
MATHEMATICS

9709/31

Paper 3

May/June 2017

MARK SCHEME

Maximum Mark: 75

Published

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously “correct” answers or results obtained from incorrect working.
- Note: B2 or A2 means that the candidate can earn 2 or 0.
B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.
- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE Any Equivalent Form (of answer is equally acceptable) / Or Equivalent

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

CAO Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)

CWO Correct Working Only – often written by a ‘fortuitous’ answer

ISW Ignore Subsequent Working

SOI Seen or implied

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks
1	EITHER: State or imply non-modular inequality $(2x+1)^2 < (3(x-2))^2$, or corresponding quadratic equation, or pair of linear equations $(2x+1) = \pm 3(x-2)$	(B1
	Make reasonable solution attempt at a 3-term quadratic e.g. $5x^2 - 40x + 35 = 0$ or solve two linear equations for x	M1
	Obtain critical values $x = 1$ and $x = 7$	A1
	State final answer $x < 1$ and $x > 7$	A1)
	OR: Obtain critical value $x = 7$ from a graphical method, or by inspection, or by solving a linear equation or inequality	(B1
	Obtain critical value $x = 1$ similarly	B2
	State final answer $x < 1$ and $x > 7$	B1)
	Total:	4
2	EITHER: State a correct unsimplified version of the x or x^2 or x^3 term in the expansion of $(1+6x)^{-\frac{1}{3}}$	(M1
	State correct first two terms $1 - 2x$	A1
	Obtain term $8x^2$	A1
	Obtain term $-\frac{112}{3}x^3 \left(37\frac{1}{3}x^3\right)$ in final answer	A1)
	OR: Differentiate expression and evaluate $f(0)$ and $f'(0)$, where $f'(x) = k(1+6x)^{-\frac{4}{3}}$	(M1
	Obtain correct first two terms $1 - 2x$	A1
	Obtain term $8x^2$	A1
	Obtain term $-\frac{112}{3}x^3$ in final answer	A1)
	Total:	4

Question	Answer	Marks
3(i)	Remove logarithms correctly and obtain $e^x = \frac{1-y}{y}$	B1
	Obtain the given answer $y = \frac{e^{-x}}{1+e^{-x}}$ following full working	B1
	Total:	2
3(ii)	State integral $k \ln(1+e^{-x})$ where $k = \pm 1$	*M1
	State correct integral $-\ln(1+e^{-x})$	A1
	Use limits correctly	DM1
	Obtain the given answer $\ln\left(\frac{2e}{e+1}\right)$ following full working	A1
	Total:	4
4(i)	Use chain rule to differentiate $x \left(\frac{dx}{d\theta} = -\frac{\sin \theta}{\cos \theta} \right)$	M1
	State $\frac{dy}{d\theta} = 3 - \sec^2 \theta$	B1
	Use $\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta}$	M1
	Obtain correct $\frac{dy}{dx}$ in any form e.g. $\frac{3 - \sec^2 \theta}{-\tan \theta}$	A1
	Obtain $\frac{dy}{dx} = \frac{\tan^2 \theta - 2}{\tan \theta}$, or equivalent	A1
	Total:	5
4(ii)	Equate gradient to -1 and obtain an equation in $\tan \theta$	M1
	Solve a 3 term quadratic $(\tan^2 \theta + \tan \theta - 2 = 0)$ in $\tan \theta$	M1
	Obtain $\theta = \frac{\pi}{4}$ and $y = \frac{3\pi}{4} - 1$ only	A1
	Total:	3

Question	Answer	Marks
5(i)	Use correct sector formula at least once and form an equation in r and x	M1
	Obtain a correct equation in any form	A1
	Rearrange in the given form	A1
	Total:	3
5(ii)	Calculate values of a relevant expression or expressions at $x = 1$ and $x = 1.5$	M1
	Complete the argument correctly with correct calculated values	A1
	Total:	2
5(iii)	Use the iterative formula correctly at least once	M1
	Obtain final answer 1.374	A1
	Show sufficient iterations to 5 d.p. to justify 1.374 to 3 d.p., or show there is a sign change in the interval (1.3745, 1.3755)	A1
	Total:	3
6(i)	State or obtain coordinates (1, 2, 1) for the mid-point of AB	B1
	Verify that the midpoint lies on m	B1
	State or imply a correct normal vector to the plane, e.g. $2\mathbf{i} + 2\mathbf{j} - \mathbf{k}$	B1
	State or imply a direction vector for the segment AB , e.g. $-4\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}$	B1
	Confirm that m is perpendicular to AB	B1
	Total:	5
6(ii)	State or imply that the perpendicular distance of m from the origin is $\frac{5}{3}$, or unsimplified equivalent	B1
	State or imply that n has an equation of the form $2x + 2y - z = k$	B1
	Obtain answer $2x + 2y - z = 2$	B1
	Total:	3

Question	Answer	Marks
7(i)	State that $u - 2w = -7 - i$	B1
	EITHER: Multiply numerator and denominator of $\frac{u}{w}$ by $3 - 4i$, or equivalent	(M1
	Simplify the numerator to $25 + 25i$ or denominator to 25	A1
	Obtain final answer $1 + i$	A1)
	OR: Obtain two equations in x and y and solve for x or for y	(M1
	Obtain $x = 1$ or $y = 1$	A1
	Obtain final answer $1 + i$	A1)
	Total:	4
7(ii)	Find the argument of $\frac{u}{w}$	M1
	Obtain the given answer	A1
	Total:	2
7(iii)	State that OB and CA are parallel	B1
	State that $CA = 2OB$, or equivalent	B1
	Total:	2
8(i)	Use $\sin(A - B)$ formula and obtain an expression in terms of $\sin x$ and $\cos x$	M1
	Collect terms and reach $\sqrt{3} \sin x - 2 \cos x$, or equivalent	A1
	Obtain $R = \sqrt{7}$	A1
	Use trig formula to find α	M1
	Obtain $\alpha = 49.11^\circ$ with no errors seen	A1
	Total:	5

Question	Answer	Marks
8(ii)	Evaluate $\sin^{-1}(1/\sqrt{7})$ to at least 1 d.p. (22.21° to 2 d.p.)	B1 FT
	Use a correct method to find a value of x in the interval $0^\circ < x < 180^\circ$	M1
	Obtain answer 71.3°	A1
	[ignore answers outside given range.]	
	Total:	3
9(i)	Carry out a relevant method to obtain A and B such that $\frac{1}{x(2x+3)} \equiv \frac{A}{x} + \frac{B}{2x+3}$, or equivalent	M1
	Obtain $A = \frac{1}{3}$ and $B = -\frac{2}{3}$, or equivalent	A1
	Total:	2
9(ii)	Separate variables and integrate one side	B1
	Obtain term $\ln y$	B1
	Integrate and obtain terms $\frac{1}{3} \ln x - \frac{1}{3} \ln(2x+3)$, or equivalent	B2 FT
	Use $x = 1$ and $y = 1$ to evaluate a constant, or as limits, in a solution containing $a \ln y$, $b \ln x$, $c \ln(2x+3)$	M1
	Obtain correct solution in any form, e.g. $\ln y = \frac{1}{3} \ln x - \frac{1}{3} \ln(2x+3) + \frac{1}{3} \ln 5$	A1
	Obtain answer $y = 1.29$ (3s.f. only)	A1
	Total:	7
10(i)	State or imply $du = -\sin x \, dx$	B1
	Using correct double angle formula, express the integral in terms of u and du	M1
	Obtain integrand $\pm(2u^2 - 1)^2$	A1
	Change limits and obtain correct integral $\int_{\frac{1}{\sqrt{2}}}^1 (2u^2 - 1)^2 du$ with no errors seen	A1
	Substitute limits in an integral of the form $au^5 + bu^3 + cu$	M1
	Obtain answer $\frac{1}{15}(7 - 4\sqrt{2})$, or exact simplified equivalent	A1
	Total:	6

Question	Answer	Marks
10(ii)	Use product rule and chain rule at least once	M1
	Obtain correct derivative in any form	A1
	Equate derivative to zero and use trig formulae to obtain an equation in $\cos x$ and $\sin x$	M1
	Use correct methods to obtain an equation in $\cos x$ or $\sin x$ only	M1
	Obtain $10\cos^2 x = 9$ or $10\sin^2 x = 1$, or equivalent	A1
	Obtain answer 0.32	A1
	Total:	6

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Question	Answer	Marks
1	Use law of the logarithm of a power or a quotient	M1
	Remove logarithms and obtain a correct equation in x . e.g. $x^2 + 1 = ex^2$	A1
	Obtain answer 0.763 and no other	A1
	Total:	3
2	<i>EITHER:</i> State or imply non-modular inequality $(x-3)^2 < (3x-4)^2$, or corresponding equation	(B1
	Make reasonable attempt at solving a three term quadratic	M1
	Obtain critical value $x = \frac{7}{4}$	A1
	State final answer $x > \frac{7}{4}$ only	A1)
	<i>OR1:</i> State the relevant critical inequality $3-x < 3x-4$, or corresponding equation	(B1
	Solve for x	M1
	Obtain critical value $x = \frac{7}{4}$	A1
	State final answer $x > \frac{7}{4}$ only	A1)
	<i>OR2:</i> Make recognizable sketches of $y = x-3 $ and $y = 3x-4$ on a single diagram	(B1
	Find x -coordinate of the intersection	M1
	Obtain $x = \frac{7}{4}$	A1
	State final answer $x > \frac{7}{4}$ only	A1)
	Total:	4

Question	Answer	Marks
3(i)	Use correct formulae to express the equation in terms of $\cos \theta$ and $\sin \theta$	M1
	Use Pythagoras and express the equation in terms of $\cos \theta$ only	M1
	Obtain correct 3-term equation, e.g. $2\cos^4 \theta + \cos^2 \theta - 2 = 0$	A1
	Total:	3
3(ii)	Solve a 3-term quadratic in $\cos^2 \theta$ for $\cos \theta$	M1
	Obtain answer $\theta = 152.1^\circ$ only	A1
	Total:	2
4(i)	State $\frac{dy}{dt} = 4 + \frac{2}{2t-1}$	B1
	Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	M1
	Obtain answer $\frac{dy}{dx} = \frac{8t-2}{2t(2t-1)}$, or equivalent e.g. $\frac{2}{t} + \frac{2}{4t^2-2t}$	A1
	Total:	3
4(ii)	Use correct method to find the gradient of the normal at $t = 1$	M1
	Use a correct method to form an equation for the normal at $t = 1$	M1
	Obtain final answer $x + 3y - 14 = 0$, or horizontal equivalent	A1
	Total:	3

Question	Answer	Marks
5(i)	State $\frac{dy}{dt} = -\frac{2y}{(1+t)^2}$, or equivalent	B1
	Separate variables correctly and attempt integration of one side	M1
	Obtain term $\ln y$, or equivalent	A1
	Obtain term $\frac{2}{(1+t)}$, or equivalent	A1
	Use $y = 100$ and $t = 0$ to evaluate a constant, or as limits in an expression containing terms of the form $a \ln y$ and $\frac{b}{1+t}$	M1
	Obtain correct solution in any form, e.g. $\ln y = \frac{2}{1+t} - 2 + \ln 100$	A1
	Total:	6
5(ii)	State that the mass of B approaches $\frac{100}{e^2}$, or exact equivalent	B1
	State or imply that the mass of A tends to zero	B1
	Total:	2

Question	Answer	Marks
6(i)	<i>EITHER:</i> Substitute $x = 2 - i$ (or $x = 2 + i$) in the equation and attempt expansions of x^2 and x^3	(M1
	Equate real and/or imaginary parts to zero	M1
	Obtain $a = -2$	A1
	Obtain $b = 10$	A1)
	<i>OR1:</i> Substitute $x = 2 - i$ in the equation and attempt expansions of x^2 and x^3	(M1
	Substitute $x = 2 + i$ in the equation and add/subtract the two equations	M1
	Obtain $a = -2$	A1
	Obtain $b = 10$	A1)
	<i>OR2:</i> Factorise to obtain $(x - 2 + i)(x - 2 - i)(x - p) \left(= (x^2 - 4x + 5)(x - p) \right)$	(M1
	Compare coefficients	M1
	Obtain $a = -2$	A1
	Obtain $b = 10$	A1)
	<i>OR3:</i> Obtain the quadratic factor $(x^2 - 4x + 5)$	(M1
	Use algebraic division to obtain a real linear factor of the form $x - p$ and set the remainder equal to zero	M1
	Obtain $a = -2$	A1
	Obtain $b = 10$	A1)
	<i>OR4:</i> Use $\alpha\beta = 5$ and $\alpha + \beta = 4$ in $\alpha\beta + \beta\gamma + \gamma\alpha = -3$	(M1
	Solve for γ and use in $\alpha\beta\gamma = -b$ and/or $\alpha + \beta + \gamma = -a$	M1
	Obtain $a = -2$	A1
	Obtain $b = 10$	A1)

Question	Answer	Marks
	<i>OR5:</i> Factorise as $(x - (2-i))(x^2 + ex + g)$ and compare coefficients to form an equation in a and b	(M1
	Equate real and/or imaginary parts to zero	M1
	Obtain $a = -2$	A1
	Obtain $b = 10$	A1)
	Total:	4
6(ii)	Show a circle with centre $2 - i$ in a relatively correct position	B1
	Show a circle with radius 1 and centre not at the origin	B1
	Show the perpendicular bisector of the line segment joining 0 to $-i$	B1
	Shade the correct region	B1
	Total:	4
7(i)	Use quotient or chain rule	M1
	Obtain given answer correctly	A1
	Total:	2
7(ii)	<i>EITHER:</i> Multiply numerator and denominator of LHS by $1 + \sin \theta$	(M1
	Use Pythagoras and express LHS in terms of $\sec \theta$ and $\tan \theta$	M1
	Complete the proof	A1)
	<i>OR1:</i> Express RHS in terms of $\cos \theta$ and $\sin \theta$	(M1
	Use Pythagoras and express RHS in terms of $\sin \theta$	M1
	Complete the proof	A1)
	<i>OR2:</i> Express LHS in terms of $\sec \theta$ and $\tan \theta$	(M1
	Multiply numerator and denominator by $\sec \theta + \tan \theta$ and use Pythagoras	M1
	Complete the proof	A1)
	Total:	3

Question	Answer	Marks
7(iii)	Use the identity and obtain integral $2 \tan \theta + 2 \sec \theta - \theta$	B2
	Use correct limits correctly in an integral containing terms $a \tan \theta$ and $b \sec \theta$	M1
	Obtain answer $2\sqrt{2} - \frac{1}{4}\pi$	A1
	Total:	4
8(i)	State or imply the form $\frac{A}{3x+2} + \frac{Bx+C}{x^2+5}$	B1
	Use a relevant method to determine a constant	M1
	Obtain one of the values $A = 2, B = 1, C = -3$	A1
	Obtain a second value	A1
	Obtain the third value	A1
	Total:	5
8(ii)	Use correct method to find the first two terms of the expansion of $(3x+2)^{-1}, (1+\frac{3}{2}x)^{-1}, (5+x^2)^{-1}$ or $(1+\frac{1}{5}x^2)^{-1}$ [Symbolic coefficients, e.g. $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$ are not sufficient]	M1
	Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction. The FT is on A, B, C from part (i)	A1FT + A1FT
	Multiply out up to the term in x^2 by $Bx + C$, where $BC \neq 0$	M1
	Obtain final answer $\frac{2}{5} - \frac{13}{10}x + \frac{237}{100}x^2$, or equivalent	A1
	Total:	5
9(i)	<i>EITHER:</i> Find $\overrightarrow{AP}$ for a general point P on l with parameter λ , e.g. $(8+3\lambda, -3-\lambda, 4+2\lambda)$	(B1
	Equate scalar product of $\overrightarrow{AP}$ and direction vector of l to zero and solve for λ	M1
	Obtain $\lambda = -\frac{5}{2}$ and foot of perpendicular $\frac{3}{2}\mathbf{i} + \frac{3}{2}\mathbf{j} + 3\mathbf{k}$	A1
	Carry out a complete method for finding the position vector of the reflection of A in l	M1
	Obtain answer $2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$	A1)

Question	Answer	Marks
	<i>OR:</i> Find $\overrightarrow{AP}$ for a general point P on l with parameter λ , e.g. $(8 + 3\lambda, -3 - \lambda, 4 + 2\lambda)$	(B1
	Differentiate $ AP ^2$ and solve for λ at minimum	M1
	Obtain $\lambda = -\frac{5}{2}$ and foot of perpendicular $\frac{3}{2}\mathbf{i} + \frac{3}{2}\mathbf{j} + 3\mathbf{k}$	A1
	Carry out a complete method for finding the position vector of the reflection of A in l	M1
	Obtain answer $2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$	(A1)
	Total:	5
9(ii)	<i>EITHER:</i> Use scalar product to obtain an equation in a , b and c , e.g. $3a - b + 2c = 0$	(B1
	Form a second relevant equation, e.g. $9a - b + 8c = 0$ and solve for one ratio, e.g. $a : b$	M1
	Obtain final answer $a : b : c = 1 : 1 : -1$ and state plane equation $x + y - z = 0$	(A1)
	<i>OR1:</i> Attempt to calculate vector product of two relevant vectors, e.g. $(3\mathbf{i} - \mathbf{j} + 2\mathbf{k}) \times (9\mathbf{i} - \mathbf{j} + 8\mathbf{k})$	(M1
	Obtain two correct components	A1
	Obtain correct answer, e.g. $-6\mathbf{i} - 6\mathbf{j} + 6\mathbf{k}$, and state plane equation $-x - y + z = 0$	(A1)
	<i>OR2:</i> Using a relevant point and relevant vectors, attempt to form a 2-parameter equation for the plane, e.g. $\mathbf{r} = 6\mathbf{i} + 6\mathbf{k} + s(3\mathbf{i} - \mathbf{j} + 2\mathbf{k}) + t(9\mathbf{i} - \mathbf{j} + 8\mathbf{k})$	(M1
	State 3 correct equations in x , y , z , s and t	A1
	Eliminate s and t and state plane equation $x + y - z = 0$, or equivalent	(A1)
	<i>OR3:</i> Using a relevant point and relevant vectors, attempt to form a determinant equation for the plane, e.g. $\begin{vmatrix} x-3 & y-1 & z-4 \\ 3 & -1 & 2 \\ 9 & -1 & 8 \end{vmatrix} = 0$	(M1
	Expand a correct determinant and obtain two correct cofactors	A1
	Obtain answer $-6x - 6y + 6z = 0$, or equivalent	(A1)
	Total:	3

Question	Answer	Marks
9(iii)	<i>EITHER:</i> Using the correct processes, divide the scalar product of $\overrightarrow{OA}$ and a normal to the plane by the modulus of the normal or make a recognisable attempt to apply the perpendicular formula	(M1)
	Obtain a correct expression in any form, e.g. $\frac{1+2-4}{\sqrt{(1^2+1^2+(-1)^2)}}$, or equivalent	A1 FT
	Obtain answer $1/\sqrt{3}$, or exact equivalent	A1)
	<i>OR1:</i> Obtain equation of the parallel plane through A , e.g. $x + y - z = -1$ [The f.t. is on the plane found in part (ii).]	(B1 FT
	Use correct method to find its distance from the origin	M1
	Obtain answer $1/\sqrt{3}$, or exact equivalent	A1)
	<i>OR2:</i> Form equation for the intersection of the perpendicular through A and the plane [FT on their $\mathbf{n}$]	(B1 FT
	Solve for λ	M1
	$ \lambda \mathbf{n} = \frac{1}{\sqrt{3}}$	A1)
	Total:	3
10(i)	Use correct product rule	M1
	Obtain correct derivative in any form $\left(y' = 2x \cos 2x - 2x^2 \sin 2x\right)$	A1
	Equate to zero and derive the given equation	A1
	Total:	3
10(ii)	Use the iterative formula correctly at least once e.g. $0.5 \rightarrow 0.55357 \rightarrow 0.53261 \rightarrow 0.54070 \rightarrow 0.53755$	M1
	Obtain final answer 0.54	A1
	Show sufficient iterations to 4 d.p. to justify 0.54 to 2 d.p., or show there is a sign change in the interval (0.535, 0.545)	A1
	Total:	3

Question	Answer	Marks
10(iii)	Integrate by parts and reach $ax^2 \sin 2x + b \int x \sin 2x \, dx$	*M1
	Obtain $\frac{1}{2}x^2 \sin 2x - \int 2x \cdot \frac{1}{2} \sin 2x \, dx$	A1
	Complete integration and obtain $\frac{1}{2}x^2 \sin 2x + \frac{1}{2}x \cos 2x - \frac{1}{4} \sin 2x$, or equivalent	A1
	Substitute limits $x = 0$, $x = \frac{1}{4}\pi$, having integrated twice	DM1
	Obtain answer $\frac{1}{32}(\pi^2 - 8)$, or exact equivalent	A1
	Total:	5

MATHEMATICS

9709/33

Paper 3

May/June 2017

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2017 series for most Cambridge IGCSE[®], Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously “correct” answers or results obtained from incorrect working.
- Note: B2 or A2 means that the candidate can earn 2 or 0.
B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.
- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE Any Equivalent Form (of answer is equally acceptable) / Or Equivalent

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

CAO Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)

CWO Correct Working Only – often written by a ‘fortuitous’ answer

ISW Ignore Subsequent Working

SOI Seen or implied

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks
1	Express the LHS in terms of either $\cos x$ and $\sin x$ or in terms of $\tan x$	B1
	Use Pythagoras	M1
	Obtain the given answer	A1
	Total:	3
2	<i>EITHER:</i> State a correct unsimplified version of the x or x^2 term in the expansion of $(1 + \frac{2}{3}x)^{-3}$ or $(3 + 2x)^{-3}$ [Symbolic binomial coefficients, e.g. $\binom{-3}{2}$, are not sufficient for M1 .]	(M1
	State correct first term $\frac{1}{27}$	B1
	Obtain term $-\frac{2}{27}x$	A1
	Obtain term $\frac{8}{81}x^2$	A1)
	<i>OR:</i> Differentiate expression and evaluate $f(0)$ and $f'(0)$, where $f'(x) = k(3 + 2x)^{-4}$	(M1
	State correct first term $\frac{1}{27}$	B1
	Obtain term $-\frac{2}{27}x$	A1
	Obtain term $\frac{8}{81}x^2$	A1)
	Total:	4
3	Rearrange as $3u^2 + 4u - 4 = 0$, or $3e^{2x} + 4e^x - 4 = 0$, or equivalent	B1
	Solve a 3-term quadratic for e^x or for u	M1
	Obtain $e^x = \frac{2}{3}$ or $u = \frac{2}{3}$	A1
	Obtain answer $x = -0.405$ and no other	A1
	Total:	4

Question	Answer	Marks
4	Integrate by parts and reach $a\theta \cos \frac{1}{2}\theta + b \int \cos \frac{1}{2}\theta \, d\theta$	*M1
	Complete integration and obtain indefinite integral $-2\theta \cos \frac{1}{2}\theta + 4 \sin \frac{1}{2}\theta$	A1
	Substitute limits correctly, having integrated twice	DM1
	Obtain final answer $(4 - \pi) / \sqrt{2}$, or exact equivalent	A1
	Total:	4
5(i)	Use the chain rule	M1
	Obtain correct derivative in any form	A1
	Use correct trigonometry to express derivative in terms of $\tan x$	M1
	Obtain $\frac{dy}{dx} = -\frac{4 \tan x}{4 + \tan^2 x}$, or equivalent	A1
	Total:	4
5(ii)	Equate derivative to -1 and solve a 3-term quadratic for $\tan x$	M1
	Obtain answer $x=1.11$ and no other in the given interval	A1
	Total:	2
6(i)	Calculate the value of a relevant expression or expressions at $x = 2.5$ and at another relevant value, e.g. $x = 3$	M1
	Complete the argument correctly with correct calculated values	A1
	Total:	2
6(ii)	State a suitable equation, e.g. $x = \pi + \tan^{-1}(1/(1-x))$ without suffices	B1
	Rearrange this as $\cot x = 1 - x$, or commence working <i>vice versa</i>	B1
	Total:	2
6(iii)	Use the iterative formula correctly at least once	M1
	Obtain final answer 2.576 only	A1
	Show sufficient iterations to 5 d.p. to justify 2.576 to 3 d.p., or show there is a sign change in the interval (2.5755, 2.5765)	A1
	Total:	3

Question	Answer	Marks
7(i)	Use correct quotient rule or product rule	M1
	Obtain correct derivative in any form	A1
	Equate derivative to zero and solve for x	M1
	Obtain $x = 2$	A1
	Total:	4
7(ii)	State or imply ordinates 1.6487..., 1.3591..., 1.4938...	B1
	Use correct formula, or equivalent, with $h = 1$ and three ordinates	M1
	Obtain answer 2.93 only	A1
	Total:	3
7(iii)	Explain why the estimate would be less than E	B1
	Total:	1
8(i)	Justify the given differential equation	B1
	Total:	1
8(ii)	Separate variables correctly and attempt to integrate one side	B1
	Obtain term kt , or equivalent	B1
	Obtain term $-\ln(50 - x)$, or equivalent	B1
	Evaluate a constant, or use limits $x = 0$, $t = 0$ in a solution containing terms $a \ln(50 - x)$ and bt	M1*
	Obtain solution $-\ln(50 - x) = kt - \ln 50$, or equivalent	A1
	Use $x = 25$, $t = 10$ to determine k	DM1
	Obtain correct solution in any form, e.g. $\ln 50 - \ln(50 - x) = \frac{1}{10}(\ln 2)t$	A1
	Obtain answer $x = 50(1 - \exp(-0.0693t))$, or equivalent	A1
	Total:	8

Question	Answer	Marks
9(i)	State or imply the form $\frac{A}{x} + \frac{B}{x^2} + \frac{C}{3x+2}$	B1
	Use a relevant method to determine a constant	M1
	Obtain one of the values $A = 3, B = -2, C = -6$	A1
	Obtain a second value	A1
	Obtain the third value [Mark the form $\frac{Ax+B}{x^2} + \frac{C}{3x+2}$ using same pattern of marks.]	A1
	Total:	5
9(ii)	Integrate and obtain terms $3 \ln x = \frac{2}{x} - 2 \ln(3x+2)$ [The FT is on A, B and C] Note: Candidates who integrate the partial fraction $\frac{3x-2}{x^2}$ by parts should obtain $3 \ln x + \frac{2}{x} - 3$ or equivalent	B3 FT
	Use limits correctly, having integrated all the partial fractions, in a solution containing terms $a \ln x + \frac{b}{x} + c \ln(3x+2)$	M1
	Obtain the given answer following full and exact working	A1
	Total:	5
10(i)	Carry out a correct method for finding a vector equation for AB	M1
	Obtain $\mathbf{r} = \mathbf{i} - 2\mathbf{j} + 2\mathbf{k} + \lambda(2\mathbf{i} + 3\mathbf{j} - \mathbf{k})$, or equivalent	A1
	Equate two pairs of components of general points on AB and l and solve for λ or for μ	M1
	Obtain correct answer for λ or μ , e.g. $\lambda = \frac{5}{7}$ or $\mu = \frac{3}{7}$	A1
	Obtain $m = 3$	A1
	Total:	5

Question	Answer	Marks
10(ii)	EITHER: Use scalar product to obtain an equation in a , b and c , e.g. $a - 2b - 4c = 0$	(B1
	Form a second relevant equation, e.g. $2a + 3b - c = 0$ and solve for one ratio, e.g. $a : b$	M1
	Obtain final answer $a : b : c = 14 : -7 : 7$	A1
	Use coordinates of a relevant point and values of a , b and c and find d	M1
	Obtain answer $14x - 7y + 7z = 42$, or equivalent	A1)
	OR 1: Attempt to calculate the vector product of relevant vectors, e.g. $(\mathbf{i} - 2\mathbf{j} - 4\mathbf{k}) \times (2\mathbf{i} + 3\mathbf{j} - \mathbf{k})$	(M1
	Obtain two correct components	A1
	Obtain correct answer, e.g. $14\mathbf{i} - 7\mathbf{j} + 7\mathbf{k}$	A1
	Substitute coordinates of a relevant point in $14x - 7y + 7z = d$, or equivalent, and find d	M1
	Obtain answer $14x - 7y + 7z = 42$, or equivalent	A1)
	OR 2: Using a relevant point and relevant vectors, form a 2-parameter equation for the plane	(M1
	State a correct equation, e.g. $\mathbf{r} = \mathbf{i} - 2\mathbf{j} + 2\mathbf{k} + s(\mathbf{i} - 2\mathbf{j} - 4\mathbf{k}) + t(2\mathbf{i} + 3\mathbf{j} - \mathbf{k})$	A1
	State 3 correct equations in x , y , z , s and t	A1
	Eliminate s and t	M1
	Obtain answer $2x - y + z = 6$, or equivalent	A1)
	OR 3: Using a relevant point and relevant vectors, form a determinant equation for the plane	(M1
	State a correct equation, e.g. $\begin{vmatrix} x-1 & y+2 & z-1 \\ 1 & -2 & -4 \\ 2 & 3 & -1 \end{vmatrix} = 0$	A1
	Attempt to expand the determinant	M1
	Obtain or imply two correct cofactors	A1
	Obtain answer $14x - 7y + 7z = 42$, or equivalent	A1)
	Total:	5

Question	Answer	Marks
11(a)	Solve for z or for w	M1
	Use $i^2 = -1$	M1
	Obtain $w = \frac{i}{2-i}$ or $z = \frac{2+i}{2-i}$	A1
	Multiply numerator and denominator by the conjugate of the denominator	M1
	Obtain $w = -\frac{1}{5} + \frac{2}{5}i$	A1
	Obtain $z = \frac{3}{5} + \frac{4}{5}i$	A1
	Total:	6
11(b)	<i>EITHER:</i> Find $\pm \left[2 + (2 - 2\sqrt{3})i \right]$	(B1
	Multiply by $2i$ (or $-2i$)	M1*
	Add result to v	DM1
	Obtain answer $4\sqrt{3} - 1 + 6i$	A1)
	<i>OR:</i> State $\frac{z-v}{v-u} = ki$, or equivalent	(M1
	State $k = 2$	A1
	Substitute and solve for z even if i omitted	M1
	Obtain answer $4\sqrt{3} - 1 + 6i$	A1)
	Total:	4