
MATHEMATICS

9709/71

Paper 7 Probability & Statistics 2 (S2)

May/June 2014

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
 Graph Paper
 List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

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Answer **all** the questions.

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- 1 The masses, in grams, of apples of a certain type are normally distributed with mean 60.4 and standard deviation 8.2. The apples are packed in bags, with each bag containing 8 randomly chosen apples. The bags are checked by Quality Control and any bag containing apples with a total mass of less than 436 g is rejected. Find the proportion of bags that are rejected. [4]
- 2 A die is biased. The mean and variance of a random sample of 70 scores on this die are found to be 3.61 and 2.70 respectively. Calculate a 95% confidence interval for the population mean score. [5]
- 3 The lengths, in centimetres, of rods produced in a factory have mean μ and standard deviation 0.2. The value of μ is supposed to be 250, but a manager claims that one machine is producing rods that are too long on average. A random sample of 40 rods from this machine is taken and the sample mean length is found to be 250.06 cm. Test at the 5% significance level whether the manager's claim is justified. [5]
- 4 The proportion of people who have a particular gene, on average, is 1 in 1000. A random sample of 3500 people in a certain country is chosen and the number of people, X , having the gene is found.
- (i) State the distribution of X and state also an appropriate approximating distribution. Give the values of any parameters in each case. Justify your choice of the approximating distribution. [3]
- (ii) Use the approximating distribution to find $P(X \leq 3)$. [2]
- 5 The score on one throw of a 4-sided die is denoted by the random variable X with probability distribution as shown in the table.

x	0	1	2	3
$P(X = x)$	0.25	0.25	0.25	0.25

- (i) Show that $\text{Var}(X) = 1.25$. [1]

The die is thrown 300 times. The score on each throw is noted and the mean, $\bar{X}$, of the 300 scores is found.

- (ii) Use a normal distribution to find $P(\bar{X} < 1.4)$. [3]
- (iii) Justify the use of the normal distribution in part (ii). [1]

- 6 Stephan is an athlete who competes in the high jump. In the past, Stephan has succeeded in 90% of jumps at a certain height. He suspects that his standard has recently fallen and he decides to carry out a hypothesis test to find out whether he is right. If he succeeds in fewer than 17 of his next 20 jumps at this height, he will conclude that his standard has fallen.
- (i) Find the probability of a Type I error. [4]
- (ii) In fact Stephan succeeds in 18 of his next 20 jumps. Which of the errors, Type I or Type II, is possible? Explain your answer. [2]

- 7 A random variable X has probability density function given by

$$f(x) = \begin{cases} \frac{k}{x} & 1 \leq x \leq a, \\ 0 & \text{otherwise,} \end{cases}$$

where k and a are positive constants.

(i) Show that $k = \frac{1}{\ln a}$. [3]

(ii) Find $E(X)$ in terms of a . [3]

(iii) Find the median of X in terms of a . [4]

- 8 (i) The following tables show the probability distributions for the random variables V and W .

v	-1	0	1	>1
$P(V = v)$	0.368	0.368	0.184	0.080

w	0	0.5	1	>1
$P(W = w)$	0.368	0.368	0.184	0.080

For each of the variables V and W state how you can tell from its probability distribution that it does NOT have a Poisson distribution. [2]

- (ii) The random variable X has the distribution $Po(\lambda)$. It is given that

$$P(X = 0) = p \quad \text{and} \quad P(X = 1) = 2.5p,$$

where p is a constant.

(a) Show that $\lambda = 2.5$. [1]

(b) Find $P(X \geq 3)$. [2]

- (iii) The random variable Y has the distribution $Po(\mu)$, where $\mu > 30$. Using a suitable approximating distribution, it is found that $P(Y > 40) = 0.5793$ correct to 4 decimal places. Find μ . [5]

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- 1 The weights, in grams, of a random sample of 8 packets of cereal are as follows.

250 248 255 244 259 250 242 258

Calculate unbiased estimates of the population mean and variance. [3]

- 2 Each day Samuel travels from A to B and from B to C . He then returns directly from C to A . The times, in minutes, for these three journeys have the independent distributions $N(20, 2^2)$, $N(18, 1.5^2)$ and $N(30, 1.8^2)$, respectively. Find the probability that, on a randomly chosen day, the total time for his two journeys from A to B and B to C is less than the time for his return journey from C to A . [5]

- 3 The number of calls per day to an enquiry desk has a Poisson distribution. In the past the mean has been 5. In order to test whether the mean has changed, the number of calls on a random sample of 10 days was recorded. The total number of calls was found to be 61. Use an approximate distribution to test at the 10% significance level whether the mean has changed. [5]

- 4 (i) The random variable W has the distribution $Po(1.5)$. Find the probability that the sum of 3 independent values of W is greater than 2. [3]

- (ii) The random variable X has the distribution $Po(\lambda)$. Given that $P(X = 0) = 0.523$, find the value of λ correct to 3 significant figures. [2]

- (iii) The random variable Y has the distribution $Po(\mu)$, where $\mu \neq 0$. Given that

$$P(Y = 3) = 24 \times P(Y = 1),$$

find μ . [3]

- 5 Mahmoud throws a coin 400 times and finds that it shows heads 184 times. The probability that the coin shows heads on any throw is denoted by p .

- (i) Calculate an approximate 95% confidence interval for p . [4]

- (ii) Mahmoud claims that the coin is not fair. Use your answer to part (i) to comment on this claim. [1]

- (iii) Mahmoud's result of 184 heads in 400 throws gives an $\alpha\%$ confidence interval for p with width 0.1. Calculate the value of α . [4]

- 6 The time, T hours, spent by people on a visit to a museum has probability density function

$$f(t) = \begin{cases} kt(16 - t^2) & 0 \leq t \leq 4, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

- (i) Show that $k = \frac{1}{64}$. [3]

- (ii) Calculate the probability that two randomly chosen people each spend less than 1 hour on a visit to the museum. [4]

- (iii) Find the mean time spent on a visit to the museum. [3]

- 7 A researcher is investigating the actual lengths of time that patients spend with the doctor at their appointments. He plans to choose a sample of 12 appointments on a particular day.

(i) Which of the following methods is preferable, and why?

- Choose the first 12 appointments of the day.
- Choose 12 appointments evenly spaced throughout the day. [2]

Appointments are scheduled to last 10 minutes. The actual lengths of time, in minutes, that patients spend with the doctor may be assumed to have a normal distribution with mean μ and standard deviation 3.4. The researcher suspects that the actual time spent is more than 10 minutes on average. To test this suspicion, he recorded the actual times spent for a random sample of 12 appointments and carried out a hypothesis test at the 1% significance level.

(ii) State the probability of making a Type I error and explain what is meant by a Type I error in this context. [2]

(iii) Given that the total length of time spent for the 12 appointments was 147 minutes, carry out the test. [5]

(iv) Give a reason why the Central Limit theorem was not needed in part (iii). [1]

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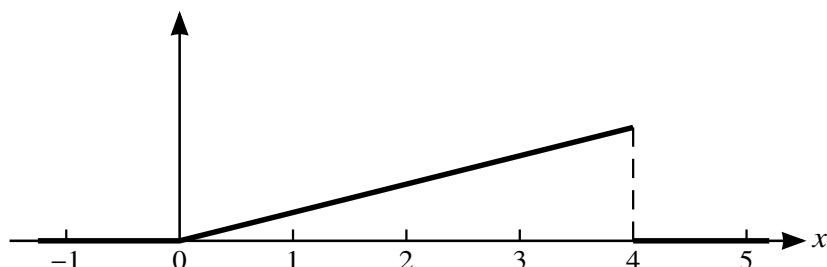
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- 1 On average 1 in 25 000 people have a rare blood condition. Use a suitable approximating distribution to find the probability that fewer than 2 people in a random sample of 100 000 have the condition. [3]

2



A random variable X takes values between 0 and 4 only and has probability density function as shown in the diagram. Calculate the median of X . [4]

- 3 A die is thrown 100 times and shows an odd number on 56 throws. Calculate an approximate 97% confidence interval for the probability that the die shows an odd number on one throw. [4]
- 4 The weights, X kilograms, of rabbits in a certain area have population mean μ kg. A random sample of 100 rabbits from this area was taken and the weights are summarised by

$$\Sigma x = 165, \quad \Sigma x^2 = 276.25.$$

Test at the 5% significance level the null hypothesis $H_0 : \mu = 1.6$ against the alternative hypothesis $H_1 : \mu \neq 1.6$. [6]

- 5 The lifetime, X years, of a certain type of battery has probability density function given by

$$f(x) = \begin{cases} \frac{k}{x^2} & 1 \leq x \leq a, \\ 0 & \text{otherwise,} \end{cases}$$

where k and a are positive constants.

- (i) State what the value of a represents in this context. [1]
- (ii) Show that $k = \frac{a}{a-1}$. [3]
- (iii) Experience has shown that the longest that any battery of this type lasts is 2.5 years. Find the mean lifetime of batteries of this type. [3]

- 6** A machine is designed to generate random digits between 1 and 5 inclusive. Each digit is supposed to appear with the same probability as the others, but Max claims that the digit 5 is appearing less often than it should. In order to test this claim the manufacturer uses the machine to generate 25 digits and finds that exactly 1 of these digits is a 5.
- (i) Carry out a test of Max's claim at the 2.5% significance level. [5]
 - (ii) Max carried out a similar hypothesis test by generating 1000 digits between 1 and 5 inclusive. The digit 5 appeared 180 times. Without carrying out the test, state the distribution that Max should use, including the values of any parameters. [2]
 - (iii) State what is meant by a Type II error in this context. [1]
- 7** A Lost Property office is open 7 days a week. It may be assumed that items are handed in to the office randomly, singly and independently.
- (i) State another condition for the number of items handed in to have a Poisson distribution. [1]
- It is now given that the number of items handed in per week has the distribution $Po(4.0)$.
- (ii) Find the probability that exactly 2 items are handed in on a particular day. [2]
 - (iii) Find the probability that at least 4 items are handed in during a 10-day period. [3]
 - (iv) Find the probability that, during a certain week, 5 items are handed in altogether, but no items are handed in on the first day of the week. [3]
- 8** In an examination, the marks in the theory paper and the marks in the practical paper are denoted by the random variables X and Y respectively, where $X \sim N(57, 13)$ and $Y \sim N(28, 5)$. You may assume that each candidate's marks in the two papers are independent. The final score of each candidate is found by calculating $X + 2.5Y$. A candidate is chosen at random. Without using a continuity correction, find the probability that this candidate
- (i) has a final score that is greater than 140, [5]
 - (ii) obtains at least 20 more marks in the theory paper than in the practical paper. [5]

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