
MATHEMATICS

9709/31

Paper 3 Pure Mathematics 3 (P3)

May/June 2016

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
 Graph Paper
 List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.

Write your Centre number, candidate number and name on all the work you hand in.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

DO NOT WRITE IN ANY BARCODES.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 75.

Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **3** printed pages and **1** blank page.

- 1 (i) Solve the equation $2|x - 1| = 3|x|$. [3]
- (ii) Hence solve the equation $2|5^x - 1| = 3|5^x|$, giving your answer correct to 3 significant figures. [2]

- 2 Find the exact value of $\int_0^{\frac{1}{2}} xe^{-2x} dx$. [5]

- 3 By expressing the equation $\operatorname{cosec} \theta = 3 \sin \theta + \cot \theta$ in terms of $\cos \theta$ only, solve the equation for $0^\circ < \theta < 180^\circ$. [5]

- 4 The variables x and y satisfy the differential equation

$$x \frac{dy}{dx} = y(1 - 2x^2),$$

and it is given that $y = 2$ when $x = 1$. Solve the differential equation and obtain an expression for y in terms of x in a form not involving logarithms. [6]

- 5 The curve with equation $y = \sin x \cos 2x$ has one stationary point in the interval $0 < x < \frac{1}{2}\pi$. Find the x -coordinate of this point, giving your answer correct to 3 significant figures. [6]

- 6 (i) By sketching a suitable pair of graphs, show that the equation

$$5e^{-x} = \sqrt{x}$$

has one root. [2]

- (ii) Show that, if a sequence of values given by the iterative formula

$$x_{n+1} = \frac{1}{2} \ln \left(\frac{25}{x_n} \right)$$

converges, then it converges to the root of the equation in part (i). [2]

- (iii) Use this iterative formula, with initial value $x_1 = 1$, to calculate the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

- 7 The equation of a curve is $x^3 - 3x^2y + y^3 = 3$.

- (i) Show that $\frac{dy}{dx} = \frac{x^2 - 2xy}{x^2 - y^2}$. [4]

- (ii) Find the coordinates of the points on the curve where the tangent is parallel to the x -axis. [5]

8 Let $f(x) = \frac{4x^2 + 12}{(x+1)(x-3)^2}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x , up to and including the term in x^2 . [5]

9 With respect to the origin O , the points A, B, C, D have position vectors given by

$$\overrightarrow{OA} = \mathbf{i} + 3\mathbf{j} + 2\mathbf{k}, \quad \overrightarrow{OB} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}, \quad \overrightarrow{OC} = 2\mathbf{i} + 4\mathbf{j} + \mathbf{k}, \quad \overrightarrow{OD} = -3\mathbf{i} + \mathbf{j} + 2\mathbf{k}.$$

(i) Find the equation of the plane containing A, B and C , giving your answer in the form $ax + by + cz = d$. [6]

(ii) The line through D parallel to OA meets the plane with equation $x + 2y - z = 7$ at the point P . Find the position vector of P and show that the length of DP is $2\sqrt{14}$. [5]

10 (a) Showing all your working and without the use of a calculator, find the square roots of the complex number $7 - (6\sqrt{2})i$. Give your answers in the form $x + iy$, where x and y are real and exact. [5]

(b) (i) On an Argand diagram, sketch the loci of points representing complex numbers w and z such that $|w - 1 - 2i| = 1$ and $\arg(z - 1) = \frac{3}{4}\pi$. [4]

(ii) Calculate the least value of $|w - z|$ for points on these loci. [2]

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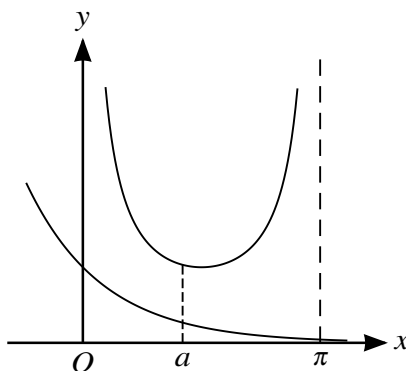
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- 1 Use logarithms to solve the equation $4^{3x-1} = 3(5^x)$, giving your answer correct to 3 decimal places. [4]
- 2 Expand $\frac{1}{\sqrt{1-2x}}$ in ascending powers of x , up to and including the term in x^3 , simplifying the coefficients. [4]
- 3 Find the exact value of $\int_0^{\frac{1}{2}\pi} x^2 \sin 2x \, dx$. [5]
- 4 The curve with equation $y = \frac{(\ln x)^2}{x}$ has two stationary points. Find the exact values of the coordinates of these points. [6]
- 5 (i) Prove the identity $\cos 4\theta - 4 \cos 2\theta \equiv 8 \sin^4 \theta - 3$. [4]
- (ii) Hence solve the equation
- $$\cos 4\theta = 4 \cos 2\theta + 3,$$
- for $0^\circ \leq \theta \leq 360^\circ$. [4]
- 6 The variables x and θ satisfy the differential equation
- $$(3 + \cos 2\theta) \frac{dx}{d\theta} = x \sin 2\theta,$$
- and it is given that $x = 3$ when $\theta = \frac{1}{4}\pi$.
- (i) Solve the differential equation and obtain an expression for x in terms of θ . [7]
- (ii) State the least value taken by x . [1]
- 7 Let $f(x) = \frac{4x^2 + 7x + 4}{(2x + 1)(x + 2)}$.
- (i) Express $f(x)$ in partial fractions. [5]
- (ii) Show that $\int_0^4 f(x) \, dx = 8 - \ln 3$. [5]

8



The diagram shows the curve $y = \operatorname{cosec} x$ for $0 < x < \pi$ and part of the curve $y = e^{-x}$. When $x = a$, the tangents to the curves are parallel.

(i) By differentiating $\frac{1}{\sin x}$, show that if $y = \operatorname{cosec} x$ then $\frac{dy}{dx} = -\operatorname{cosec} x \cot x$. [3]

(ii) By equating the gradients of the curves at $x = a$, show that

$$a = \tan^{-1} \left(\frac{e^a}{\sin a} \right). \quad [2]$$

(iii) Verify by calculation that a lies between 1 and 1.5. [2]

(iv) Use an iterative formula based on the equation in part (ii) to determine a correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

9 The points A , B and C have position vectors, relative to the origin O , given by $\overrightarrow{OA} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$, $\overrightarrow{OB} = 4\mathbf{j} + \mathbf{k}$ and $\overrightarrow{OC} = 2\mathbf{i} + 5\mathbf{j} - \mathbf{k}$. A fourth point D is such that the quadrilateral $ABCD$ is a parallelogram.

(i) Find the position vector of D and verify that the parallelogram is a rhombus. [5]

(ii) The plane p is parallel to OA and the line BC lies in p . Find the equation of p , giving your answer in the form $ax + by + cz = d$. [5]

10 (a) Showing all necessary working, solve the equation $iz^2 + 2z - 3i = 0$, giving your answers in the form $x + iy$, where x and y are real and exact. [5]

(b) (i) On a sketch of an Argand diagram, show the locus representing complex numbers satisfying the equation $|z| = |z - 4 - 3i|$. [2]

(ii) Find the complex number represented by the point on the locus where $|z|$ is least. Find the modulus and argument of this complex number, giving the argument correct to 2 decimal places. [3]

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- 1 Solve the inequality $2|x - 2| > |3x + 1|$. [4]

- 2 The variables x and y satisfy the relation $3^y = 4^{2-x}$.

(i) By taking logarithms, show that the graph of y against x is a straight line. State the exact value of the gradient of this line. [3]

(ii) Calculate the exact x -coordinate of the point of intersection of this line with the line with equation $y = 2x$, simplifying your answer. [2]

- 3 (i) Express $(\sqrt{5}) \cos x + 2 \sin x$ in the form $R \cos(x - \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$, giving the value of α correct to 2 decimal places. [3]

(ii) Hence solve the equation

$$(\sqrt{5}) \cos \frac{1}{2}x + 2 \sin \frac{1}{2}x = 1.2,$$

for $0^\circ < x < 360^\circ$. [3]

- 4 The parametric equations of a curve are

$$x = t + \cos t, \quad y = \ln(1 + \sin t),$$

where $-\frac{1}{2}\pi < t < \frac{1}{2}\pi$.

(i) Show that $\frac{dy}{dx} = \sec t$. [5]

(ii) Hence find the x -coordinates of the points on the curve at which the gradient is equal to 3. Give your answers correct to 3 significant figures. [3]

- 5 The variables x and y satisfy the differential equation

$$\frac{dy}{dx} = e^{-2y} \tan^2 x,$$

for $0 \leq x < \frac{1}{2}\pi$, and it is given that $y = 0$ when $x = 0$. Solve the differential equation and calculate the value of y when $x = \frac{1}{4}\pi$. [8]

- 6 The curve with equation $y = x^2 \cos \frac{1}{2}x$ has a stationary point at $x = p$ in the interval $0 < x < \pi$.

(i) Show that p satisfies the equation $\tan \frac{1}{2}p = \frac{4}{p}$. [3]

(ii) Verify by calculation that p lies between 2 and 2.5. [2]

(iii) Use the iterative formula $p_{n+1} = 2 \tan^{-1}\left(\frac{4}{p_n}\right)$ to determine the value of p correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

7 Let $I = \int_0^1 \frac{x^5}{(1+x^2)^3} dx$.

(i) Using the substitution $u = 1 + x^2$, show that $I = \int_1^2 \frac{(u-1)^2}{2u^3} du$. [3]

(ii) Hence find the exact value of I . [5]

8 The points A and B have position vectors, relative to the origin O , given by $\overrightarrow{OA} = \mathbf{i} + \mathbf{j} + \mathbf{k}$ and $\overrightarrow{OB} = 2\mathbf{i} + 3\mathbf{k}$. The line l has vector equation $\mathbf{r} = 2\mathbf{i} - 2\mathbf{j} - \mathbf{k} + \mu(-\mathbf{i} + 2\mathbf{j} + \mathbf{k})$.

(i) Show that the line passing through A and B does not intersect l . [4]

(ii) Show that the length of the perpendicular from A to l is $\frac{1}{\sqrt{2}}$. [5]

9 Throughout this question the use of a calculator is not permitted.

The complex numbers $-1 + 3i$ and $2 - i$ are denoted by u and v respectively. In an Argand diagram with origin O , the points A , B and C represent the numbers u , v and $u + v$ respectively.

(i) Sketch this diagram and state fully the geometrical relationship between OB and AC . [4]

(ii) Find, in the form $x + iy$, where x and y are real, the complex number $\frac{u}{v}$. [3]

(iii) Prove that angle $AOB = \frac{3}{4}\pi$. [2]

10 Let $f(x) = \frac{10x - 2x^2}{(x+3)(x-1)^2}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x , up to and including the term in x^2 . [5]

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