

CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Level

MARK SCHEME for the May/June 2014 series

9709 MATHEMATICS

9709/31

Paper 3, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2014 series for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level components and some Ordinary Level components.

Page 2	Mark Scheme	Syllabus	Paper
	GCE A LEVEL – May/June 2014	9709	31

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- The symbol ∇ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously “correct” answers or results obtained from incorrect working.
- Note: B2 or A2 means that the candidate can earn 2 or 0.
B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.
- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

Page 3	Mark Scheme	Syllabus	Paper
	GCE A LEVEL – May/June 2014	9709	31

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)
CWO	Correct Working Only – often written by a ‘fortuitous’ answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through ✓” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Page 4	Mark Scheme	Syllabus	Paper
	GCE A LEVEL – May/June 2014	9709	31

- 1 (i) State $\sin 2\alpha = 2\sin\alpha \cos\alpha$ and $\sec\alpha = 1/\cos\alpha$ B1
Obtain $2\sin\alpha$ B1 [2]
- (ii) Use $\cos 2\beta = 2\cos^2\beta - 1$ or equivalent to produce correct equation in $\cos\beta$ B1
Solve three-term quadratic equation for $\cos\beta$ M1
Obtain $\cos\beta = \frac{1}{3}$ only A1 [3]
- 2 State $\frac{du}{dx} = 3\sec^2 x$ or equivalent B1
Express integral in terms of u and du (accept unsimplified and without limits) M1
Obtain $\int \frac{1}{3} u^{\frac{1}{2}} du$ A1
Integrate $Cu^{\frac{1}{2}}$ to obtain $\frac{2C}{3} u^{\frac{3}{2}}$ M1
Obtain $\frac{14}{9}$ A1 [5]
- 3 Obtain $\frac{2}{2t+3}$ for derivative of x B1
Use quotient of product rule, or equivalent, for derivative of y M1
Obtain $\frac{5}{(2t+3)^2}$ or unsimplified equivalent A1
Obtain $t = -1$ B1
Use $\frac{dy}{dx} = \frac{dy}{dt} / \frac{dx}{dt}$ in algebraic or numerical form M1
Obtain gradient $\frac{5}{2}$ A1 [6]
- 4 Separate variables correctly and recognisable attempt at integration of at least one side M1
Obtain $\ln y$, or equivalent B1
Obtain $k \ln(2 + e^{3x})$ B1
Use $y(0) = 36$ to find constant in $y = A(2 + e^{3x})^k$ or $\ln y = k \ln(2 + e^{3x}) + c$ or equivalent M1*
Obtain equation correctly without logarithms from $\ln y = \ln(A(2 + e^{3x})^k)$ *M1
Obtain $y = 4(2 + e^{3x})^2$ A1 [6]

Page 5	Mark Scheme	Syllabus	Paper
	GCE A LEVEL – May/June 2014	9709	31

5	(i) <u>Either</u>	Multiply numerator and denominator by $\sqrt{3} + i$ and use $i^2 = -1$	M1	
		Obtain correct numerator $18 + 18\sqrt{3}i$ or correct denominator 4	B1	
		Obtain $\frac{9}{2} + \frac{9}{2}\sqrt{3}i$ or $(18 + 18\sqrt{3}i)/4$	A1	
		Obtain modulus or argument	M1	
		Obtain $9e^{\frac{1}{3}\pi i}$	A1	[5]
	<u>OR</u>	Obtain modulus and argument of numerator or denominator, or both moduli or both arguments	M1	
		Obtain moduli and argument 18 and $\frac{1}{6}\pi$ or 2 and $-\frac{1}{6}\pi$		
		or moduli 18 and 2 or arguments $\frac{1}{6}\pi$ and $-\frac{1}{6}\pi$ (allow degrees)	B1	
		Obtain $18e^{\frac{1}{6}\pi i} \div 2e^{-\frac{1}{6}\pi i}$ or equivalent	A1	
		Divide moduli and subtract arguments	M1	
		Obtain $9e^{\frac{1}{3}\pi i}$	A1	[5]
6	(ii)	State $3e^{\frac{1}{6}\pi i}$, following through their answer to part (i)	B1✓	
		State $3e^{\frac{1}{6}\pi i \pm \frac{1}{2}\pi i}$, following through their answer to part (i)	B1✓	
		Obtain $3e^{-\frac{5}{6}\pi i}$		
			B1	[3]
	(i)	Use law for the logarithm for a product or quotient or exponentiation AND for a power	M1	
		Obtain $(4x - 5)^2(x + 1) = 27$	B1	
		Obtain given equation correctly $16x^3 - 24x^2 - 15x - 2 = 0$	A1	[3]
	(ii)	Obtain $x = 2$ is root or $(x - 2)$ is a factor, or likewise with $x = -\frac{1}{4}$	B1	
		Divide by $(x - 2)$ to reach a quotient of the form $16x^2 + kx$	M1	
		Obtain quotient $16x^2 + 8x + 1$	A1	
		Obtain $(x - 2)(4x + 1)^2$ or $(x - 2), (4x + 1), (4x + 1)$	A1	[4]
	(iii)	State $x = 2$ only	A1	[1]
7	(i)	Obtain $2x - 3y + 6z$ for LHS of equation	B1	
		Obtain $2x - 3y + 6z = 23$	B1	[2]
	(ii) <u>Either</u>	Use correct formula to find perpendicular distance	M1	
		Obtain unsimplified value $\frac{\pm 23}{\sqrt{2^2 + (-3)^2 + 6^2}}$, following answer to (i)	A1✓	
		Obtain $\frac{23}{7}$ or equivalent	A1	[3]

Page 6	Mark Scheme	Syllabus	Paper
	GCE A LEVEL – May/June 2014	9709	31

8	<u>OR 1</u>	Use scalar product of $(4, -1, 2)$ and a vector normal to the plane	M1	
		Use unit normal to plane to obtain $\pm \frac{(8+3+12)}{\sqrt{49}}$	A1	
		Obtain $\frac{23}{7}$ or equivalent	A1	[3]
	<u>OR 2</u>	Find parameter intersection of p and $\mathbf{r} = \mu(2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k})$	M1	
		Obtain $\mu = \frac{23}{49}$ [and $\left(\frac{46}{49}, -\frac{69}{49}, \frac{138}{49}\right)$ as foot of perpendicular]	A1	
		Obtain distance $\frac{23}{7}$ or equivalent	A1	[3]
	(iii) <u>Either</u>	Recognise that plane is $2x - 3y + 6z = k$ and attempt use of formula for perpendicular distance to plane at least once	M1	
		Obtain $\frac{ 23-k }{7} = 14$ or equivalent	A1	
		Obtain $2x - 3y + 6z = 121$ and $2x - 3y + 6z = -75$	A1	[3]
	<u>OR</u>	Recognise that plane is $2x - 3y + 6z = k$ and attempt to find at least one point on q using l with $\lambda = \pm 2$	M1	
		Obtain $2x - 3y + 6z = 121$	A1	
		Obtain $2x - 3y + 6z = -75$	A1	[3]
	(i)	Sketch $y = \operatorname{cosec} x$ for at least $0, x, \pi$	B1	
		Sketch $y = x(\pi - x)$ for at least $0, x, \pi$	B1	
		Justify statement concerning two roots, with evidence of 1 and $\frac{1}{4}\pi^2$ for y -values on graph via scales	B1	[3]
	(ii)	Use $\operatorname{cosec} x = \frac{1}{\sin x}$ and commence rearrangement	M1	
		Obtain given equation correctly, showing sufficient detail	A1	[2]
	(iii) (a)	Use the iterative formula correctly at least once	M1	
		Obtain final answer 0.66	A1	
		Show sufficient iterations to 4 decimal places to justify answer or show a sign change in the interval $(0.655, 0.665)$	A1	[3]
	(b)	Obtain 2.48	B1	[1]

Page 7	Mark Scheme	Syllabus	Paper
	GCE A LEVEL – May/June 2014	9709	31

- 9 (i) Either State or imply partial fractions are of form $\frac{A}{3-x} + \frac{B}{1+2x} + \frac{C}{(1+2x)^2}$ B1
- Use any relevant method to obtain a constant M1
- Obtain $A = 1$ A1
- Obtain $B = \frac{3}{2}$ A1
- Obtain $C = -\frac{1}{2}$ A1 [5]
- Or State or imply partial fractions are of form $\frac{A}{3-x} + \frac{Dx+E}{(1+2x)^2}$ B1
- Use any relevant method to obtain a constant M1
- Obtain $A = 1$ A1
- Obtain $D = 3$ A1
- Obtain $E = 1$ A1 [5]
- (ii) Obtain the first two terms of one of the expansion of $(3-x)^{-1}, \left(1-\frac{1}{3}x\right)^{-1}$
- $(1+2x)^{-1}$ and $(1+2x)^{-2}$ M1
- Obtain correct unsimplified expansion up to the term in x^2 of each partial fraction, following in each case the value of A, B, C A1✓✓
A1✓✓
A1✓✓
- Obtain answer $\frac{4}{3} - \frac{8}{9}x + \frac{1}{27}x^2$ A1 [5]
- [If A, D, E approach used in part (i), give M1A1✓✓A1✓✓ for the expansions, M1 for multiplying out fully and A1 for final answer]
- 10 (i) Use of product or quotient rule M1
- Obtain $-5e^{-\frac{1}{2}x} \sin 4x + 40e^{-\frac{1}{2}x} \cos 4x$ A1
- Equate $\frac{dy}{dx}$ to zero and obtain $\tan 4x = k$ or $R \cos(4x \pm \alpha)$ M1
- Obtain $\tan 4x = 8$ or $\sqrt{65} \cos\left(4x \pm \tan^{-1} \frac{1}{8}\right)$ A1
- Obtain 0.362 or 20.7° A1
- Obtain 1.147 or 65.7° A1 [6]
- (ii) State or imply that x -coordinates of T_n are increasing by $\frac{1}{4}\pi$ or 45° B1
- Attempt solution of inequality (or equation) of form $x_1 + (n-1)k\pi \leq 25$ M1
- Obtain $n > \frac{4}{\pi}(25 - 0.362) + 1$, following through on their value of x_1 A1✓✓
- $n = 33$ A1 [4]

CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Level

MARK SCHEME for the May/June 2014 series

9709 MATHEMATICS

9709/32

Paper 3, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2014 series for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level components and some Ordinary Level components.

Page 2	Mark Scheme	Syllabus	Paper
	GCE A LEVEL – May/June 2014	9709	32

Mark Scheme Notes

Marks are of the following three types:

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
 - When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
 - The symbol ∇ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously “correct” answers or results obtained from incorrect working.
 - **Note:** B2 or A2 means that the candidate can earn 2 or 0.
B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.
- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

Page 3	Mark Scheme	Syllabus	Paper
	GCE A LEVEL – May/June 2014	9709	32

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)
CWO	Correct Working Only – often written by a “fortuitous” answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

- MR – 1** A penalty of MR – 1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR – 2 penalty may be applied in particular cases if agreed at the coordination meeting.
- PA – 1** This is deducted from A or B marks in the case of premature approximation. The PA – 1 penalty is usually discussed at the meeting.

Page 4	Mark Scheme	Syllabus	Paper
	GCE A LEVEL – May/June 2014	9709	32

- 1 EITHER: State or imply non-modular inequality $(x + 2a)^2 > (3(x - a))^2$, or corresponding quadratic equation, or pair of linear equations $(x + 2a) = \pm 3(x - a)$ B1
 Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations for x M1
 Obtain critical values $x = \frac{1}{4}a$ and $x = \frac{5}{2}a$ A1
 State answer $\frac{1}{4}a < x < \frac{5}{2}a$ A1
 OR: Obtain critical value $x = \frac{5}{2}a$ from a graphical method, or by inspection, or by solving a linear equation or inequality B1
 Obtain critical value $x = \frac{1}{4}a$ similarly B2
 State answer $\frac{1}{4}a < x < \frac{5}{2}a$ B1 4
 [Do not condone $\leq$ for $<$.]
- 2 Remove logarithms and obtain $5 - e^{-2x} = e^{\frac{1}{2}}$, or equivalent B1
 Obtain a correct value for e^{-2x} , e^{2x} , e^{-x} or e^x , e.g. $e^{2x} = 1/(5 - e^{\frac{1}{2}})$ B1
 Use correct method to solve an equation of the form $e^{2x} = a$, $e^{-2x} = a$, $e^x = a$ or $e^{-x} = a$ where $a > 0$. [The M1 is dependent on the correct removal of logarithms.] M1
 Obtain answer $x = -0.605$ only. A1 4
- 3 Use $\cos(A + B)$ formula to obtain an equation in $\cos x$ and $\sin x$ M1
 Use trig formula to obtain an equation in $\tan x$ (or $\cos x$ or $\sin x$) M1
 Obtain $\tan x = \sqrt{3} - 4$, or equivalent (or find $\cos x$ or $\sin x$) A1
 Obtain answer $x = -66.2^\circ$ A1
 Obtain answer $x = 113.8^\circ$ and no others in the given interval A1 5
 [Ignore answers outside the given interval. Treat answers in radians as a misread $(-1.16, 1.99)$.]
 [The other solution methods are *via* $\cos x = \pm 1/\sqrt{1 + (\sqrt{3} - 4)^2}$ and $\sin x = \pm(\sqrt{3} - 4)/\sqrt{1 + (\sqrt{3} - 4)^2}$.]
- 4 (i) State $\frac{dx}{dt} = 1 - \sec^2 t$, or equivalent B1
 Use chain rule M1
 Obtain $\frac{dy}{dt} = -\frac{\sin t}{\cos t}$, or equivalent A1
 Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$ M1
 Obtain the given answer correctly. A1 5
 (ii) State or imply $t = \tan^{-1}(\frac{1}{2})$ B1
 Obtain answer $x = -0.0364$ B1 2

Page 5	Mark Scheme	Syllabus	Paper
	GCE A LEVEL – May/June 2014	9709	32

5	(i) Differentiate $f(x)$ and obtain $f'(x) = (x-2)^2 g'(x) + 2(x-2)g(x)$ Conclude that $(x-2)$ is a factor of $f'(x)$	B1 B1	2
	(ii) <i>EITHER:</i> Substitute $x = 2$, equate to zero and state a correct equation, e.g. $32 + 16a + 24 + 4b + a = 0$ Differentiate polynomial, substitute $x = 2$ and equate to zero or divide by $(x-2)$ and equate constant remainder to zero Obtain a correct equation, e.g. $80 + 32a + 36 + 4b = 0$	B1 M1* A1	
	<i>OR1:</i> Identify given polynomial with $(x-2)^2(x^3 + Ax^2 + Bx + C)$ and obtain an equation in a and/or b Obtain a correct equation, e.g. $\frac{1}{4}a - 4(4+a) + 4 = 3$ Obtain a second correct equation, e.g. $-\frac{3}{4}a + 4(4+a) = b$	M1* A1 A1	
	<i>OR2:</i> Divide given polynomial by $(x-2)^2$ and obtain an equation in a and b Obtain a correct equation, e.g. $29 + 8a + b = 0$ Obtain a second correct equation, e.g. $176 + 47a + 4b = 0$ Solve for a or for b Obtain $a = -4$ and $b = 3$	M1* A1 A1 M1(dep*) A1	5
6	(i) Use correct arc formula and form an equation in r and x Obtain a correct equation in any form Rearrange in the given form	M1 A1 A1	3
	(ii) Consider sign of a relevant expression at $x = 1$ and $x = 1.5$, or compare values of relevant expressions at $x = 1$ and $x = 1.5$ Complete the argument correctly with correct calculated values	M1 A1	2
	(iii) Use the iterative formula correctly at least once Obtain final answer 1.21 Show sufficient iterations to 4 d.p. to justify 1.21 to 2 d.p., or show there is a sign change in the interval (1.205, 1.215)	M1 A1 A1	3
7	(a) <i>EITHER:</i> Substitute and expand $(-1 + \sqrt{5}i)^3$ completely Use $i^2 = -1$ correctly at least once Obtain $a = -12$ State that the other complex root is $-1 - \sqrt{5}i$	M1 M1 A1 B1	
	<i>OR1:</i> State that the other complex root is $-1 - \sqrt{5}i$ State the quadratic factor $z^2 + 2z + 6$ Divide the cubic by a 3-term quadratic, equate remainder to zero and solve for a or, using a 3-term quadratic, factorise the cubic and determine a Obtain $a = -12$	B1 B1 M1 A1	
	<i>OR2:</i> State that the other complex root is $-1 - \sqrt{5}i$ State or show the third root is 2 Use a valid method to determine a Obtain $a = -12$	B1 B1 M1 A1	
	<i>OR3:</i> Substitute and use De Moivre to cube $\sqrt{6}\text{cis}(114.1^\circ)$, or equivalent Find the real and imaginary parts of the expression Obtain $a = -12$ State that the other complex root is $-1 - \sqrt{5}i$	M1 M1 A1 B1	4

Page 6	Mark Scheme	Syllabus	Paper
	GCE A LEVEL – May/June 2014	9709	32
	(b) <i>EITHER:</i> Substitute $w = \cos 2\theta + i \sin 2\theta$ in the given expression	B1	
	Use double angle formulae throughout	M1	
	Express numerator and denominator in terms of $\cos \theta$ and $\sin \theta$ only	A1	
	Obtain given answer correctly	A1	
	<i>OR:</i> Substitute $w = e^{2i\theta}$ in the given expression	B1	
	Divide numerator and denominator by $e^{i\theta}$, or equivalent	M1	
	Express numerator and denominator in terms of $\cos \theta$ and $\sin \theta$ only	A1	
	Obtain the given answer correctly	A1	4
8	(i) Use product rule	M1	
	Obtain derivative in any correct form	A1	
	Differentiate first derivative using the product rule	M1	
	Obtain second derivative in any correct form, e.g. $-\frac{1}{2} \sin \frac{1}{2}x - \frac{1}{4}x \cos \frac{1}{2}x - \frac{1}{2} \sin \frac{1}{2}x$	A1	
	Verify the given statement	A1	5
	(ii) Integrate and reach $kx \sin \frac{1}{2}x + \int \sin \frac{1}{2}x \, dx$	M1*	
	Obtain $2x \sin \frac{1}{2}x - 2 \int \sin \frac{1}{2}x \, dx$, or equivalent	A1	
	Obtain indefinite integral $2x \sin \frac{1}{2}x + 4 \cos \frac{1}{2}x$	A1	
	Use correct limits $x = 0, x = \pi$ correctly	M1(dep*)	
	Obtain answer $2\pi - 4$, or exact equivalent	A1	5
9	(i) State or imply $\frac{dN}{dt} = kN(1 - 0.01N)$ and obtain the given answer $k = 0.02$	B1	1
	(ii) Separate variables and attempt integration of at least one side	M1	
	Integrate and obtain term $0.02t$, or equivalent	A1	
	Carry out a relevant method to obtain A or B such that $\frac{1}{N(1 - 0.01N)} \equiv \frac{A}{N} + \frac{B}{1 - 0.01N}$, or		
	equivalent	M1*	
	Obtain $A = 1$ and $B = 0.01$, or equivalent	A1	
	Integrate and obtain terms $\ln N - \ln(1 - 0.01N)$, or equivalent	A1 ^h	
	Evaluate a constant or use limits $t = 0, N = 20$ in a solution with terms $a \ln N$ and $b \ln(1 - 0.01N)$, $ab \neq 0$	M1(dep*)	
	Obtain correct answer in any form, e.g. $\ln N - \ln(1 - 0.01N) = 0.02t + \ln 25$	A1	
	Rearrange and obtain $t = 50 \ln(4N/(100 - N))$, or equivalent	A1	8
	(iii) Substitute $N = 40$ and obtain $t = 49.0$	B1	1

Page 7	Mark Scheme	Syllabus	Paper
	GCE A LEVEL – May/June 2014	9709	32

10 (i)	<i>EITHER:</i> State or imply $\overrightarrow{AB}$ and $\overrightarrow{AC}$ correctly in component form	B1	4
	Using the correct processes evaluate the scalar product $\overrightarrow{AB} \cdot \overrightarrow{AC}$, or equivalent	M1	
	Using the correct process for the moduli divide the scalar product by the product of the moduli	M1	
	Obtain answer $\frac{20}{21}$	A1	
	<i>OR:</i> Use correct method to find lengths of all sides of triangle ABC	M1	
	Apply cosine rule correctly to find the cosine of angle BAC	M1	
	Obtain answer $\frac{20}{21}$	A1	
(ii)	State an exact value for the sine of angle BAC , e.g. $\frac{\sqrt{41}}{21}$	B1✓	3
	Use correct area formula to find the area of triangle ABC	M1	
	Obtain answer $\frac{1}{2}\sqrt{41}$, or exact equivalent	A1	
	[SR: Allow use of a vector product, e.g. $\overrightarrow{AB} \times \overrightarrow{AC} = -6\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ B1✓. Using correct process for the modulus, divide the modulus by 2 M1. Obtain answer $\frac{1}{2}\sqrt{41}$ A1.]		
(iii)	<i>EITHER:</i> State or obtain $b = 0$	B1	5
	Equate scalar product of normal vector and $\overrightarrow{BC}$ (or $\overrightarrow{CB}$) to zero	M1	
	Obtain $a + b - 4c = 0$ (or $a - 4c = 0$)	A1	
	Substitute a relevant point in $4x + z = d$ and evaluate d	M1	
	Obtain answer $4x + z = 9$, or equivalent	A1	
	<i>OR1:</i> Attempt to calculate vector product of relevant vectors, e.g. $(\mathbf{j}) \times (\mathbf{i} + \mathbf{j} - 4\mathbf{k})$	M1	
	Obtain two correct components of the product	A1	
	Obtain correct product, e.g. $-4\mathbf{i} - \mathbf{k}$	A1	
	Substitute a relevant point in $4x + z = d$ and evaluate d	M1	
	Obtain $4x + z = 9$, or equivalent	A1	
	<i>OR2:</i> Attempt to form 2-parameter equation for the plane with relevant vectors	M1	
	State a correct equation, e.g. $\mathbf{r} = 2\mathbf{i} + 4\mathbf{j} + \mathbf{k} + \lambda(\mathbf{j}) + \mu(\mathbf{i} + \mathbf{j} - 4\mathbf{k})$	A1	
	State 3 equations in x, y, z, λ and μ	A1	
	Eliminate μ	M1	
	Obtain answer $4x + z = 9$, or equivalent	A1	
	<i>OR3:</i> State or obtain $b = 0$	B1	
	Substitute for B and C in the plane equation and obtain $2a + c = d$ and $3a - 3c = d$ (or $2a + 4b + c = d$ and $3a + 5b - 3c = d$)	B1	
	Solve for one ratio, e.g. $a : d$	M1	
	Obtain $a : c : d$, or equivalent	M1	
	Obtain answer $4x + z = 9$, or equivalent	A1	
	<i>OR4:</i> Attempt to form a determinant equation for the plane with relevant vectors	M1	
	State a correct equation, e.g. $\begin{vmatrix} x-2 & y-4 & z-1 \\ 0 & 1 & 0 \\ 1 & 1 & -4 \end{vmatrix} = 0$	A1	
	Attempt to use a correct method to expand the determinant	M1	
	Obtain two correct terms of a 3-term expansion, or equivalent	A1	
	Obtain answer $4x + z = 9$, or equivalent	A1	

CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Level

MARK SCHEME for the May/June 2014 series

9709 MATHEMATICS

9709/33

Paper 3, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2014 series for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level components and some Ordinary Level components.

Page 2	Mark Scheme	Syllabus	Paper
	GCE A LEVEL – May/June 2014	9709	33

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- The symbol ∇ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously “correct” answers or results obtained from incorrect working.
- Note: B2 or A2 means that the candidate can earn 2 or 0.
B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.
- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

Page 3	Mark Scheme	Syllabus	Paper
	GCE A LEVEL – May/June 2014	9709	33

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)
CWO	Correct Working Only – often written by a “fortuitous” answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through ✓” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Page 4	Mark Scheme	Syllabus	Paper
	GCE A LEVEL – May/June 2014	9709	33
1	Use law of the logarithm of a quotient or product or $2 = \log_{10} 100$ Remove logarithms and obtain $x + 9 = 100x$, or equivalent Obtain answer $x = \frac{1}{11}$	M1 A1 A1	3
2	State a correct unsimplified version of the x or x^2 or x^3 term State correct first two terms $1 - x$ Obtain the next two terms $2x^2 - \frac{14}{3}x^3$ [Symbolic binomial coefficients, e.g. $\left(-\frac{1}{3}\right)_3$ are not sufficient for the M mark.]	M1 A1 A1 + A1	4
3	(i) Use $\tan(A \pm B)$ formula and obtain an equation in $\tan x$ Using $\tan 60^\circ = \sqrt{3}$, obtain a horizontal equation in $\tan x$ in any correct form Reduce the equation to the given form (ii) Solve the given quadratic for $\tan x$ Obtain a correct answer, e.g. $x = 21.6^\circ$ Obtain a second answer, e.g. $x = 128.4^\circ$, and no others [Ignore answers outside the given interval. Treat answers in radians as a misread (0.377, 2.24).]	M1 A1 A1 M1 A1 A1	3 3
4	(i) Consider sign of $x - 10/(e^{2x} - 1)$ at $x = 1$ and $x = 2$ Complete the argument correctly with correct calculated values (ii) State or imply $\alpha = \frac{1}{2} \ln(1 + 10/\alpha)$ Rearrange this as $\alpha = 10/(e^{2\alpha} - 1)$ or work <i>vice versa</i> (iii) Use the iterative formula correctly at least once Obtain final answer 1.14 Show sufficient iterations to 4 d.p. to justify 1.14 to 2 d.p., or show there is a sign change in the interval (1.135, 1.145)	M1 A1 B1 B1 M1 A1 A1	2 2 3
5	Separate variables correctly and attempt integration of at least one side Obtain term in the form $a\sqrt{(2x+1)}$ Express $1/(\cos^2 \theta)$ as $\sec^2 \theta$ Obtain term of the form $k \tan \theta$ Evaluate a constant, or use limits $x = 0, \theta = \frac{1}{4}\pi$ in a solution with terms $a\sqrt{(2x+1)}$ and $k \tan \theta$, $ak \neq 0$ Obtain correct solution in any form, e.g. $\sqrt{(2x+1)} = \frac{1}{2} \tan \theta + \frac{1}{2}$ Rearrange and obtain $x = \frac{1}{8}(\tan \theta + 1)^2 - \frac{1}{2}$, or equivalent	B1 M1 B1 M1 M1 A1 A1	7

Page 5	Mark Scheme	Syllabus	Paper
	GCE A LEVEL – May/June 2014	9709	33
6	Obtain correct derivative of RHS in any form Obtain correct derivative of LHS in any form Set $\frac{dy}{dx}$ equal to zero and obtain a horizontal equation Obtain a correct equation, e.g. $x^2 + y^2 = 1$, from correct work By substitution in the curve equation, or otherwise, obtain an equation in x^2 or y^2 Obtain $x = \frac{1}{2}\sqrt{3}$ Obtain $y = \frac{1}{2}$	B1 B1 M1 A1 M1 A1 A1	7
7	(a) EITHER: Multiply numerator and denominator by $1 - 4i$, or equivalent, and use $i^2 = -1$ Simplify numerator to $-17 - 17i$, or denominator to 17 Obtain final answer $-1 - i$ OR: Using $i^2 = -1$, obtain two equations in x and y , and solve for x or for y Obtain $x = -1$ or $y = -1$, or equivalent Obtain final answer $-1 - i$	M1 A1 A1 M1 A1 A1	3
	(b) (i) Show a point representing $2 + i$ in relatively correct position Show a circle with centre $2 + i$ and radius 1 Show the perpendicular bisector of the line segment joining i and 2 Shade the correct region	B1 B1✓ B1 B1	4
	(ii) State or imply that the angle between the tangents from the origin to the circle is required Obtain answer 0.927 radians (or 53.1°)	M1 A1	2
8	(i) Use a correct method for finding a constant Obtain one of $A = 3$, $B = 3$, $C = 0$ Obtain a second value Obtain a third value	M1 A1 A1 A1	4
	(ii) Integrate and obtain term $-3 \ln(2 - x)$ Integrate and obtain term of the form $k \ln(2 + x^2)$ Obtain term $\frac{3}{2} \ln(2 + x^2)$ Substitute limits correctly in an integral of the form $a \ln(2 - x) + b \ln(2 + x^2)$, where $ab \neq 0$ Obtain given answer after full and correct working	B1✓ M1 A1✓ M1 A1	5
9	(i) Substitute for x and dx throughout using $u = \sin x$ and $du = \cos x \, dx$, or equivalent Obtain integrand e^{2u} Obtain indefinite integral $\frac{1}{2} e^{2u}$ Use limits $u = 0$, $u = 1$ correctly, or equivalent Obtain answer $\frac{1}{2}(e^2 - 1)$, or exact equivalent	M1 A1 A1 M1 A1	5

Page 6	Mark Scheme	Syllabus	Paper
	GCE A LEVEL – May/June 2014	9709	33

	(ii) Use chain rule or product rule	M1	6
	Obtain correct terms of the derivative in any form, e.g. $2\cos x e^{2\sin x} \cos x - e^{2\sin x} \sin x$	A1 + A1	
	Equate derivative to zero and obtain a quadratic equation in $\sin x$	M1	
	Solve a 3-term quadratic and obtain a value of x	M1	
	Obtain answer 0.896	A1	
10	(i) Express general point of l in component form, e.g. $(1 + 3\lambda, 2 - 2\lambda, -1 + 2\lambda)$	B1	3
	Substitute in given equation of p and solve for λ	M1	
	Obtain final answer $-\frac{1}{2}\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$, or equivalent, from $\lambda = -\frac{1}{2}$	A1	
	(ii) State or imply a vector normal to the plane, e.g. $2\mathbf{i} + 3\mathbf{j} - 5\mathbf{k}$	B1	4
	Using the correct process, evaluate the scalar product of a direction vector for l and a normal for p	M1	
	Using the correct process for the moduli, divide the scalar product by the product of the moduli and find the inverse sine or cosine of the result	M1	
	Obtain answer 23.2° (or 0.404 radians)	A1	
	(iii) EITHER: State $2a + 3b - 5c = 0$ or $3a - 2b + 2c = 0$	B1	
	Obtain two relevant equations and solve for one ratio, e.g. $a : b$	M1	
	Obtain $a : b : c = 4 : 19 : 13$, or equivalent	A1	
	Substitute coordinates of a relevant point in $4x + 19y + 13z = d$, and evaluate d	M1	5
	Obtain answer $4x + 19y + 13z = 29$, or equivalent	A1	
	OR1: Attempt to calculate vector product of relevant vectors, e.g. $(2\mathbf{i} + 3\mathbf{j} - 5\mathbf{k}) \times (3\mathbf{i} - 2\mathbf{j} + 2\mathbf{k})$	M1	
	Obtain two correct components of the product	A1	
	Obtain correct product, e.g. $-4\mathbf{i} - 19\mathbf{j} - 13\mathbf{k}$	A1	
	Substitute coordinates of a relevant point in $4x + 19y + 13z = d$	M1	
	Obtain answer $4x + 19y + 13z = 29$, or equivalent	A1	
	OR2: Attempt to form a 2-parameter equation with relevant vectors	M1	
	State a correct equation, e.g. $\mathbf{r} = \mathbf{i} + 2\mathbf{j} - \mathbf{k} + \lambda(2\mathbf{i} + 3\mathbf{j} - 5\mathbf{k}) + \mu(3\mathbf{i} - 2\mathbf{j} + 2\mathbf{k})$	A1	
	State 3 equations in x, y, z, λ and μ	A1	
	Eliminate λ and μ	M1	
	Obtain answer $4x + 19y + 13z = 29$, or equivalent	A1	
	OR3: Using a relevant point and relevant direction vectors, form a determinant equation for the plane	M1	
	State a correct equation, e.g. $\begin{vmatrix} x-1 & y-2 & z+1 \\ 2 & 3 & -5 \\ 3 & -2 & 2 \end{vmatrix} = 0$	A1	
	Attempt to expand the determinant	M1	
	Obtain correct values of two cofactors	A1	
	Obtain answer $4x + 19y + 13z = 29$, or equivalent	A1	