}i$	A1
	Obtain the other root, e.g. $-\sqrt{6} - \sqrt{2}i$	A1
		3
7(ii)	Represent both roots in relatively correct positions	B1ft
		1
7(iii)	State or imply correct value of a relevant length or angle, e.g. OA , OB , AB , angle between OA or OB and the real axis	B1ft
	Carry out a complete method for finding angle OAB	M1
	Obtain $AOB = 60^\circ$ correctly	A1
		3
7(iv)	Give a complete justification of the given statement	B1
		1

Question	Answer	Marks
8(i)	Integrate by parts and reach $\int x e^{-\frac{1}{2}x} + m \int e^{-\frac{1}{2}x} dx$	M1*
	Obtain $-2xe^{-\frac{1}{2}x} + 2 \int e^{-\frac{1}{2}x} dx$	A1
	Complete the integration and obtain $-2xe^{-\frac{1}{2}x} - 4e^{-\frac{1}{2}x}$, or equivalent	A1
	Having integrated twice, use limits and equate result to 2	M1(dep*)
	Obtain the given equation correctly	A1
		5
8(ii)	Calculate values of a relevant expression or pair of expressions at $a = 3$ and $a = 3.5$	M1
	Complete the argument correctly with correct calculated values	A1
		2
8(iii)	Use the iterative formula $a_{n+1} = 2 \ln(a_n + 2)$ correctly at least once	M1
	Obtain final answer 3.36	A1
	Show sufficient iterations to 4 d.p. to justify 3.36 to 2 d.p., or show there is a sign change in the interval (3.355, 3.365)	A1
		3

Question	Answer	Marks
9(i)	State or imply the form $A + \frac{B}{x-1} + \frac{C}{3x+2}$	B1
	State or obtain $A = 4$	B1
	Use a correct method to obtain a constant	M1
	Obtain one of $B = 3, C = -1$	A1
	Obtain the other value	A1
		5
9(ii)	Use correct method to find the first two terms of the expansion of $(x-1)^{-1}$ or $(3x+2)^{-1}$, or equivalent	M1
	Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction	A1ft + A1ft
	Add the value of A to the sum of the expansions	M1
	Obtain final answer $\frac{1}{2} - \frac{9}{4}x - \frac{33}{8}x^2$	A1
		5

Question	Answer	Marks
10(a)	<i>EITHER:</i> Find $\overrightarrow{PQ}$ (or $\overrightarrow{QP}$) for a general point Q on l , e.g. $(1+\mu)\mathbf{i} + (4+2\mu)\mathbf{j} + (4+3\mu)\mathbf{k}$	B1
	Calculate the scalar product of $\overrightarrow{PQ}$ and a direction vector for l and equate to zero	M1
	Solve and obtain correct solution e.g. $\mu = -\frac{3}{2}$	A1
	Carry out method to calculate PQ	M1
	Obtain answer 1.22	A1
	<i>OR1:</i> Find $\overrightarrow{PQ}$ (or $\overrightarrow{QP}$) for a general point Q on l	B1
	Use a correct method to express PQ^2 (or PQ) in terms of μ	M1
	Obtain a correct expression in any form	A1
	Carry out a complete method for finding its minimum	M1
	Obtain answer 1.22	A1
	<i>OR2:</i> Calling $(4, 2, 5)$ A , state $\overrightarrow{PA}$ (or $\overrightarrow{AP}$) in component form, e.g. $\mathbf{i} + 4\mathbf{j} + 4\mathbf{k}$	B1
	Use a scalar product to find the projection of $\overrightarrow{PA}$ (or $\overrightarrow{AP}$) on l	M1
	Obtain correct answer $21/\sqrt{14}$, or equivalent	A1
	Use Pythagoras to find the perpendicular	M1
	Obtain answer 1.22	A1
	<i>OR3:</i> State $\overrightarrow{PA}$ (or $\overrightarrow{AP}$) in component form	B1
	Calculate vector product of $\overrightarrow{PA}$ and a direction vector for l	M1
	Obtain correct answer, e.g. $4\mathbf{i} + \mathbf{j} - 2\mathbf{k}$	A1
	Divide modulus of the product by that of the direction vector	M1
	Obtain answer 1.22	A1
		5

Question	Answer	Marks
10(ii)	<i>EITHER:</i> Use scalar product to obtain a relevant equation in a , b and c , e.g. $a + 2b + 3c = 0$	B1
	Obtain a second relevant equation, e.g. using $\overrightarrow{PA}$ $a + 4b + 4c = 0$, and solve for one ratio	M1
	Obtain $a : b : c = 4 : 1 : -2$, or equivalent	A1
	Substitute a relevant point and values of a , b , c in general equation and find d	M1
	Obtain correct answer, $4x + y - 2z = 8$, or equivalent	A1
	<i>OR1:</i> Attempt to calculate vector product of relevant vectors, e.g. $(\mathbf{i} + 4\mathbf{j} + 4\mathbf{k}) \times (\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$	M1
	Obtain two correct components	A1
	Obtain correct answer, e.g. $4\mathbf{i} + \mathbf{j} - 2\mathbf{k}$	A1
	Substitute a relevant point and find d	M1
	Obtain correct answer, $4x + y - 2z = 8$, or equivalent	A1
	<i>OR2:</i> Using a relevant point and relevant vectors form a 2-parameter equation for the plane	M1
	State a correct equation, e.g. $\mathbf{r} = 4\mathbf{i} + 2\mathbf{j} + 5\mathbf{k} + \lambda(\mathbf{i} + 4\mathbf{j} + 4\mathbf{k}) + \mu(\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$	A1
	State three correct equations in x , y , z , λ and μ	A1
	Eliminate λ and μ	M1
	Obtain correct answer $4x + y - 2z = 8$, or equivalent	A1
		5

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PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks	Guidance
1	<i>EITHER:</i> State or imply non-modular equation $3^2(2^x - 1)^2 = (2^x)^2$, or pair of equations $3(2^x - 1) = \pm 2^x$	M1	$8(2^x)^2 - 18(2^x) + 9 = 0$
	Obtain $2^x = \frac{3}{2}$ and $2^x = \frac{3}{4}$ or equivalent	A1	
	<i>OR:</i> Obtain $2^x = \frac{3}{2}$ by solving an equation	B1	
	Obtain $2^x = \frac{3}{4}$ by solving an equation	B1	
	Use correct method for solving an equation of the form $2^x = a$, where $a > 0$	M1	
	Obtain final answers $x = 0.585$ and $x = -0.415$ only	A1	The question requires 3 s.f. Do not ISW if they go on to reject one value
		4	

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Question	Answer	Marks	Guidance
2	Use correct $\tan(A \pm B)$ formula and obtain an equation in $\tan \theta$	M1	$\frac{1}{\tan \theta} + \frac{1 - \tan \theta \tan 45}{\tan \theta + \tan 45} = 2$ Allow M1 with $\tan 45^\circ$ $= \frac{1}{\tan \theta} + \frac{1 - \tan \theta}{\tan \theta + 1}$
	Obtain a correct equation in any form	A1	With values substituted
	Reduce to $3 \tan^2 \theta = 1$, or equivalent	A1	
	Obtain answer $x = 30^\circ$	A1	One correct solution
	Obtain answer $x = 150^\circ$	A1	Second correct solution and no others in range
	OR: use correct $\sin(A \pm B)$ and $\cos(A \pm B)$ to form equation in $\sin \theta$ and $\cos \theta$ M1A1		
	Reduce to $\tan^2 \theta = \frac{1}{3}$, $\sin^2 \theta = \frac{1}{4}$, $\cos^2 \theta = \frac{3}{4}$ or $\cot^2 \theta = 3$ A1 etc.		
		5	

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Question	Answer	Marks	Guidance
3(i)	Fully justify the given statement	B1	Some indication of use of gradient of curve = gradient of tangent (<i>PT</i>) and no errors seen /no incorrect statements
		1	
3(ii)	Separate variables and attempt integration of at least one side Obtain terms $\ln y$ and $\frac{1}{2}x$	B1 B1	Must be working from $\int \frac{1}{y} dy = \int k dx$ B marks are not available for fortuitously correct answers
	Use $x = 4, y = 3$ to evaluate a constant or as limits in a solution with terms $a \ln y$ and bx , where $ab \neq 0$	M1	
	Obtain correct solution in any form	A1	$\ln y = \frac{1}{2}x + \ln 3 - 2$
	Obtain answer $y = 3e^{\frac{1}{2}x-2}$, or equivalent	A1	Accept $y = e^{\frac{1}{2}x + \ln 3 - 2}$, $y = e^{\frac{x-1.80}{2}}$, $y = 3\sqrt{e^{x-4}}$ $ y = \dots$ scores A0
		5	

Question	Answer	Marks	Guidance
4(i)	Use correct double angle formulae and express LHS in terms of $\cos x$ and $\sin x$	M1	$\frac{2\sin x - 2\sin x \cos x}{1 - (2\cos^2 x - 1)}$
	Obtain a correct expression	A1	
	Complete method to get correct denominator e.g. by factorising to remove a factor of $1 - \cos x$	M1	
	Obtain the given RHS correctly <i>OR (working R to L):</i>	A1	
	$\frac{\sin x}{1 + \cos x} \times \frac{1 - \cos x}{1 - \cos x} = \frac{\sin x - \sin x \cos x}{1 - \cos^2 x}$ $= \frac{2\sin x - 2\sin x \cos x}{2 - 2\cos^2 x}$ M1A1		Given answer so check working carefully
	$= \frac{2\sin x - \sin 2x}{1 - \cos 2x}$ M1A1		
		4	
4(ii)	State integral of the form $a \ln(1 + \cos x)$	M1*	If they use the substitution $u = 1 + \cos x$ allow M1A1 for $-\ln u$
	Obtain integral $-\ln(1 + \cos x)$	A1	
	Substitute correct limits in correct order	M1(dep)*	
	Obtain answer $\ln\left(\frac{3}{2}\right)$, or equivalent	A1	
		4	

Question	Answer	Marks	Guidance
5(i)	State or imply $3y^2 \frac{dy}{dx}$ as derivative of y^3	B1	
	State or imply $6xy + 3x^2 \frac{dy}{dx}$ as derivative of $3x^2y$ OR State or imply $2x(x + 3y) + x^2 \left(1 + 3 \frac{dy}{dx}\right)$ as derivative of $x^2(x + 3y)$	B1	$3x^2 + 6xy + 3x^2 \frac{dy}{dx} - 3y^2 \frac{dy}{dx} = 0$
	Equate derivative of the LHS to zero and solve for $\frac{dy}{dx}$	M1	Given answer so check working carefully
	Obtain the given answer	A1	
		4	
5(ii)	Equate derivative to -1 and solve for y	M1*	
	Use their $y = -2x$ or equivalent to obtain an equation in x or y	M1(dep*)	
	Obtain answer $(1, -2)$	A1	
	Obtain answer $(\sqrt[3]{3}, 0)$	B1	Must be exact e.g. $e^{\frac{1}{3}\ln 3}$ but ISW if decimals after exact value seen
		4	

Question	Answer	Marks	Guidance
6(i)	Use correct method for finding the area of a segment and area of semicircle and form an equation in θ	M1	e.g. $\frac{\pi a^2}{4} = \frac{1}{2}a^2\theta - \frac{1}{2}a^2 \sin \theta$
	State a correct equation in any form	A1	Given answer so check working carefully
	Obtain the given answer correctly	A1	
		3	
6(ii)	Calculate values of a relevant expression or pair of expressions at $\theta = 2.2$ and $\theta = 2.4$	M1	e.g. $f(\theta) = \frac{\pi}{2} + \sin \theta \quad \begin{cases} f(2.2) = 2.37... > 2.2 \\ f(2.4) = 2.24... < 2.4 \end{cases}$ or $f(\theta) = \theta - \frac{\pi}{2} - \sin \theta \quad \begin{cases} f(2.2) = -0.17... < 0 \\ f(2.4) = +0.15... > 0 \end{cases}$
	Complete the argument correctly with correct calculated values	A1	
		2	

Question	Answer	Marks	Guidance																														
6(iii)	Use $\theta_{n+1} = \frac{1}{2}\pi + \sin \theta_n$ correctly at least once	M1	e.g. <table><tr><td>2.2</td><td>2.3</td><td>2.4</td></tr><tr><td>2.3793</td><td>2.3165</td><td>2.2463</td></tr><tr><td>2.2614</td><td>2.3054</td><td>2.3512</td></tr><tr><td>2.3417</td><td>2.3129</td><td>2.2814</td></tr><tr><td>2.2881</td><td>2.3079</td><td>2.3288</td></tr><tr><td>2.3244</td><td></td><td>2.2970</td></tr><tr><td>2.3000</td><td></td><td>2.3185</td></tr><tr><td>2.3165</td><td></td><td>2.3041</td></tr><tr><td>2.3054</td><td></td><td>2.3138</td></tr><tr><td>2.3129</td><td></td><td>2.3072</td></tr></table>	2.2	2.3	2.4	2.3793	2.3165	2.2463	2.2614	2.3054	2.3512	2.3417	2.3129	2.2814	2.2881	2.3079	2.3288	2.3244		2.2970	2.3000		2.3185	2.3165		2.3041	2.3054		2.3138	2.3129		2.3072
	2.2	2.3		2.4																													
	2.3793	2.3165		2.2463																													
	2.2614	2.3054		2.3512																													
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	2.2881	2.3079		2.3288																													
	2.3244			2.2970																													
	2.3000			2.3185																													
	2.3165			2.3041																													
	2.3054			2.3138																													
	2.3129			2.3072																													
	Obtain final answer 2.31	A1																															
Show sufficient iterations to 4 d.p. to justify 2.31 to 2 d.p. or show there is a sign change in the interval (2.305, 2.315)	A1																																
		3																															

Question	Answer	Marks	Guidance
7(i)	Substitute in uv , expand the product and use $i^2 = -1$	M1	
	Obtain answer $uv = -11 - 5\sqrt{3}i$	A1	
	<i>EITHER:</i> Substitute in u/v and multiply numerator and denominator by the conjugate of v , or equivalent	M1	
	Obtain numerator $-7 + 7\sqrt{3}i$ or denominator 7	A1	
	Obtain final answer $-1 + \sqrt{3}i$	A1	
	<i>OR:</i> Substitute in u/v , equate to $x + iy$ and solve for x or for y	M1	$\begin{cases} -3\sqrt{3} = \sqrt{3}x - 2y \\ 1 = 2x + \sqrt{3}y \end{cases}$
	Obtain $x = -1$ or $y = \sqrt{3}$	A1	
	Obtain final answer $-1 + \sqrt{3}i$	A1	
		5	

Question	Answer	Marks	Guidance
7(ii)	Show the points A and B representing u and v in relatively correct positions	B1	
	Carry out a complete method for finding angle AOB , e.g. calculate $\arg(u/v)$ If using $\theta = \tan^{-1}(-\sqrt{3})$ must refer to $\arg\left(\frac{u}{v}\right)$	M1	$\text{OR: } \tan a = \frac{-1}{3\sqrt{3}}, \tan b = \frac{2}{\sqrt{3}} \Rightarrow \tan(a-b) = \frac{\frac{-1}{3\sqrt{3}} - \frac{2}{\sqrt{3}}}{1 - \frac{2}{9}}$ $= -\sqrt{3}$ $\Rightarrow \theta = \frac{2\pi}{3}$ $\text{OR: } \cos \theta = \frac{\begin{pmatrix} -3\sqrt{3} \\ 1 \end{pmatrix} \begin{pmatrix} \sqrt{3} \\ 2 \end{pmatrix}}{\sqrt{7}\sqrt{28}} = \frac{-9+2}{14} = \frac{-1}{2}$ $\Rightarrow \theta = \frac{2\pi}{3}$ $\text{OR: } \cos \theta = \frac{28+7-49}{2\sqrt{28}\sqrt{7}} = -\frac{1}{2} \Rightarrow \theta = \frac{2\pi}{3}$
	Prove the given statement	A1	Given answer so check working carefully
		3	

Question	Answer	Marks	Guidance
8(i)	Use correct product or quotient rule	M1	$\frac{dy}{dx} = -\frac{1}{3}(x+1)e^{-\frac{1}{3}x} + e^{-\frac{1}{3}x}$ or $\frac{dy}{dx} = \frac{e^{\frac{1}{3}x} - (x+1)\frac{1}{3}e^{\frac{1}{3}x}}{e^{\frac{2}{3}x}}$
	Obtain complete correct derivative in any form	A1	
	Equate derivative to zero and solve for x	M1	
	Obtain answer $x = 2$ with no errors seen	A1	
		4	
8(ii)	Integrate by parts and reach $a(x+1)e^{-\frac{1}{3}x} + b\int e^{-\frac{1}{3}x} dx$	M1*	
	Obtain $-3(x+1)e^{-\frac{1}{3}x} + 3\int e^{-\frac{1}{3}x} dx$, or equivalent	A1	$-3xe^{-\frac{1}{3}x} + \int 3e^{-\frac{1}{3}x} dx - 3e^{-\frac{1}{3}x}$
	Complete integration and obtain $-3(x+1)e^{-\frac{1}{3}x} - 9e^{-\frac{1}{3}x}$, or equivalent	A1	
	Use correct limits $x = -1$ and $x = 0$ in the correct order , having integrated twice	M1(dep*)	
	Obtain answer $9e^{\frac{1}{3}} - 12$, or equivalent	A1	
		5	

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Question	Answer	Marks	Guidance
9(i)	Use a correct method to find a constant	M1	
	Obtain one of the values $A = -3$, $B = 1$, $C = 2$	A1	
	Obtain a second value	A1	
	Obtain the third value	A1	
		4	
9(ii)	Use a correct method to find the first two terms of the expansion of $(3-x)^{-1}$, $\left(1-\frac{1}{3}x\right)^{-1}$, $(2+x^2)^{-1}$ or $\left(1+\frac{1}{2}x^2\right)^{-1}$	M1	Symbolic binomial coefficients are not sufficient for the M1.
	Obtain correct unsimplified expansions up to the term in x^3 of each partial fraction	A1Ft + A1Ft	The ft is on A , B and C . $-1\left(1+\frac{x}{3}+\frac{x^2}{9}+\frac{x^3}{27}\dots\right) + \frac{x+2}{2}\left(1-\frac{x^2}{2}\dots\right)$ $-1-\frac{x}{3}-\frac{x^2}{9}-\frac{x^3}{27}+1-\frac{x^2}{2}+\frac{x}{2}-\frac{x^3}{4}$
	Multiply out their expansion, up to the terms in x^3 , by $Bx + C$, where $BC \neq 0$	M1	
	Obtain final answer $\frac{1}{6}x - \frac{11}{18}x^2 - \frac{31}{108}x^3$, or equivalent	A1	
		5	

Question	Answer	Marks	Guidance
10(i)	Equate at least two pairs of components and solve for s or for t	M1	$\begin{cases} s = \frac{-4}{3} \\ t = \frac{-5}{3} \\ -5 \neq \frac{-1}{3} \end{cases} \quad \text{or} \quad \begin{cases} s = -6 \\ t = -11 \\ 7 \neq -7 \end{cases} \quad \text{or} \quad \begin{cases} s = \frac{-2}{5} \\ t = \frac{-13}{5} \\ \frac{6}{5} \neq \frac{-8}{5} \end{cases}$
	Obtain correct answer for s or t , e.g. $s = -6$, $t = -11$	A1	
	Verify that all three equations are not satisfied and the lines fail to intersect	A1	
	State that the lines are not parallel	B1	
		4	
10(ii)	<i>EITHER:</i> Use scalar product to obtain a relevant equation in a , b and c , e.g. $2a + 3b - c = 0$	B1	
	Obtain a second equation, e.g. $a + 2b + c = 0$, and solve for one ratio, e.g. $a : b$	M1	
	Obtain $a : b : c$ and state correct answer, e.g. $5\mathbf{i} - 3\mathbf{j} + \mathbf{k}$, or equivalent	A1	
	<i>OR:</i> Attempt to calculate vector product of relevant vectors, e.g. $(2\mathbf{i} + 3\mathbf{j} - \mathbf{k}) \times (\mathbf{i} + 2\mathbf{j} + \mathbf{k})$	M1	
	Obtain two correct components	A1	
	Obtain correct answer, e.g. $5\mathbf{i} - 3\mathbf{j} + \mathbf{k}$	A1	
		3	

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Question	Answer	Marks	Guidance
10(iii)	<i>EITHER:</i> State position vector or coordinates of the mid-point of a line segment joining points on l and m , e.g. $\frac{3}{2}\mathbf{i} + \mathbf{j} + \frac{5}{2}\mathbf{k}$	B1	<i>OR:</i> Use the result of (ii) to form equations of planes containing l and m B1
	Use the result of (ii) and the mid-point to find d	M1	Use average of distances to find equation of p . M1
	Obtain answer $5x - 3y + z = 7$, or equivalent	A1	Obtain answer $5x - 3y + z = 7$, or equivalent A1
	<i>OR:</i> Using the result of part (ii), form an equation in d by equating perpendicular distances to the plane of a point on l and a point on m	M1	
	State a correct equation, e.g. $\left \frac{14-d}{\sqrt{35}} \right = \left \frac{-d}{\sqrt{35}} \right $	A1	
	Solve for d and obtain answer $5x - 3y + z = 7$, or equivalent	A1	
		3	

MATHEMATICS

9709/33

Paper 3

May/June 2018

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

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This document consists of **9** printed pages.



Cambridge Assessment
International Education

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously ‘correct’ answers or results obtained from incorrect working.
- Note: B2 or A2 means that the candidate can earn 2 or 0.
B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.
- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE Any Equivalent Form (of answer is equally acceptable) / Or Equivalent

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

CAO Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)

CWO Correct Working Only – often written by a ‘fortuitous’ answer

ISW Ignore Subsequent Working

SOI Seen or implied

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become ‘follow through’ marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks
1	Obtain a correct unsimplified version of the x or x^2 term of the expansion of $(4-3x)^{-\frac{1}{2}}$ or $\left(1-\frac{3}{4}x\right)^{-\frac{1}{2}}$	M1
	State correct first term 2	B1
	Obtain the next two terms $\frac{3}{4}x + \frac{27}{64}x^2$	A1 + A1
	Total:	4

Question	Answer	Marks
2	State or imply $u^2 = u + 5$, or equivalent in 5^x	B1
	Solve for u , or 5^x	M1
	Obtain root $\frac{1}{2}(1 + \sqrt{21})$, or decimal in $[2.79, 2.80]$	A1
	Use correct method for finding x from a positive root	M1
	Obtain answer $x = 0.638$ and no other answer	A1
	Total:	5

Question	Answer	Marks
3	Integrate by parts and reach $ax \sin 3x + b \int \sin 3x dx$	M1*
	Obtain $\frac{1}{3}x \sin 3x - \frac{1}{3} \int \sin 3x dx$, or equivalent	A1
	Complete the integration and obtain $\frac{1}{3}x \sin 3x + \frac{1}{9} \cos 3x$, or equivalent	A1
	Substitute limits correctly having integrated twice and obtained $ax \sin 3x + b \cos 3x$	M1(dep*)
	Obtain answer $\frac{1}{18}(\pi - 2)$ OE	A1
	Total:	5

Question	Answer	Marks
4(i)	Use the quotient or product rule	M1
	Obtain correct derivative in any form	A1
	Equate derivative to zero and obtain the given equation	A1
	Total:	3
4(ii)	Sketch a relevant graph, e.g. $y = \ln x$	B1
	Sketch a second relevant graph, e.g. $y = 1 + \frac{3}{x}$, and justify the given statement	B1
	Total:	2
4(iii)	Use iterative formula $x_{n+1} = \frac{3+x}{\ln x_n}$ correctly at least once	M1
	Obtain final answer 4.97	A1
	Show sufficient iterations to 4 d.p. to justify 4.97 to 2 d.p. or show there is a sign change in the interval (4.965, 4.975)	A1
	Total:	3

Question	Answer	Marks
5(i)	Attempt cubic expansion and equate to 1	M1
	Obtain a correct equation	A1
	Use Pythagoras and double angle formula in the expansion	M1
	Obtain the given result correctly	A1
	Total:	4
5(ii)	Use the identity and carry out a method for finding a root	M1
	Obtain answer 20.9°	A1
	Obtain a second answer, e.g. 69.1°	A1FT
	Obtain the remaining answers, e.g. 110.9° and 159.1° , and no others in the given interval	A1FT
	Total:	4

Question	Answer	Marks
6(i)	Carry out relevant method to find A and B such that $\frac{1}{4-y^2} \equiv \frac{A}{2+y} + \frac{B}{2-y}$	M1
	Obtain $A = B = \frac{1}{4}$	A1
	Total:	2
6(ii)	Separate variables correctly and integrate at least one side to obtain one of the terms $a \ln x$, $b \ln(2+y)$ or $c \ln(2-y)$	M1
	Obtain term $\ln x$	B1
	Integrate and obtain terms $\frac{1}{4} \ln(2+y) - \frac{1}{4} \ln(2-y)$	A1FT
	Use $x = 1$ and $y = 1$ to evaluate a constant, or as limits, in a solution containing at least two terms of the form $a \ln x$, $b \ln(2+y)$ and $c \ln(2-y)$	M1
	Obtain a correct solution in any form, e.g. $\ln x = \frac{1}{4} \ln(2+y) - \frac{1}{4} \ln(2-y) - \frac{1}{4} \ln 3$	A1
	Rearrange as $\frac{2(3x^4 - 1)}{(3x^4 + 1)}$, or equivalent	A1
	Total:	6

Question	Answer	Marks
7(i)	State answer $R = \sqrt{5}$	B1
	Use trig formulae to find $\tan \alpha$	M1
	Obtain $\tan \alpha = 2$	A1
	Total:	3
7(ii)	State that the integrand is $3\sec^2(\theta - \alpha)$	B1FT
	State correct indefinite integral $3\tan(\theta - \alpha)$	B1FT
	Substitute limits correctly	M1
	Use $\tan(A \pm B)$ formula	M1
	Obtain the given exact answer correctly	A1
	Total:	5

Question	Answer	Marks
8(i)	State or imply $3y^2 \frac{dy}{dx}$ as derivative of y^3	B1
	State or imply $3y^2 + 6xy \frac{dy}{dx}$ as derivative of $3xy^2$	B1
	Equate derivative of LHS to zero and solve for $\frac{dy}{dx}$	M1
	Obtain the given answer	A1
	Total:	4
8(ii)	Equate denominator to zero and solve for y	M1*
	Obtain $y = 0$ and $x = a$	A1
	Obtain $y = ax$ and substitute in curve equation to find x or to find y	M1(dep*)
	Obtain $x = -a$	A1
	Obtain $y = 2a$	A1
	Total:	5

Question	Answer	Marks
9(a)	Substitute and obtain a correct equation in x and y	B1
	Use $i^2 = -1$ and equate real and imaginary parts	M1
	Obtain two correct equations in x and y , e.g. $3x - y = 1$ and $3y - x = 5$	A1
	Solve and obtain answer $z = 1 + 2(i)$	A1
	Total:	4
9(b)	Show a circle with radius 3	B1
	Show the line $y = 2$ extending in both quadrants	B1
	Shade the correct region	B1
	Carry out a complete method for finding the greatest value of $\arg z$	M1
	Obtain answer 2.41	A1
	Total:	5

Question	Answer	Marks
10(i)	Carry out a correct method for finding a vector equation for AB	M1
	Obtain $\mathbf{r} = 2\mathbf{i} + \mathbf{j} + 3\mathbf{k} + \lambda(2\mathbf{i} - 2\mathbf{k})$, or equivalent	A1
	Equate pair(s) of components AB and l and solve for λ or μ	M1(dep*)
	Obtain correct answer for λ or μ	A1
	Verify that all three component equations are not satisfied	A1
	Total:	5
10(ii)	State or imply a direction vector for AP has components $(2 + t, 5 + 2t, -3 - 2t)$	B1
	State or imply that $\cos 120^\circ$ equals the scalar product of $\overrightarrow{AP}$ and $\overrightarrow{AB}$ divided by the product of their moduli	M1
	Carry out the correct processes for finding the scalar product and the product of the moduli in terms of t , and obtain an equation in terms of t	M1
	Obtain the given equation correctly	A1
	Solve the quadratic and use a root to find a position vector for P	M1
	Obtain position vector $2\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$ from $t = -2$, having rejected the root $t = -\frac{2}{3}$	A1
	Total:	6