
MATHEMATICS

9709/31

Paper 3 Pure Mathematics 3 (P3)

May/June 2014

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
 Graph Paper
 List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.

Write your Centre number, candidate number and name on all the work you hand in.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 75.

Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **3** printed pages and **1** blank page.

- 1 (i) Simplify $\sin 2\alpha \sec \alpha$. [2]

- (ii) Given that $3 \cos 2\beta + 7 \cos \beta = 0$, find the exact value of $\cos \beta$. [3]

- 2 Use the substitution $u = 1 + 3 \tan x$ to find the exact value of

$$\int_0^{\frac{1}{4}\pi} \frac{\sqrt{(1 + 3 \tan x)}}{\cos^2 x} dx. \quad [5]$$

- 3 The parametric equations of a curve are

$$x = \ln(2t + 3), \quad y = \frac{3t + 2}{2t + 3}.$$

- Find the gradient of the curve at the point where it crosses the y -axis. [6]

- 4 The variables x and y are related by the differential equation

$$\frac{dy}{dx} = \frac{6ye^{3x}}{2 + e^{3x}}.$$

- Given that $y = 36$ when $x = 0$, find an expression for y in terms of x . [6]

- 5 The complex number z is defined by $z = \frac{9\sqrt{3} + 9i}{\sqrt{3} - i}$. Find, showing all your working,

- (i) an expression for z in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leq \pi$, [5]

- (ii) the two square roots of z , giving your answers in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leq \pi$. [3]

- 6 It is given that $2 \ln(4x - 5) + \ln(x + 1) = 3 \ln 3$.

- (i) Show that $16x^3 - 24x^2 - 15x - 2 = 0$. [3]

- (ii) By first using the factor theorem, factorise $16x^3 - 24x^2 - 15x - 2$ completely. [4]

- (iii) Hence solve the equation $2 \ln(4x - 5) + \ln(x + 1) = 3 \ln 3$. [1]

- 7 The straight line l has equation $\mathbf{r} = 4\mathbf{i} - \mathbf{j} + 2\mathbf{k} + \lambda(2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k})$. The plane p passes through the point $(4, -1, 2)$ and is perpendicular to l .

- (i) Find the equation of p , giving your answer in the form $ax + by + cz = d$. [2]

- (ii) Find the perpendicular distance from the origin to p . [3]

- (iii) A second plane q is parallel to p and the perpendicular distance between p and q is 14 units. Find the possible equations of q . [3]

- 8 (i) By sketching each of the graphs $y = \operatorname{cosec} x$ and $y = x(\pi - x)$ for $0 < x < \pi$, show that the equation

$$\operatorname{cosec} x = x(\pi - x)$$

has exactly two real roots in the interval $0 < x < \pi$. [3]

- (ii) Show that the equation $\operatorname{cosec} x = x(\pi - x)$ can be written in the form $x = \frac{1 + x^2 \sin x}{\pi \sin x}$. [2]

- (iii) The two real roots of the equation $\operatorname{cosec} x = x(\pi - x)$ in the interval $0 < x < \pi$ are denoted by α and β , where $\alpha < \beta$.

- (a) Use the iterative formula

$$x_{n+1} = \frac{1 + x_n^2 \sin x_n}{\pi \sin x_n}$$

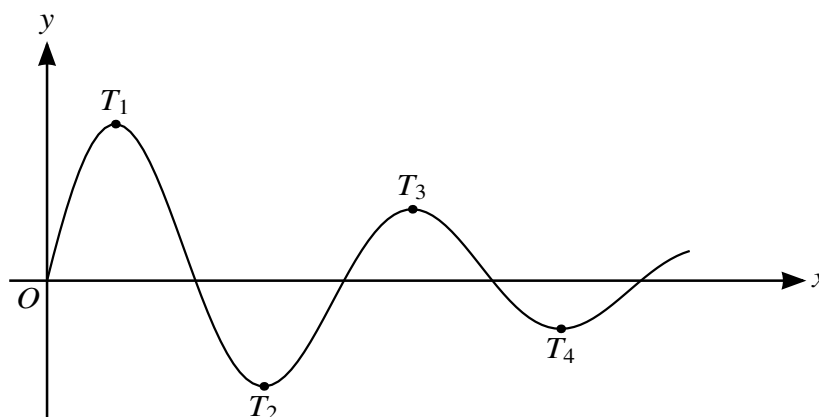
to find α correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

- (b) Deduce the value of β correct to 2 decimal places. [1]

- 9 (i) Express $\frac{4 + 12x + x^2}{(3 - x)(1 + 2x)^2}$ in partial fractions. [5]

- (ii) Hence obtain the expansion of $\frac{4 + 12x + x^2}{(3 - x)(1 + 2x)^2}$ in ascending powers of x , up to and including the term in x^2 . [5]

10



The diagram shows the curve $y = 10e^{-\frac{1}{2}x} \sin 4x$ for $x \geq 0$. The stationary points are labelled $T_1, T_2, T_3, \dots$ as shown.

- (i) Find the x -coordinates of T_1 and T_2 , giving each x -coordinate correct to 3 decimal places. [6]
- (ii) It is given that the x -coordinate of T_n is greater than 25. Find the least possible value of n . [4]

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- 1 Find the set of values of x satisfying the inequality

$$|x + 2a| > 3|x - a|,$$

where a is a positive constant.

[4]

- 2 Solve the equation

$$2 \ln(5 - e^{-2x}) = 1,$$

giving your answer correct to 3 significant figures.

[4]

- 3 Solve the equation

$$\cos(x + 30^\circ) = 2 \cos x,$$

giving all solutions in the interval $-180^\circ < x < 180^\circ$.

[5]

- 4 The parametric equations of a curve are

$$x = t - \tan t, \quad y = \ln(\cos t),$$

for $-\frac{1}{2}\pi < t < \frac{1}{2}\pi$.

(i) Show that $\frac{dy}{dx} = \cot t$.

[5]

- (ii) Hence find the x -coordinate of the point on the curve at which the gradient is equal to 2. Give your answer correct to 3 significant figures.

[2]

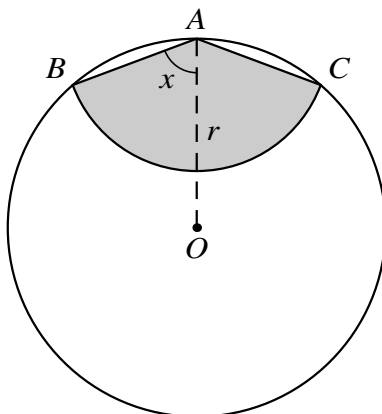
- 5 (i) The polynomial $f(x)$ is of the form $(x - 2)^2 g(x)$, where $g(x)$ is another polynomial. Show that $(x - 2)$ is a factor of $f'(x)$.

[2]

- (ii) The polynomial $x^5 + ax^4 + 3x^3 + bx^2 + a$, where a and b are constants, has a factor $(x - 2)^2$. Using the factor theorem and the result of part (i), or otherwise, find the values of a and b .

[5]

6



In the diagram, A is a point on the circumference of a circle with centre O and radius r . A circular arc with centre A meets the circumference at B and C . The angle OAB is equal to x radians. The shaded region is bounded by AB , AC and the circular arc with centre A joining B and C . The perimeter of the shaded region is equal to half the circumference of the circle.

(i) Show that $x = \cos^{-1} \left(\frac{\pi}{4 + 4x} \right)$. [3]

(ii) Verify by calculation that x lies between 1 and 1.5. [2]

(iii) Use the iterative formula

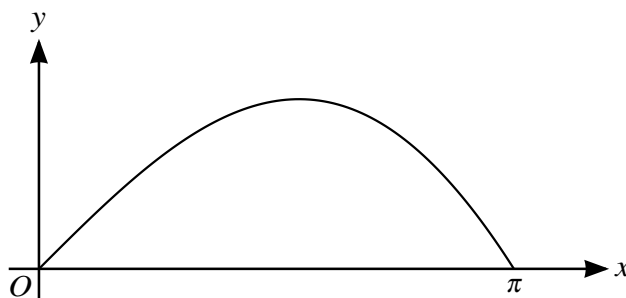
$$x_{n+1} = \cos^{-1} \left(\frac{\pi}{4 + 4x_n} \right)$$

to determine the value of x correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

7 (a) It is given that $-1 + (\sqrt{5})i$ is a root of the equation $z^3 + 2z + a = 0$, where a is real. Showing your working, find the value of a , and write down the other complex root of this equation. [4]

(b) The complex number w has modulus 1 and argument 2θ radians. Show that $\frac{w-1}{w+1} = i \tan \theta$. [4]

8



The diagram shows the curve $y = x \cos \frac{1}{2}x$ for $0 \leq x \leq \pi$.

(i) Find $\frac{dy}{dx}$ and show that $4 \frac{d^2y}{dx^2} + y + 4 \sin \frac{1}{2}x = 0$. [5]

(ii) Find the exact value of the area of the region enclosed by this part of the curve and the x -axis. [5]

- 9 The population of a country at time t years is N millions. At any time, N is assumed to increase at a rate proportional to the product of N and $(1 - 0.01N)$. When $t = 0$, $N = 20$ and $\frac{dN}{dt} = 0.32$.

(i) Treating N and t as continuous variables, show that they satisfy the differential equation

$$\frac{dN}{dt} = 0.02N(1 - 0.01N). \quad [1]$$

(ii) Solve the differential equation, obtaining an expression for t in terms of N . [8]

(iii) Find the time at which the population will be double its value at $t = 0$. [1]

- 10 Referred to the origin O , the points A , B and C have position vectors given by

$$\overrightarrow{OA} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}, \quad \overrightarrow{OB} = 2\mathbf{i} + 4\mathbf{j} + \mathbf{k} \quad \text{and} \quad \overrightarrow{OC} = 3\mathbf{i} + 5\mathbf{j} - 3\mathbf{k}.$$

(i) Find the exact value of the cosine of angle BAC . [4]

(ii) Hence find the exact value of the area of triangle ABC . [3]

(iii) Find the equation of the plane which is parallel to the y -axis and contains the line through B and C . Give your answer in the form $ax + by + cz = d$. [5]

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1 Solve the equation $\log_{10}(x+9) = 2 + \log_{10} x$. [3]

2 Expand $(1+3x)^{-\frac{1}{3}}$ in ascending powers of x , up to and including the term in x^3 , simplifying the coefficients. [4]

3 (i) Show that the equation

$$\tan(x - 60^\circ) + \cot x = \sqrt{3}$$

can be written in the form

$$2 \tan^2 x + (\sqrt{3}) \tan x - 1 = 0. \quad [3]$$

(ii) Hence solve the equation

$$\tan(x - 60^\circ) + \cot x = \sqrt{3},$$

for $0^\circ < x < 180^\circ$. [3]

4 The equation $x = \frac{10}{e^{2x} - 1}$ has one positive real root, denoted by α .

(i) Show that α lies between $x = 1$ and $x = 2$. [2]

(ii) Show that if a sequence of positive values given by the iterative formula

$$x_{n+1} = \frac{1}{2} \ln \left(1 + \frac{10}{x_n} \right)$$

converges, then it converges to α . [2]

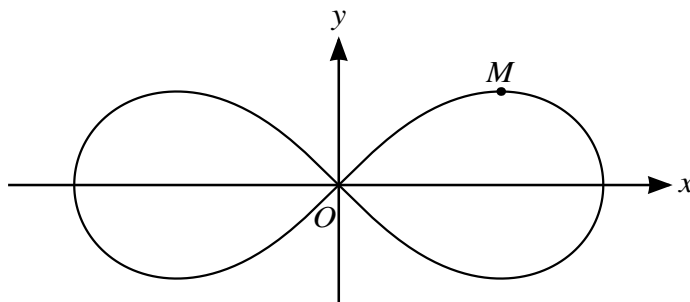
(iii) Use this iterative formula to determine α correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

5 The variables x and θ satisfy the differential equation

$$2 \cos^2 \theta \frac{dx}{d\theta} = \sqrt{(2x+1)},$$

and $x = 0$ when $\theta = \frac{1}{4}\pi$. Solve the differential equation and obtain an expression for x in terms of θ . [7]

6



The diagram shows the curve $(x^2 + y^2)^2 = 2(x^2 - y^2)$ and one of its maximum points M . Find the coordinates of M . [7]

- 7 (a) The complex number $\frac{3 - 5i}{1 + 4i}$ is denoted by u . Showing your working, express u in the form $x + iy$, where x and y are real. [3]

- (b) (i) On a sketch of an Argand diagram, shade the region whose points represent complex numbers satisfying the inequalities $|z - 2 - i| \leq 1$ and $|z - i| \leq |z - 2|$. [4]

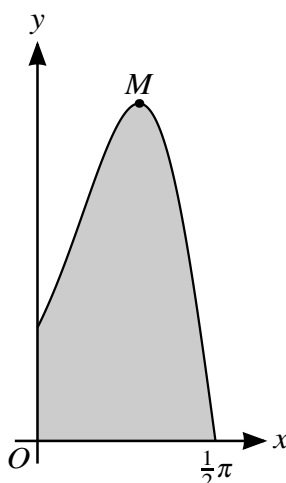
- (ii) Calculate the maximum value of $\arg z$ for points lying in the shaded region. [2]

8 Let $f(x) = \frac{6 + 6x}{(2 - x)(2 + x^2)}$.

- (i) Express $f(x)$ in the form $\frac{A}{2 - x} + \frac{Bx + C}{2 + x^2}$. [4]

- (ii) Show that $\int_{-1}^1 f(x) dx = 3 \ln 3$. [5]

9



The diagram shows the curve $y = e^{2 \sin x} \cos x$ for $0 \leq x \leq \frac{1}{2}\pi$, and its maximum point M .

- (i) Using the substitution $u = \sin x$, find the exact value of the area of the shaded region bounded by the curve and the axes. [5]

- (ii) Find the x -coordinate of M , giving your answer correct to 3 decimal places. [6]

10 The line l has equation $\mathbf{r} = \mathbf{i} + 2\mathbf{j} - \mathbf{k} + \lambda(3\mathbf{i} - 2\mathbf{j} + 2\mathbf{k})$ and the plane p has equation $2x + 3y - 5z = 18$.

(i) Find the position vector of the point of intersection of l and p . [3]

(ii) Find the acute angle between l and p . [4]

(iii) A second plane q is perpendicular to the plane p and contains the line l . Find the equation of q , giving your answer in the form $ax + by + cz = d$. [5]