
MATHEMATICS

9709/31

Paper 3

May/June 2016

MARK SCHEME

Maximum Mark: 75

Published

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Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- The symbol $\Rightarrow$ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously “correct” answers or results obtained from incorrect working.
- Note: B2 or A2 means that the candidate can earn 2 or 0.
B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.
- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

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AEF	Any Equivalent Form (of answer is equally acceptable)
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)
CWO	Correct Working Only – often written by a ‘fortuitous’ answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through ✓” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

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- 1 (i) *EITHER*: State or imply non-modular equation $(2(x-1))^2 = (3x)^2$, or pair of linear equations
 $2(x-1) = \pm 3x$ **B1**
Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations **M1**
Obtain answers $x = -2$ and $x = \frac{2}{5}$ **A1**
- OR*: Obtain answer $x = -2$ by inspection or by solving a linear equation **(B1**
Obtain answer $x = \frac{2}{5}$ similarly **B2)**
[3]
- (ii) Use correct method for solving an equation of the form $5^x = a$ or $5^{x+1} = a$, where $a > 0$ **M1**
Obtain answer $x = -0.569$ only **A1**
[2]
- 2 Integrate by parts and reach $axe^{-2x} + b \int e^{-2x} dx$ **M1**
Obtain $-\frac{1}{2}xe^{-2x} + \frac{1}{2} \int e^{-2x} dx$, or equivalent **A1**
Complete the integration correctly, obtaining $-\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x}$, or equivalent **A1**
Use limits $x = 0$ and $x = \frac{1}{2}$ correctly, having integrated twice **M1**
Obtain answer $\frac{1}{4} - \frac{1}{2}e^{-1}$, or exact equivalent **A1**
[5]
- 3 Correctly restate the equation in terms of $\sin \theta$ and $\cos \theta$ **B1**
Using Pythagoras obtain a horizontal equation in $\cos \theta$ **M1**
Reduce the equation to a correct quadratic in $\cos \theta$, e.g. $3\cos^2 \theta - \cos \theta - 2 = 0$ **A1**
Solve a 3-term quadratic for $\cos \theta$ **M1**
Obtain answer $\theta = 131.8^\circ$ only **A1**
[5]
- [Ignore answers outside the given interval.]
- 4 Separate variables and attempt integration of at least one side **M1***
Obtain term $\ln y$ **A1**
Obtain terms $\ln x - x^2$ **A1**
Use $x = 1$ and $y = 2$ to evaluate a constant, or as limits **DM1***
Obtain correct solution in any form, e.g. $\ln y = \ln x - x^2 + \ln 2 + 1$ **A1**
Obtain correct expression for y , free of logarithms, i.e. $y = 2x \exp(1 - x^2)$ **A1**
[6]

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- 5** Use product rule **M1**
Obtain correct derivative in any form, e.g. $\cos x \cos 2x - 2 \sin x \sin 2x$ **A1**
Equate derivative to zero and use double angle formulae **M1**
Remove factor of $\cos x$ and reduce equation to one in a single trig function **M1**
Obtain $6 \sin^2 x = 1$, $6 \cos^2 x = 5$ or $5 \tan^2 x = 1$ **A1**
Solve and obtain $x = 0.421$ **A1**
[6]
- [Alternative: Use double angle formula M1. Use chain rule to differentiate M1. Obtain correct derivative
e.g. $\cos \theta - 6 \sin^2 \theta \cos \theta$ **A1**, then as above.]
- 6 (i)** Make recognizable sketch of a relevant graph **B1**
Sketch the other relevant graph and justify the given statement **B1**
[2]
- (ii)** State $x = \frac{1}{2} \ln(25/x)$ **B1**
Rearrange this in the form $5e^{-x} = \sqrt{x}$ **B1**
[2]
- (iii)** Use the iterative formula correctly at least once **M1**
Obtain final answer 1.43 **A1**
Show sufficient iterations to 4 d.p. to justify 1.43 to 2 d.p., or show there is a sign change in the interval (1.425, 1.435) **A1**
[3]
- 7 (i)** State or imply $6xy + 3x^2 \frac{dy}{dx}$ as derivative of $3x^2y$ **B1**
State $3y^2 \frac{dy}{dx}$ as derivative of y^3 **B1**
Equate attempted derivative of the LHS to zero and solve for $\frac{dy}{dx}$ **M1**
Obtain the given answer **A1**
[4]
- (ii)** Equate numerator to zero **M1***
Obtain $x = 2y$, or equivalent **A1**
Obtain an equation in x or y **DM1***
Obtain the point $(-2, -1)$ **A1**
State the point $(0, 1.44)$ **B1**
[5]

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- 8 (i) State or imply the form $\frac{A}{x+1} + \frac{B}{x-3} + \frac{C}{(x-3)^2}$ **B1**
- Use a correct method to determine a constant **M1**
- Obtain one of the values $A = 1, B = 3, C = 12$ **A1**
- Obtain a second value **A1**
- Obtain a third value **A1**
- [5]

[Mark the form $\frac{A}{x+1} + \frac{Dx+E}{(x-3)^2}$, where $A=1, D=3, E=3$, B1M1A1A1A1 as above.]

- (ii) Use correct method to find the first two terms of the expansion of $(x+1)^{-1}, (x-3)^{-1}, (1-\frac{1}{3}x)^{-1}$,
- $(x-3)^{-2}$, or $(1-\frac{1}{3}x)^{-2}$ **M1**
- Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction **A1✓ + A1✓ + A1✓**
- Obtain final answer $\frac{4}{3} - \frac{4}{9}x + \frac{4}{3}x^2$, or equivalent **A1**
- [5]

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- 9 (i) *EITHER*: Obtain a vector parallel to the plane, e.g. $\overrightarrow{AB} = \mathbf{i} - 2\mathbf{j} - 3\mathbf{k}$ **B1**
 Use scalar product to obtain an equation in a, b, c e.g. $a - 2b - 3c = 0, a + b - c = 0,$
 or $3b + 2c = 0$ **M1**
 State two correct equations **A1**
 Solve to obtain ratio $a : b : c$ **M1**
 Obtain $a : b : c = 5 : -2 : 3$ **A1**
 Obtain equation $5x - 2y + 3z = 5$, or equivalent **A1**
- OR1*: Substitute for two points, e.g. A and B , and obtain $a + 3b + 2c = d$ and $2a + b - c = d$ **(B1)**
 Substitute for another point, e.g. C , to obtain a third equation and eliminate one unknown entirely from all three equations **M1**
 Obtain two correct equations in three unknowns, e.g. in a, b, c **A1**
 Solve to obtain their ratio **M1**
 Obtain $a : b : c = 5 : -2 : 3, a : c : d = 5 : 3 : 5, a : b : d = 5 : -2 : 5$, or $b : c : d = -2 : 3 : 5$ **A1**
 Obtain equation $5x - 2y + 3z = 5$, or equivalent **A1)**
- OR2*: Obtain a vector parallel to the plane, e.g. $\overrightarrow{AC} = \mathbf{i} + \mathbf{j} - \mathbf{k}$ **(B1)**
 Obtain a second such vector and calculate their vector product, e.g. $(\mathbf{i} - 2\mathbf{j} - 3\mathbf{k}) \times (\mathbf{i} + \mathbf{j} - \mathbf{k})$ **M1**
 Obtain two correct components of the product **A1**
 Obtain correct answer e.g. $5\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$ **A1**
 Substitute in $5x - 2y + 3z = d$ to find d **M1**
 Obtain equation $5x - 2y + 3z = 5$, or equivalent **A1)**
- OR3*: Obtain a vector parallel to the plane, e.g. $\overrightarrow{BC} = 3\mathbf{j} + 2\mathbf{k}$ **(B1)**
 Obtain a second such vector and form correctly a 2-parameter equation for the plane **M1**
 Obtain a correct equation, e.g. $\mathbf{r} = \mathbf{i} + 3\mathbf{j} + 2\mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} - 3\mathbf{k}) + \mu(3\mathbf{j} + 2\mathbf{k})$ **A1**
 State three correct equations in x, y, z, λ, μ **A1**
 Eliminate λ and μ **M1**
 Obtain equation $3x - 2y + 3z = 5$, or equivalent **A1)**
[6]
- (ii) Correctly form an equation for the line through D parallel to OA **M1**
 Obtain a correct equation e.g. $\mathbf{r} = -3\mathbf{i} + \mathbf{j} + 2\mathbf{k} + \lambda(\mathbf{i} + 3\mathbf{j} + 2\mathbf{k})$ **A1**
 Substitute components in the equation of the plane and solve for λ **M1**
 Obtain $\lambda = 2$ and position vector $-\mathbf{i} + 7\mathbf{j} + 6\mathbf{k}$ for P **A1**
 Obtain the given answer correctly **A1**
[5]
- 10 (a) Square $x + iy$ and equate real and imaginary parts to 7 and $-6\sqrt{2}$ respectively **M1**
 Obtain equations $x^2 - y^2 = 7$ and $2xy = -6\sqrt{2}$ **A1**
 Eliminate one variable and find an equation in the other **M1**
 Obtain $x^4 - 7x^2 - 18 = 0$ or $y^4 + 7y^2 - 18 = 0$, or 3-term equivalent **A1**
 Obtain answers $\pm(3 - i\sqrt{2})$ **A1**
[5]

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- (b) (i) Show point representing $1 + 2i$ **B1**
Show circle with radius 1 and centre $1 + 2i$ **B1**
Show a half line from the point representing 1 **B1**
Show line making the correct angle with the real axis **B1**
[4]
- (ii) State or imply the relevance of the perpendicular from $1 + 2i$ to the line **M1**
Obtain answer $\sqrt{2} - 1$ (or 0.414) **A1**
[2]

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- 1 Use law of the logarithm of a product, power or quotient M1*
Obtain a correct linear equation, e.g. $(3x - 1)\ln 4 = \ln 3 + x \ln 5$ A1
Solve a linear equation for x DM1*
Obtain answer $x = 0.975$ A1 [4]
- 2 State a correct un-simplified version of the x or x^2 or x^3 term M1
State correct first two terms $1 + x$ A1
Obtain the next two terms $\frac{3}{2}x^2 + \frac{5}{2}x^3$ A1 A1 [4]
[Symbolic binomial coefficients, e.g. $\binom{-\frac{1}{2}}{3}$ are not sufficient for the M mark.]
- 3 Integrate by parts and reach $ax^2 \cos 2x + b \int x \cos 2x \, dx$ M1*
Obtain $-\frac{1}{2}x^2 \cos 2x + \int x \cos 2x$, or equivalent A1
Complete the integration and obtain $-\frac{1}{2}x^2 \cos 2x + \frac{1}{2}x \sin 2x + \frac{1}{4} \cos 2x$, or equivalent A1
Use limits correctly having integrated twice DM1*
Obtain answer $\frac{1}{8}(\pi^2 - 4)$, or exact equivalent, with no errors seen A1 [5]
- 4 State or imply derivative of $(\ln x)^2$ is $\frac{2 \ln x}{x}$ B1
Use correct quotient or product rule M1
Obtain correct derivative in any form, e.g. $\frac{2 \ln x}{x^2} - \frac{(\ln x)^2}{x^2}$ A1
Equate derivative (or its numerator) to zero and solve for $\ln x$ M1
Obtain the point $(1, 0)$ with no errors seen A1
Obtain the point $(e^2, 4e^{-2})$ A1 [6]
- 5 (i) EITHER: Express $\cos 4\theta$ in terms of $\cos 2\theta$ and/or $\sin 2\theta$ B1
Use correct double angle formulae to express LHS in terms of $\sin \theta$ and/or $\cos \theta$ M1
Obtain a correct expression in terms of $\sin \theta$ alone A1
Reduce correctly to the given form A1
OR: Use correct double angle formula to express RHS in terms of $\cos 2\theta$ M1
Express $\cos^2 2\theta$ in terms of $\cos 4\theta$ B1
Obtain a correct expression in terms of $\cos 4\theta$ and $\cos 2\theta$ A1
Reduce correctly to the given form A1 [4]
- (ii) Use the identity and carry out a method for finding a root M1
Obtain answer 68.5° A1
Obtain a second answer, e.g. 291.5° A1[✓]
Obtain the remaining answers, e.g. 111.5° and 248.5° , and no others in the given interval A1[✓] [4]
[Ignore answers outside the given interval. Treat answers in radians as a misread.]

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- 6 (i) Separate variables correctly and attempt integration of at least one side **B1**
Obtain term $\ln x$ **B1**
Obtain term of the form $k \ln(3 + \cos 2\theta)$, or equivalent **M1**
Obtain term $-\frac{1}{2} \ln(3 + \cos 2\theta)$, or equivalent **A1**
Use $x = 3$, $\theta = \frac{1}{4}\pi$ to evaluate a constant or as limits in a solution
with terms $a \ln x$ and $b \ln(3 + \cos 2\theta)$, where $ab \neq 0$ **M1**
State correct solution in any form, e.g. $\ln x = -\frac{1}{2} \ln(3 + \cos 2\theta) + \frac{3}{2} \ln 3$ **A1**
Rearrange in a correct form, e.g. $x = \sqrt{\left(\frac{27}{3 + \cos 2\theta}\right)}$ **A1** [7]
- (ii) State answer $x = 3\sqrt{3}/2$, or exact equivalent (accept decimal answer in [2.59, 2.60]) **B1** [1]
- 7 (i) State or imply the form $A + \frac{B}{2x+1} + \frac{C}{x+2}$ **B1**
State or obtain $A = 2$ **B1**
Use a correct method for finding a constant **M1**
Obtain one of $B = 1$, $C = -2$ **A1**
Obtain the other value **A1** [5]
- (ii) Integrate and obtain terms $2x + \frac{1}{2} \ln(2x+1) - 2 \ln(x+2)$ **B3[†]**
Substitute correct limits correctly in an integral with terms $a \ln(2x+1)$
and $b \ln(x+2)$, where $ab \neq 0$ **M1**
Obtain the given answer after full and correct working **A1** [5]
- 8 (i) Use correct quotient or chain rule **M1**
Obtain correct derivative in any form **A1**
Obtain the given answer correctly **A1** [3]
- (ii) State a correct equation, e.g. $-e^{-a} = -\operatorname{cosec} a \cot a$ **B1**
Rearrange it correctly in the given form **B1** [2]
- (iii) Calculate values of a relevant expression or pair of expressions at $x = 1$ and $x = 1.5$ **M1**
Complete the argument correctly with correct calculated values **A1** [2]
- (iv) Use the iterative formula correctly at least once **M1**
Obtain final answer 1.317 **A1**
Show sufficient iterations to 5 d.p. to justify 1.317 to 3 d.p., or show there is a sign change in the interval (1.3165, 1.3175) **A1** [3]

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- 9 (i) *Either* state or imply $\overrightarrow{AB}$ or $\overrightarrow{BC}$ in component form, *or* state position vector of midpoint of $\overrightarrow{AC}$ B1
- Use a correct method for finding the position vector of D M1
Obtain answer $3\mathbf{i} + 3\mathbf{j} + \mathbf{k}$, or equivalent A1
- EITHER:* Using the correct process for the moduli, compare lengths of a pair of adjacent sides,
e.g. AB and BC M1
Show that $ABCD$ has a pair of adjacent sides that are equal A1
- OR:* Calculate scalar product $\overrightarrow{AC} \cdot \overrightarrow{BD}$ or equivalent M1
Show that $ABCD$ has perpendicular diagonals A1 [5]
- (ii) *EITHER:* State $a + 2b + 3c = 0$ or $2a + b - 2c = 0$ B1
Obtain two relevant equations and solve for one ratio, e.g. $a : b$ M1
Obtain $a : b : c = -7 : 8 : -3$, or equivalent A1
Substitute coordinates of a relevant point in $-7x + 8y - 3z = d$, and evaluate M1
Obtain answer $-7x + 8y - 3z = 29$, or equivalent A1
- OR1:* Attempt to calculate vector product of relevant vectors,
e.g. $(\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}) \times (2\mathbf{i} + \mathbf{j} - 2\mathbf{k})$ M1
Obtain two correct components of the product A1
Obtain correct product, e.g. $-7\mathbf{i} + 8\mathbf{j} - 3\mathbf{k}$ A1
Substitute coordinates of a relevant point in $-7x + 8y - 3z = d$ and evaluate d M1
Obtain answer $-7x + 8y - 3z = 29$ or equivalent A1
- OR2:* Attempt to form a 2-parameter equation with relevant vectors M1
State a correct equation, e.g. $\mathbf{r} = 2\mathbf{i} + 5\mathbf{j} - \mathbf{k} + \lambda(\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}) + \mu(2\mathbf{i} + \mathbf{j} - 2\mathbf{k})$ A1
State 3 equations in x, y, z, λ and μ A1
Eliminate λ and μ M1
Obtain answer $-7x + 8y - 3z = 29$, or equivalent A1
- OR3:* Using a relevant point and relevant direction vectors, form a determinant equation for the plane M1
- State a correct equation, e.g. $\begin{vmatrix} x-2 & y-5 & z+1 \\ 1 & 2 & 3 \\ 2 & 1 & -2 \end{vmatrix} = 0$ A1
- Attempt to expand the determinant M1
Obtain correct values of two cofactors A1
Obtain answer $-7x + 8y - 3z = 29$, or equivalent A1 [5]

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- 10 (a)** *EITHER*: Use quadratic formula to solve for z **M1**
- Use $i^2 = -1$ **M1**
- Obtain a correct answer in any form, simplified as far as $(-2 \pm i\sqrt{8}) / 2i$ **A1**
- Multiply numerator and denominator by i , or equivalent **M1**
- Obtain final answers $\sqrt{2} + i$ and $-\sqrt{2} + i$ **A1**
- OR*: Substitute $x + iy$ and equate real and imaginary parts to zero **M1**
- Use $i^2 = -1$ **M1**
- Obtain $-2xy + 2x = 0$ and $x^2 - y^2 + 2y - 3 = 0$, or equivalent **A1**
- Solve for x and y **M1**
- Obtain final answers $\sqrt{2} + i$ and $-\sqrt{2} + i$ **A1** [5]
- (b) (i)** *EITHER*: Show the point representing $4 + 3i$ in relatively correct position **B1**
- Show the perpendicular bisector of the line segment joining this point to the origin **B1**[✓] [2]
- OR*: Obtain correct Cartesian equation of the locus in any form, e.g.
 $8x + 6y = 25$ **B1**
Show this line **B1**[✓]
[This f.t. is dependent on using a correct method to determine the equation.]
- (ii)** State or imply the relevant point is represented by $2 + 1.5i$ or is at $(2, 1.5)$ **B1**
- Obtain modulus 2.5 **B1**[✓]
- Obtain argument 0.64 (or 36.9°) (allow decimals in $[0.64, 0.65]$ or $[36.8, 36.9]$) **B1**[✓] [3]

MATHEMATICS

9709/33

Paper 3

May/June 2016

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2016 series for most Cambridge IGCSE[®], Cambridge International A and AS Level components and some Cambridge O Level components.



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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- The symbol ∇ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously “correct” answers or results obtained from incorrect working.
- Note: B2 or A2 means that the candidate can earn 2 or 0.
B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.
- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

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The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)
CWO	Correct Working Only – often written by a “fortuitous” answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through ✓” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

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- 1 *EITHER*: State or imply non-modular inequality $(2(x-2))^2 > (3x+1)^2$, or corresponding quadratic equation, or pair of linear equations $2(x-2) = \pm(3x+1)$ **B1**
 Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations for x **M1**
 Obtain critical values $x = -5$ and $x = \frac{3}{5}$ **A1**
 State final answer $-5 < x < \frac{3}{5}$ **A1**
- OR*: Obtain critical value $x = -5$ from a graphical method, or by inspection, or by solving a linear equation or inequality **(B1)**
 Obtain critical value $x = \frac{3}{5}$ similarly **B2**
 State final answer $-5 < x < \frac{3}{5}$ **(B1)**
 [Do not condone $\leq$ for $<$.] **[4]**
- 2 (i) State or imply $y \ln 3 = (2-x) \ln 4$ **B1**
 State that this is of the form $ay = bx + c$ and thus a straight line, or equivalent **B1**
 State gradient is $-\frac{\ln 4}{\ln 3}$, or exact equivalent **B1**
[3]
- (ii) Substitute $y = 2x$ and solve for x , using a log law correctly at least once **M1**
 Obtain answer $x = \ln 4 / \ln 6$, or exact equivalent **A1**
[2]
- 3 (i) State answer $R = 3$ **B1**
 Use trig formula to find **M1**
 Obtain $\alpha = 41.81^\circ$ with no errors seen **A1**
[3]
- (ii) Evaluate $\cos^{-1}(0.4)$ to at least 1 d.p. (66.42° to 2 d.p.) **B1^{ft}**
 Carry out an appropriate method to find a value of x in the given range **M1**
 Obtain answer 216.5° only **A1**
 [Ignore answers outside the given interval.] **[3]**
- 4 (i) State $\frac{dx}{dt} = 1 - \sin t$ **B1**
 Use chain rule to find the derivative of y **M1**
 Obtain $\frac{dy}{dt} = \frac{\cos t}{1 + \sin t}$, or equivalent **A1**
 Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$ **M1**
 Obtain the given answer correctly **A1**
[5]
- (ii) State or imply $t = \cos^{-1}(\frac{1}{3})$ **B1**
 Obtain answers $x = 1.56$ and $x = -0.898$ **B1 + B1**
[3]

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- 5** Separate variables and make reasonable attempt at integration of either integral **M1**
Obtain term $\frac{1}{2}e^{2y}$ **B1**
Use Pythagoras **M1**
Obtain terms $\tan x - x$ **A1**
Evaluate a constant or use $x = 0, y = 0$ as limits in a solution containing terms
 $ae^{\pm 2y}$ and $b \tan x, (ab \neq 0)$ **M1**
Obtain correct solution in any form, e.g. $\frac{1}{2}e^{2y} = \tan x - x + \frac{1}{2}$ **A1**
Set $x = \frac{1}{4}\pi$ and use correct method to solve an equation of the form $e^{\pm 2y} = a$ or $e^{\pm y} = a$, where
 $a > 0$ **M1**
Obtain answer $y = 0.179$ **A1**
[8]
- 6** (i) Use the product rule **M1**
Obtain correct derivative in any form **A1**
Equate 2-term derivative to zero and obtain the given answer correctly **A1**
[3]
- (ii) Use calculations to consider the sign of a relevant expression at $p = 2$ and $p = 2.5$, or
compare values of relevant expressions at $p = 2$ and $p = 2.5$ **M1**
Complete the argument correctly with correct calculated values **A1**
[2]
- (iii) Use the iterative formula correctly at least once **M1**
Obtain final answer 2.15 **A1**
Show sufficient iterations to 4 d.p. to justify 2.15 to 2 d.p., or show there is a sign change
in the interval (2.145, 2.155) **A1**
[3]
- 7** (i) State or imply $du = 2x \, dx$, or equivalent **B1**
Substitute for x and dx throughout **M1**
Reduce to the given form and justify the change in limits **A1**
[3]
- (ii) Convert integrand to a sum of integrable terms and attempt integration **M1**
Obtain integral $\frac{1}{2} \ln u + \frac{1}{u} - \frac{1}{4u^2}$, or equivalent **A1 + A1**
(deduct A1 for each error or omission)
Substitute limits in an integral containing two terms of the form $a \ln u$ and bu^{-2} **M1**
Obtain answer $\frac{1}{2} \ln 2 - \frac{5}{16}$, exact simplified equivalent **A1**
[5]

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- 8 (i) State a correct equation for AB in any form, e.g. $\mathbf{r} = \mathbf{i} + \mathbf{j} + \mathbf{k} + \lambda(\mathbf{i} - \mathbf{j} + 2\mathbf{k})$, or equivalent **B1**
Equate at least two pairs of components of AB and l and solve for λ or for μ **M1**
Obtain correct answer for λ or for μ , e.g. $\lambda = -1$ or $\mu = 2$ **A1**
Show that not all three equations are not satisfied and that the lines do not intersect **A1**
[4]
- (ii) *EITHER:* Find $\overrightarrow{AP}$ (or $\overrightarrow{PA}$) for a general point P on l , e.g. $(1 - \mu)\mathbf{i} + (-3 + 2\mu)\mathbf{j} + (-2 + \mu)\mathbf{k}$ **B1**
Calculate the scalar product of $\overrightarrow{AP}$ and a direction vector for l and equate to zero **M1**
Solve and obtain $\mu = \frac{3}{2}$ **A1**
Carry out a method to calculate AP when $\mu = \frac{3}{2}$ **M1**
Obtain the given answer $\frac{1}{\sqrt{2}}$ correctly **A1**
- OR 1:* Find $\overrightarrow{AP}$ (or $\overrightarrow{PA}$) for a general point P on l **(B1)**
Use correct method to express AP^2 (or AP) in terms of μ **M1**
Obtain a correct expression in any form, e.g. $(1 - \mu)^2 + (-3 + 2\mu)^2 + (-2 + \mu)^2$ **A1**
- Carry out a complete method for finding its minimum **M1**
Obtain the given answer correctly **A1)**
- OR 2:* Calling $(2, -2, -1)$ C , state $\overrightarrow{AC}$ (or $\overrightarrow{CA}$) in component form, e.g. $\mathbf{i} - 3\mathbf{j} - 2\mathbf{k}$ **(B1)**
Use a scalar product to find the projection of $\overrightarrow{AC}$ (or $\overrightarrow{CA}$) on l **M1**
Obtain correct answer in any form, e.g. $\frac{9}{\sqrt{6}}$ **A1**
Use Pythagoras to find the perpendicular **M1**
Obtain the given answer correctly **A1)**
- OR 3:* State $\overrightarrow{AC}$ (or $\overrightarrow{CA}$) in component form **(B1)**
Calculate vector product of $\overrightarrow{AC}$ and a direction vector for l , e.g. $(\mathbf{i} - 3\mathbf{j} - 2\mathbf{k}) \times (-\mathbf{i} + 2\mathbf{j} + \mathbf{k})$ **M1**
Obtain correct answer in any form, e.g. $\mathbf{i} + \mathbf{j} - \mathbf{k}$ **A1**
Divide modulus of the product by that of the direction vector **M1**
Obtain the given answer correctly **A1)**
[5]
- 9 (i) *EITHER:* Multiply numerator and denominator of $\frac{u}{v}$ by $2 + i$, or equivalent **M1**
Simplify the numerator to $-5 + 5i$ or denominator to 5 **A1**
Obtain final answer $-1 + i$ **A1**
- OR:* Obtain two equations in x and y and solve for x or for y **(M1)**
Obtain $x = -1$ or $y = 1$ **A1**
Obtain final answer $-1 + i$ **A1)**
[3]
- (ii) Obtain $u + v = 1 + 2i$ **B1**
In an Argand diagram show points A, B, C representing u, v and $u + v$ respectively **B1**
State that OB and AC are parallel **B1**
State that $OB = AC$ **B1**
[4]

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- (iii) Carry out an appropriate method for finding angle AOB , e.g. find $\arg(u/v)$ **M1**
 Show sufficient working to justify the given answer $\frac{3}{4}\pi$ **A1**
 [2]

- 10 (i) State or imply the form $\frac{A}{x+3} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$ **B1**
 Use a correct method to determine a constant **M1**
 Obtain one of the values $A = -3, B = 1, C = 2$ **A1**
 Obtain a second value **A1**
 Obtain the third value **A1**
 [Mark the form $\frac{A}{x+3} + \frac{Dx+E}{(x-1)^2}$, where $A = -3, D = 1, E = 1$, B1M1A1A1A1 as above.] **[5]**

- (ii) Use a correct method to find the first two terms of the expansion of $(x+3)^{-1}, (1+\frac{1}{3}x)^{-1},$
 $(x-1)^{-1}, (1-x)^{-1}, (x-1)^{-2},$ or $(1-x)^{-2}$ **M1**
 Obtain correct unsimplified expressions up to the term in x^2 of each partial fraction **A1✓ + A1✓ + A1✓**
 Obtain final answer $\frac{10}{3}x + \frac{44}{9}x^2$, or equivalent **A1**
[5]