

CANDIDATE
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MATHEMATICS

9709/31

Paper 3 Pure Mathematics 3 (P3)

May/June 2017

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name in the spaces at the top of this page.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

DO NOT WRITE IN ANY BARCODES.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 75.

This document consists of **19** printed pages and **1** blank page.



1 Solve the inequality $|2x + 1| < 3|x - 2|$.

[4]

[illegible]

- 2 Expand $\frac{1}{\sqrt[3]{1+6x}}$ in ascending powers of x , up to and including the term in x^3 , simplifying the coefficients. [4]

[illegible]

- 3** It is given that $x = \ln(1 - y) - \ln y$, where $0 < y < 1$.

(i) Show that $y = \frac{e^{-x}}{1 + e^{-x}}$. [2]

[illegible]

(ii) Hence show that $\int_0^1 y \, dx = \ln\left(\frac{2e}{e+1}\right)$. [4]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

4 The parametric equations of a curve are

$$x = \ln \cos \theta, \quad y = 3\theta - \tan \theta,$$

where $0 \leq \theta < \frac{1}{2}\pi$.

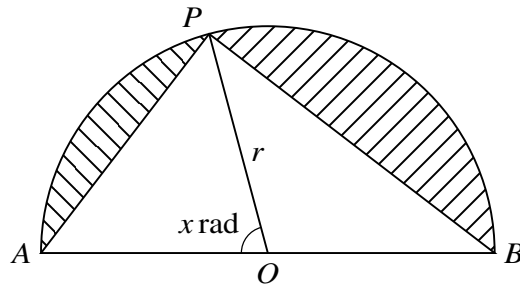
(i) Express $\frac{dy}{dx}$ in terms of $\tan \theta$.

[5]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- (ii) Find the exact y-coordinate of the point on the curve at which the gradient of the normal is equal to 1. [3]

[illegible]



The diagram shows a semicircle with centre O , radius r and diameter AB . The point P on its circumference is such that the area of the minor segment on AP is equal to half the area of the minor segment on BP . The angle AOP is x radians.

- (ii) Show that x satisfies the equation $x = \frac{1}{3}(\pi + \sin x)$. [3]

This image shows a single page of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page. There are no margins, text, or other markings on the paper.

(ii) Verify by calculation that x lies between 1 and 1.5.

[2]

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(iii) Use an iterative formula based on the equation in part (i) to determine x correct to 3 decimal places. Give the result of each iteration to 5 decimal places.

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- 6** The plane with equation $2x + 2y - z = 5$ is denoted by m . Relative to the origin O , the points A and B have coordinates $(3, 4, 0)$ and $(-1, 0, 2)$ respectively.

(i) Show that the plane m bisects AB at right angles.

[5]

[illegible]

(ii) Find the equation of p , giving your answer in the form $ax + by + cz = d$. [3]

[illegible]

(i) Showing all your working, find in the form $x + iy$, where x and y are real, the complex numbers $u - 2w$ and $\frac{u}{w}$. [4]

[illegible]

(ii) Prove that angle $AOB = \frac{1}{4}\pi$. [2]

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(iii) State fully the geometrical relation between the line segments OB and CA . [2]

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- 8** (i) By first expanding $2 \sin(x - 30^\circ)$, express $2 \sin(x - 30^\circ) - \cos x$ in the form $R \sin(x - \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. Give the exact value of R and the value of α correct to 2 decimal places. [5]

[illegible]

(ii) Hence solve the equation

$$2 \sin(x - 30^\circ) - \cos x = 1,$$

for $0^\circ < x < 180^\circ$.

[3]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- 9 (i) Express $\frac{1}{x(2x+3)}$ in partial fractions.

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- (ii) The variables x and y satisfy the differential equation

$$x(2x+3)\frac{dy}{dx} = y,$$

and it is given that $y = 1$ when $x = 1$. Solve the differential equation and calculate the value of y when $x = 9$, giving your answer correct to 3 significant figures. [7]

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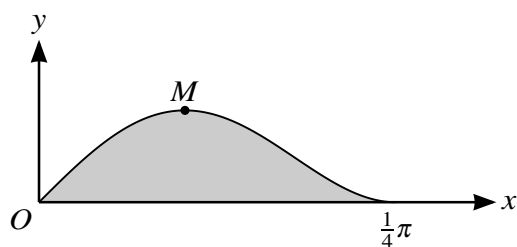
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(i) Using the substitution $u = \cos x$, find by integration the exact area of the shaded region bounded by the curve and the x -axis. [6]

[illegible]

[illegible]

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- 1** Solve the equation $\ln(x^2 + 1) = 1 + 2 \ln x$, giving your answer correct to 3 significant figures. [3]

[illegible]

2 Solve the inequality $|x - 3| < 3x - 4$.

[4]

[illegible]

- 3** (i) Express the equation $\cot \theta - 2 \tan \theta = \sin 2\theta$ in the form $a \cos^4 \theta + b \cos^2 \theta + c = 0$, where a , b and c are constants to be determined. [3]

[illegible]

(ii) Hence solve the equation $\cot \theta - 2 \tan \theta = \sin 2\theta$ for $90^\circ < \theta < 180^\circ$.

[2]

This image shows a full page of a handwriting practice worksheet. It consists of approximately 20 horizontal rows. Each row is defined by two parallel dotted lines, creating a series of uniform gaps for letter height. The entire page is otherwise blank, with no margins, text, or other markings.

4 The parametric equations of a curve are

$$x = t^2 + 1, \quad y = 4t + \ln(2t - 1).$$

(i) Express $\frac{dy}{dx}$ in terms of t .

[3]

This image shows a single page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- (ii) Find the equation of the normal to the curve at the point where $t = 1$. Give your answer in the form $ax + by + c = 0$. [3]

This image shows a full page of primary-ruled paper. It consists of numerous horizontal dashed lines spaced evenly apart, providing a guide for handwriting practice. The lines are black and extend across the entire width of the page. There are no margins, text, or other markings present.

- $x = \frac{10}{(1+t)^2}$. At the beginning of the process $y = 100$.

(i) Form a differential equation in y and t , and solve this differential equation. [6]

[illegible]

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- (ii) Find the exact value approached by the mass of B as t becomes large. State what happens to the mass of A as t becomes large. [2]

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(i) It is given that u is a root of the equation $x^3 + ax^2 - 3x + b = 0$, where the constants a and b are real. Find the values of a and b . [4]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- (ii) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying both the inequalities $|z - u| < 1$ and $|z| < |z + i|$. [4]

- 7 (i) Prove that if $y = \frac{1}{\cos \theta}$ then $\frac{dy}{d\theta} = \sec \theta \tan \theta$. [2]

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- (ii) Prove the identity $\frac{1 + \sin \theta}{1 - \sin \theta} \equiv 2 \sec^2 \theta + 2 \sec \theta \tan \theta - 1$. [3]

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(iii) Hence find the exact value of $\int_0^{\frac{1}{4}\pi} \frac{1 + \sin \theta}{1 - \sin \theta} d\theta$. [4]

8 Let $f(x) = \frac{5x^2 - 7x + 4}{(3x + 2)(x^2 + 5)}$.

(i) Express $f(x)$ in partial fractions.

[5]

[illegible]

- (ii)** Hence obtain the expansion of $f(x)$ in ascending powers of x , up to and including the term in x^2 . [5]

This image shows a full page of a handwriting practice worksheet. It consists of multiple sets of three horizontal dashed lines, providing a guide for letter height and placement. The lines are evenly spaced across the entire page, leaving ample room for writing practice. There is no text or other markings on the page.

- 9 Relative to the origin O , the point A has position vector given by $\overrightarrow{OA} = \mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$. The line l has equation $\mathbf{r} = 9\mathbf{i} - \mathbf{j} + 8\mathbf{k} + \mu(3\mathbf{i} - \mathbf{j} + 2\mathbf{k})$.
- (i) Find the position vector of the foot of the perpendicular from A to l . Hence find the position vector of the reflection of A in l . [5]

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page, typical of notebook or legal stationery. There are no margins, text, or other markings on the page.

- (ii) Find the equation of the plane through the origin which contains l . Give your answer in the form $ax + by + cz = d$. [3]

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- (iii) Find the exact value of the perpendicular distance of A from this plane. [3]

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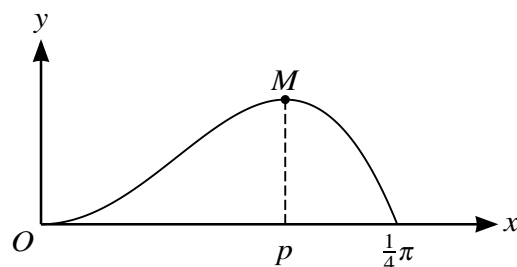
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The diagram shows the curve $y = x^2 \cos 2x$ for $0 \leq x \leq \frac{1}{4}\pi$. The curve has a maximum point at M where $x = p$.

- (i) Show that p satisfies the equation $p = \frac{1}{2} \tan^{-1} \left(\frac{1}{p} \right)$. [3]

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- (ii) Use the iterative formula $p_{n+1} = \frac{1}{2} \tan^{-1} \left(\frac{1}{p_n} \right)$ to determine the value of p correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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- (iii) Find, showing all necessary working, the exact area of the region bounded by the curve and the x -axis. [5]

[illegible]

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1 Prove the identity $\frac{\cot x - \tan x}{\cot x + \tan x} \equiv \cos 2x$.

[3]

[illegible]

- 2** Expand $(3 + 2x)^{-3}$ in ascending powers of x up to and including the term in x^2 , simplifying the coefficients. [4]

[illegible]

- 3 Using the substitution $u = e^x$, solve the equation $4e^{-x} = 3e^x + 4$. Give your answer correct to 3 significant figures. [4]

[illegible]

4 Find the exact value of $\int_0^{\frac{1}{2}\pi} \theta \sin \frac{1}{2}\theta \, d\theta$.

[4]

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- 5** A curve has equation $y = \frac{2}{3} \ln(1 + 3 \cos^2 x)$ for $0 \leq x \leq \frac{1}{2}\pi$.

(i) Express $\frac{dy}{dx}$ in terms of $\tan x$.

[4]

[illegible]

- (ii)** Hence find the x -coordinate of the point on the curve where the gradient is -1 . Give your answer correct to 3 significant figures. [2]

This image shows a full page of a handwriting practice worksheet. It consists of multiple sets of three horizontal dashed lines spaced evenly down the page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.

6 The equation $\cot x = 1 - x$ has one root in the interval $0 < x < \pi$, denoted by α .

(i) Show by calculation that α is greater than 2.5.

[2]

[illegible]

(ii) Show that, if a sequence of values in the interval $0 < x < \pi$ given by the iterative formula $x_{n+1} = \pi + \tan^{-1}\left(\frac{1}{1-x_n}\right)$ converges, then it converges to α . [2]

[illegible]

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- (iii) Use this iterative formula to determine α correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

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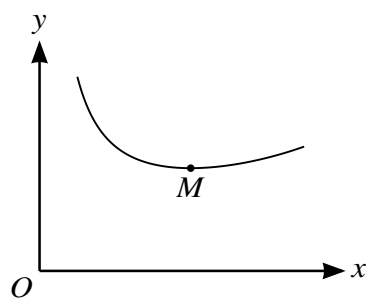
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(i) Find the x -coordinate of M .

[4]

[illegible]

- (ii) Use the trapezium rule with two intervals to estimate the value of

$$\int_1^3 \frac{e^{\frac{1}{2}x}}{x} dx,$$

giving your answer correct to 2 decimal places.

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- (iii) The estimate found in part (ii) is denoted by E . Explain, without further calculation, whether another estimate found using the trapezium rule with four intervals would be greater than E or less than E . [1]

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- 8** In a certain chemical reaction, a compound A is formed from a compound B . The masses of A and B at time t after the start of the reaction are x and y respectively and the sum of the masses is equal to 50 throughout the reaction. At any time the rate of increase of the mass of A is proportional to the mass of B at that time.

- (i) Explain why $\frac{dx}{dt} = k(50 - x)$, where k is a constant. [1]

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It is given that $x = 0$ when $t = 0$, and $x = 25$ when $t = 10$.

- (ii) Solve the differential equation in part (i) and express x in terms of t . [8]

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9 Let $f(x) = \frac{3x^2 - 4}{x^2(3x + 2)}$.

(i) Express $f(x)$ in partial fractions.

[5]

[illegible]

(ii) Hence show that $\int_1^2 f(x) \, dx = \ln\left(\frac{25}{8}\right) - 1$. [5]

This image shows a full page of primary-ruled paper. It features multiple sets of horizontal dashed lines spaced evenly down the page, providing a guide for handwriting practice. The lines are thin and black, set against a plain white background. There are no margins, text, or other markings on the page.

- 10** The points A and B have position vectors given by $\overrightarrow{OA} = \mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$ and $\overrightarrow{OB} = 3\mathbf{i} + \mathbf{j} + \mathbf{k}$. The line l has equation $\mathbf{r} = 2\mathbf{i} + \mathbf{j} + m\mathbf{k} + \mu(\mathbf{i} - 2\mathbf{j} - 4\mathbf{k})$, where m is a constant.

(i) Given that the line l intersects the line passing through A and B , find the value of m . [5]

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- (ii) Find the equation of the plane which is parallel to $\mathbf{i} - 2\mathbf{j} - 4\mathbf{k}$ and contains the points A and B .
Give your answer in the form $ax + by + cz = d$. [5]

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11 Throughout this question the use of a calculator is not permitted.

- (a) The complex numbers z and w satisfy the equations

$$z + (1 + i)w = i \quad \text{and} \quad (1 - i)z + iw = 1.$$

Solve the equations for z and w , giving your answers in the form $x + iy$, where x and y are real.

[6]

This image shows a full page of a worksheet designed for handwriting practice. It features 20 evenly spaced, horizontal dashed lines across the entire page, providing a guide for letter height and placement. The background is plain white, and there are no margins or additional markings.

[illegible]

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