

MATHEMATICS

9709/51

Paper 5 Mechanics 2 **(M2)**

May/June 2015

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
 Graph Paper
 List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

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Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

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DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

Where a numerical value for the acceleration due to gravity is needed, use 10 m s^{-2} .

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

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The total number of marks for this paper is 50.

Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of 4 printed pages.

- 1 One end of a light elastic string of natural length 0.7 m is attached to a fixed point A on a smooth horizontal surface. The other end of the string is attached to a particle P of mass 0.3 kg which is held at a point B on the horizontal surface, where $AB = 1.2$ m. It is given that P is released from rest at B and that when $AP = 0.9$ m, the particle has speed 4 m s^{-1} . Calculate the modulus of elasticity of the string. [3]

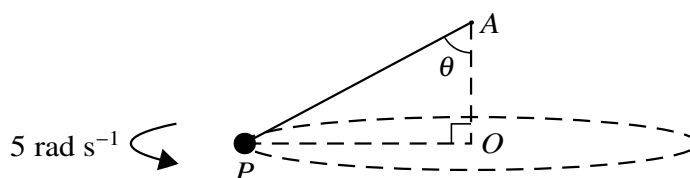
- 2 A stone is projected from a point O on horizontal ground. The equation of the trajectory of the stone is

$$y = 1.2x - 0.15x^2,$$

where x m and y m are respectively the horizontal and vertically upwards displacements of the stone from O . Find

- (i) the greatest height of the stone, [2]
 (ii) the distance from O of the point where the stone strikes the ground. [2]

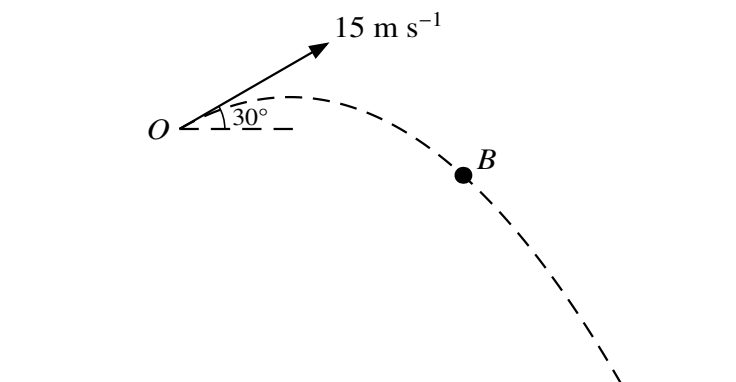
3



One end of a light inextensible string is attached to a fixed point A and the other end of the string is attached to a particle P . The particle P moves with constant angular speed 5 rad s^{-1} in a horizontal circle which has its centre O vertically below A . The string makes an angle θ with the vertical (see diagram). The tension in the string is three times the weight of P .

- (i) Show that the length of the string is 1.2 m. [3]
 (ii) Find the speed of P . [4]

4



A small ball B is projected from a point O above horizontal ground, with initial speed 15 m s^{-1} at an angle of projection of 30° above the horizontal (see diagram). The ball strikes the ground 3 s after projection.

(i) Calculate the speed and direction of motion of the ball immediately before it strikes the ground. [5]

(ii) Find the height of O above the ground. [2]

5 A particle P of mass 0.3 kg is attached to one end of a light elastic string of natural length 0.9 m and modulus of elasticity 18 N . The other end of the string is attached to a fixed point O which is 3 m above the ground.

(i) Find the extension of the string when P is in the equilibrium position. [2]

P is projected vertically downwards from the equilibrium position with initial speed 6 m s^{-1} . At the instant when the tension in the string is 12 N the string breaks. P continues to descend vertically.

(ii) (a) Calculate the height of P above the ground at the instant when the string breaks. [2]

(b) Find the speed of P immediately before it strikes the ground. [4]

6 A particle P of mass 0.1 kg moves with decreasing speed in a straight line on a smooth horizontal surface. A horizontal resisting force of magnitude $0.2e^{-x} \text{ N}$ acts on P , where $x \text{ m}$ is the displacement of P from a fixed point O on the line. The velocity of P is $v \text{ m s}^{-1}$ when its displacement from O is $x \text{ m}$.

(i) Show that

$$v \frac{dv}{dx} = ke^{-x},$$

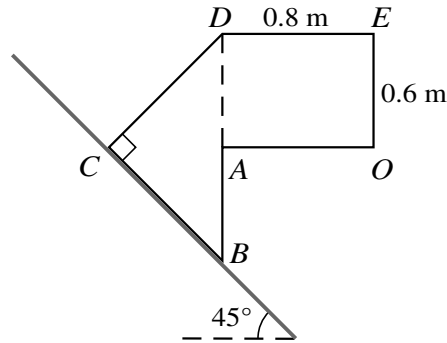
where k is a constant to be found. [2]

P passes through O with velocity 2.2 m s^{-1} .

(ii) Calculate the value of x at the instant when the velocity of P is 2 m s^{-1} . [4]

(iii) Show that the speed of P does not fall below 0.917 m s^{-1} , correct to 3 significant figures. [2]

[Question 7 is printed on the next page.]



The diagram shows the cross-section $OABCDE$ through the centre of mass of a uniform prism on a rough inclined plane. The portion $ADEO$ is a rectangle in which $AD = OE = 0.6$ m and $DE = AO = 0.8$ m; the portion BCD is an isosceles triangle in which angle BCD is a right angle, and A is the mid-point of BD . The plane is inclined at 45° to the horizontal, BC lies along a line of greatest slope of the plane and DE is horizontal.

- (i) Calculate the distance of the centre of mass of the prism from BD . [3]

The weight of the prism is 21 N, and it is held in equilibrium by a horizontal force of magnitude P N acting along ED .

- (ii) (a) Find the smallest value of P for which the prism does not topple. [2]
 (b) It is given that the prism is about to slip for this smallest value of P . Calculate the coefficient of friction between the prism and the plane. [3]

The value of P is gradually increased until the prism ceases to be in equilibrium.

- (iii) Show that the prism topples before it begins to slide, stating the value of P at which equilibrium is broken. [5]

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- 1 A particle P of mass 0.6 kg is on the rough surface of a horizontal disc with centre O . The distance OP is 0.4 m . The disc and P rotate with angular speed 3 rad s^{-1} about a vertical axis which passes through O . Find the magnitude of the frictional force which the disc exerts on the particle, and state the direction of this force. [3]

- 2 One end of a light elastic string of natural length 0.5 m and modulus of elasticity 30 N is attached to a fixed point O . The other end of the string is attached to a particle P which hangs in equilibrium vertically below O , with $OP = 0.8\text{ m}$.

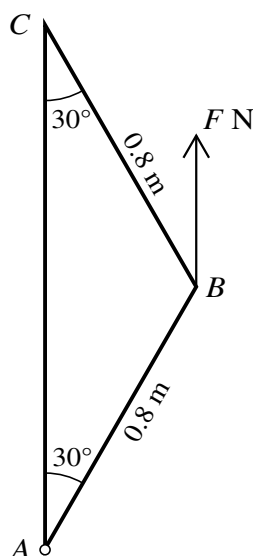
(i) Show that the mass of P is 1.8 kg . [2]

The particle is pulled vertically downwards and released from rest from the point where $OP = 1.2\text{ m}$.

(ii) Find the speed of P at the instant when the string first becomes slack. [3]

- 3 A triangular frame ABC consists of two uniform rigid rods each of length 0.8 m and weight 3 N , and a longer uniform rod of weight 4 N . The triangular frame has $AB = BC$, and angle $BAC = \text{angle } BCA = 30^\circ$.

(i) Calculate the distance of the centre of mass of the frame from AC . [3]



The vertex A of the frame is attached to a smooth hinge at a fixed point. The frame is held in equilibrium with AC vertical by a vertical force of magnitude $F\text{ N}$ applied to the frame at B (see diagram).

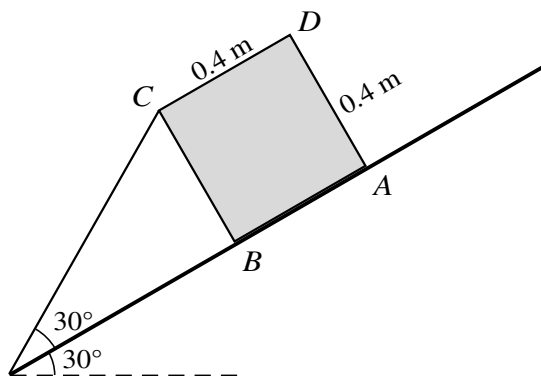
(ii) Calculate F , and state the magnitude and direction of the force acting on the frame at the hinge. [3]

- 4 One end of a light inextensible string of length 0.5 m is attached to a fixed point A . The other end of the string is attached to a particle P of weight 6 N. Another light inextensible string of length 0.5 m connects P to a fixed point B which is 0.8 m vertically below A . The particle P moves with constant speed in a horizontal circle with centre at the mid-point of AB . Both strings are taut.

(i) Calculate the speed of P when the tension in the string BP is 2 N. [5]

(ii) Show that the angular speed of P must exceed 5 rad s^{-1} . [3]

5



A uniform solid cube with edges of length 0.4 m rests in equilibrium on a rough plane inclined at an angle of 30° to the horizontal. $ABCD$ is a cross-section through the centre of mass of the cube, with AB along a line of greatest slope. B lies below the level of A . One end of a light elastic string with modulus of elasticity 12 N and natural length 0.4 m is attached to C . The other end of the string is attached to a point below the level of B on the same line of greatest slope, such that the string makes an angle of 30° with the plane (see diagram). The cube is on the point of toppling. Find

(i) the tension in the string, [3]

(ii) the weight of the cube. [4]

[Questions 6 and 7 are printed on the next page.]

6

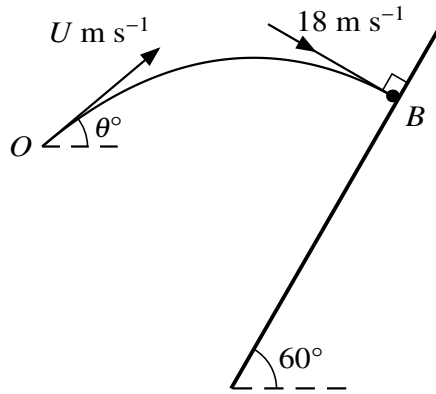


Fig. 1

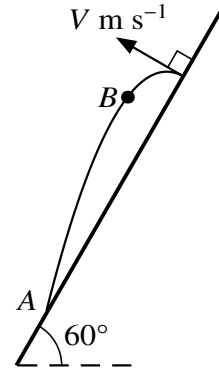


Fig. 2

A small ball B is projected with speed $U \text{ m s}^{-1}$ at an angle of θ° above the horizontal from a point O . At time 2 s after the instant of projection, B strikes a smooth wall which slopes at 60° to the horizontal. The speed of B is 18 m s^{-1} and its direction of motion is perpendicular to the wall at the instant of impact (see Fig. 1). B bounces off the wall with speed $V \text{ m s}^{-1}$ in a direction perpendicular to the wall. At time 0.8 s after B bounces off the wall, B strikes the wall again at a lower point A (see Fig. 2).

(i) Find U and θ . [5]

(ii) By considering the motion of B after it bounces off the wall, calculate V . [4]

- 7 A force of magnitude $0.4t \text{ N}$, applied at an angle of 30° above the horizontal, acts on a particle P , where $t \text{ s}$ is the time since the force starts to act. P is at rest on rough horizontal ground when $t = 0$. The mass of P is 0.2 kg and the coefficient of friction between P and the ground is μ .

(i) Given that P is about to slip when $t = 2$, find μ and the value of t for the instant when P loses contact with the ground. [5]

(ii) While P is moving on the ground, it has velocity $v \text{ m s}^{-1}$ at time $t \text{ s}$. Show that

$$\frac{dv}{dt} = 2.165t - 4.330,$$

where the coefficients are correct to 4 significant figures. [3]

(iii) Calculate the speed of P when it loses contact with the ground. [4]

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- 1** A uniform semicircular lamina has diameter AB of length 0.8 m.

(i) Find the distance of the centre of mass of the lamina from AB . [2]

The lamina rests in a vertical plane, with the point B of the lamina in contact with a rough horizontal surface and with A vertically above B . Equilibrium is maintained by a force of magnitude 6 N in the plane of the lamina, applied to the lamina at A and acting at an angle of 20° below the horizontal.

(ii) Calculate the mass of the lamina. [3]

- 2** A particle P is projected with speed $V \text{ m s}^{-1}$ at an angle of 60° above the horizontal from a point O on horizontal ground. P is moving at an angle of 45° above the horizontal at the instant 1.5 s after projection.

(i) Find V . [3]

(ii) Hence calculate the horizontal and vertical displacements of P from O at the instant 1.5 s after projection. [2]

- 3** One end of a light elastic string of natural length 0.4 m and modulus of elasticity 20 N is attached to a fixed point A on a smooth plane inclined at 30° to the horizontal. The other end of the string is attached to a particle P of mass 0.5 kg which rests in equilibrium on the plane.

(i) Calculate the extension of the string. [2]

P is projected down the plane from the equilibrium position with speed 5 m s^{-1} . The extension of the string is e m when the speed of the particle is 2 m s^{-1} for the first time.

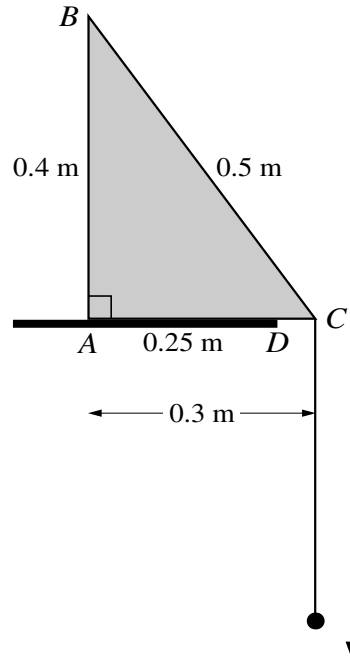
(ii) Find e . [4]

- 4** A small ball B is projected from a point 1.5 m above horizontal ground with initial speed 29 m s^{-1} at an angle of 30° above the horizontal.

(i) Show that B strikes the ground 3 s after projection. [2]

(ii) Find the speed and direction of motion of B immediately before it strikes the ground. [4]

5



A uniform triangular prism of weight 20 N rests on a horizontal table. ABC is the cross-section through the centre of mass of the prism, where $BC = 0.5$ m, $AB = 0.4$ m, $AC = 0.3$ m and angle $BAC = 90^\circ$. The vertical plane ABC is perpendicular to the edge of the table. The point D on AC is at the edge of the table, and $AD = 0.25$ m. One end of a light elastic string of natural length 0.6 m and modulus of elasticity 48 N is attached to C and a particle of mass 2.5 kg is attached to the other end of the string. The particle is released from rest at C and falls vertically (see diagram).

(i) Show that the tension in the string is 60 N at the instant when the prism topples. [3]

(ii) Calculate the speed of the particle at the instant when the prism topples. [5]

- 6 A cyclist and her bicycle have a total mass of 60 kg. The cyclist rides in a horizontal straight line, and exerts a constant force in the direction of motion of 150 N. The motion is opposed by a resistance of magnitude $12v$ N, where v m s⁻¹ is the cyclist's speed at time t s after passing through a fixed point A .

(i) Show that $5 \frac{dv}{dt} = 12.5 - v$. [2]

(ii) Given that the cyclist passes through A with speed 11.5 m s⁻¹, solve this differential equation to show that $v = 12.5 - e^{-0.2t}$. [4]

(iii) Express the displacement of the cyclist from A in terms of t . [3]

- 7 A particle P of mass 0.7 kg is attached to one end of a light inextensible string of length 0.5 m. The other end of the string is attached to a fixed point A which is h m above a smooth horizontal surface. P moves in contact with the surface with uniform circular motion about the point on the surface which is vertically below A .

(i) Given that $h = 0.14$, find an inequality for the angular speed of P . [4]

(ii) Given instead that the magnitude of the force exerted by the surface on P is 1.4 N and that the speed of P is 2.5 m s⁻¹, calculate the tension in the string and the value of h . [7]

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