
MATHEMATICS

9709/31

Paper 3 Pure Mathematics 3 (P3)

May/June 2015

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
 Graph Paper
 List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.

Write your Centre number, candidate number and name on all the work you hand in.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 75.

Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **3** printed pages and **1** blank page.

- 1 Use logarithms to solve the equation $2^{5x} = 3^{2x+1}$, giving the answer correct to 3 significant figures. [4]

- 2 Use the trapezium rule with three intervals to find an approximation to

$$\int_0^3 |3^x - 10| \, dx. \quad [4]$$

- 3 Show that, for small values of x^2 ,

$$(1 - 2x^2)^{-2} - (1 + 6x^2)^{\frac{2}{3}} \approx kx^4,$$

where the value of the constant k is to be determined. [6]

- 4 The equation of a curve is

$$y = 3 \cos 2x + 7 \sin x + 2.$$

Find the x -coordinates of the stationary points in the interval $0 \leq x \leq \pi$. Give each answer correct to 3 significant figures. [7]

- 5 (a) Find $\int (4 + \tan^2 2x) \, dx$. [3]

(b) Find the exact value of $\int_{\frac{1}{4}\pi}^{\frac{1}{2}\pi} \frac{\sin(x + \frac{1}{6}\pi)}{\sin x} \, dx$. [5]

- 6 The straight line l_1 passes through the points $(0, 1, 5)$ and $(2, -2, 1)$. The straight line l_2 has equation $\mathbf{r} = 7\mathbf{i} + \mathbf{j} + \mathbf{k} + \mu(\mathbf{i} + 2\mathbf{j} + 5\mathbf{k})$.

(i) Show that the lines l_1 and l_2 are skew. [6]

(ii) Find the acute angle between the direction of the line l_2 and the direction of the x -axis. [3]

- 7 Given that $y = 1$ when $x = 0$, solve the differential equation

$$\frac{dy}{dx} = 4x(3y^2 + 10y + 3),$$

obtaining an expression for y in terms of x . [9]

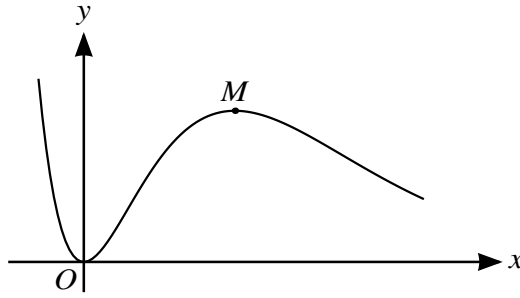
- 8 The complex number w is defined by $w = \frac{22 + 4i}{(2 - i)^2}$.

(i) Without using a calculator, show that $w = 2 + 4i$. [3]

(ii) It is given that p is a real number such that $\frac{1}{4}\pi \leq \arg(w + p) \leq \frac{3}{4}\pi$. Find the set of possible values of p . [3]

(iii) The complex conjugate of w is denoted by w^* . The complex numbers w and w^* are represented in an Argand diagram by the points S and T respectively. Find, in the form $|z - a| = k$, the equation of the circle passing through S , T and the origin. [3]

9



The diagram shows the curve $y = x^2 e^{2-x}$ and its maximum point M .

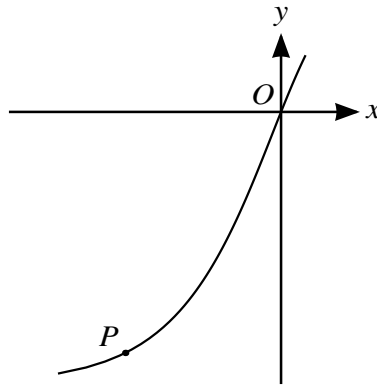
(i) Show that the x -coordinate of M is 2.

[3]

(ii) Find the exact value of $\int_0^2 x^2 e^{2-x} dx$.

[6]

10



The diagram shows part of the curve with parametric equations

$$x = 2 \ln(t + 2), \quad y = t^3 + 2t + 3.$$

(i) Find the gradient of the curve at the origin.

[5]

(ii) At the point P on the curve, the value of the parameter is p . It is given that the gradient of the curve at P is $\frac{1}{2}$.

(a) Show that $p = \frac{1}{3p^2 + 2} - 2$.

[1]

(b) By first using an iterative formula based on the equation in part (a), determine the coordinates of the point P . Give the result of each iteration to 5 decimal places and each coordinate of P correct to 2 decimal places.

[4]

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- 1 Use the trapezium rule with three intervals to estimate the value of

$$\int_0^{\frac{1}{2}\pi} \ln(1 + \sin x) \, dx,$$

giving your answer correct to 2 decimal places. [3]

- 2 Using the substitution $u = 4^x$, solve the equation $4^x + 4^2 = 4^{x+2}$, giving your answer correct to 3 significant figures. [4]

- 3 A curve has equation $y = \cos x \cos 2x$. Find the x -coordinate of the stationary point on the curve in the interval $0 < x < \frac{1}{2}\pi$, giving your answer correct to 3 significant figures. [6]

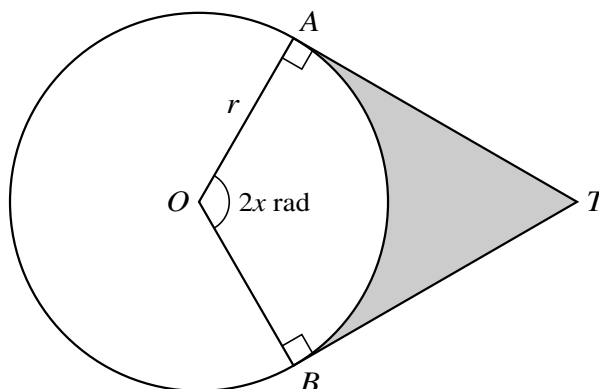
- 4 (i) Express $3 \sin \theta + 2 \cos \theta$ in the form $R \sin(\theta + \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$, stating the exact value of R and giving the value of α correct to 2 decimal places. [3]

(ii) Hence solve the equation

$$3 \sin \theta + 2 \cos \theta = 1,$$

for $0^\circ < \theta < 180^\circ$. [3]

5



The diagram shows a circle with centre O and radius r . The tangents to the circle at the points A and B meet at T , and the angle AOB is $2x$ radians. The shaded region is bounded by the tangents AT and BT , and by the minor arc AB . The perimeter of the shaded region is equal to the circumference of the circle.

- (i) Show that x satisfies the equation

$$\tan x = \pi - x. \quad [3]$$

- (ii) This equation has one root in the interval $0 < x < \frac{1}{2}\pi$. Verify by calculation that this root lies between 1 and 1.3. [2]

- (iii) Use the iterative formula

$$x_{n+1} = \tan^{-1}(\pi - x_n)$$

to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

6 Let $I = \int_0^1 \frac{\sqrt{x}}{2 - \sqrt{x}} dx$.

(i) Using the substitution $u = 2 - \sqrt{x}$, show that $I = \int_1^2 \frac{2(2-u)^2}{u} du$. [4]

(ii) Hence show that $I = 8 \ln 2 - 5$. [4]

7 The complex number u is given by $u = -1 + (4\sqrt{3})i$.

(i) Without using a calculator and showing all your working, find the two square roots of u . Give your answers in the form $a + ib$, where the real numbers a and b are exact. [5]

(ii) On an Argand diagram, sketch the locus of points representing complex numbers z satisfying the relation $|z - u| = 1$. Determine the greatest value of $\arg z$ for points on this locus. [4]

8 Let $f(x) = \frac{5x^2 + x + 6}{(3 - 2x)(x^2 + 4)}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x , up to and including the term in x^2 . [5]

9 The number of organisms in a population at time t is denoted by x . Treating x as a continuous variable, the differential equation satisfied by x and t is

$$\frac{dx}{dt} = \frac{xe^{-t}}{k + e^{-t}},$$

where k is a positive constant.

(i) Given that $x = 10$ when $t = 0$, solve the differential equation, obtaining a relation between x , k and t . [6]

(ii) Given also that $x = 20$ when $t = 1$, show that $k = 1 - \frac{2}{e}$. [2]

(iii) Show that the number of organisms never reaches 48, however large t becomes. [2]

10 The points A and B have position vectors given by $\overrightarrow{OA} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$ and $\overrightarrow{OB} = \mathbf{i} + \mathbf{j} + 5\mathbf{k}$. The line l has equation $\mathbf{r} = \mathbf{i} + \mathbf{j} + 2\mathbf{k} + \mu(3\mathbf{i} + \mathbf{j} - \mathbf{k})$.

(i) Show that l does not intersect the line passing through A and B . [5]

(ii) Find the equation of the plane containing the line l and the point A . Give your answer in the form $ax + by + cz = d$. [6]

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- 1 Solve the equation $\ln(x + 4) = 2 \ln x + \ln 4$, giving your answer correct to 3 significant figures. [4]
- 2 Solve the inequality $|x - 2| > 2x - 3$. [4]
- 3 Solve the equation $\cot 2x + \cot x = 3$ for $0^\circ < x < 180^\circ$. [6]

- 4 The curve with equation $y = \frac{e^{2x}}{4 + e^{3x}}$ has one stationary point. Find the exact values of the coordinates of this point. [6]

- 5 The parametric equations of a curve are

$$x = a \cos^4 t, \quad y = a \sin^4 t,$$

where a is a positive constant.

- (i) Express $\frac{dy}{dx}$ in terms of t . [3]

- (ii) Show that the equation of the tangent to the curve at the point with parameter t is

$$x \sin^2 t + y \cos^2 t = a \sin^2 t \cos^2 t. \quad [3]$$

- (iii) Hence show that if the tangent meets the x -axis at P and the y -axis at Q , then

$$OP + OQ = a,$$

where O is the origin. [2]

- 6 It is given that $\int_0^a x \cos x \, dx = 0.5$, where $0 < a < \frac{1}{2}\pi$.

- (i) Show that a satisfies the equation $\sin a = \frac{1.5 - \cos a}{a}$. [4]

- (ii) Verify by calculation that a is greater than 1. [2]

- (iii) Use the iterative formula

$$a_{n+1} = \sin^{-1} \left(\frac{1.5 - \cos a_n}{a_n} \right)$$

to determine the value of a correct to 4 decimal places, giving the result of each iteration to 6 decimal places. [3]

- 7 The number of micro-organisms in a population at time t is denoted by M . At any time the variation in M is assumed to satisfy the differential equation

$$\frac{dM}{dt} = k(\sqrt{M}) \cos(0.02t),$$

where k is a constant and M is taken to be a continuous variable. It is given that when $t = 0$, $M = 100$.

- (i) Solve the differential equation, obtaining a relation between M , k and t . [5]
- (ii) Given also that $M = 196$ when $t = 50$, find the value of k . [2]
- (iii) Obtain an expression for M in terms of t and find the least possible number of micro-organisms. [2]

- 8 The complex number $1 - i$ is denoted by u .

- (i) Showing your working and without using a calculator, express

$$\frac{i}{u}$$

in the form $x + iy$, where x and y are real. [2]

- (ii) On an Argand diagram, sketch the loci representing complex numbers z satisfying the equations $|z - u| = |z|$ and $|z - i| = 2$. [4]
- (iii) Find the argument of each of the complex numbers represented by the points of intersection of the two loci in part (ii). [3]

- 9 Two planes have equations $x + 3y - 2z = 4$ and $2x + y + 3z = 5$. The planes intersect in the straight line l .

- (i) Calculate the acute angle between the two planes. [4]
- (ii) Find a vector equation for the line l . [6]

10 Let $f(x) = \frac{11x + 7}{(2x - 1)(x + 2)^2}$.

- (i) Express $f(x)$ in partial fractions. [5]
- (ii) Show that $\int_1^2 f(x) dx = \frac{1}{4} + \ln\left(\frac{9}{4}\right)$. [5]

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