



UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS

9709/51

Paper 5 Mechanics 2 (M2)

May/June 2013

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.

Write your Centre number, candidate number and name on all the work you hand in.

Write in dark blue or black pen.

You may use a soft pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

Where a numerical value for the acceleration due to gravity is needed, use 10 m s^{-2} .

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 50.

Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **3** printed pages and **1** blank page.



- 1 A small ball is projected with speed 20 m s^{-1} at an angle of 45° above the horizontal from a point O on horizontal ground. At time t s after projection, the horizontal and vertically upwards displacements of the ball from O are x m and y m respectively.

(i) Express x and y in terms of t . [2]

(ii) Show that the equation of the trajectory of the ball is $y = x - \frac{1}{40}x^2$. [2]

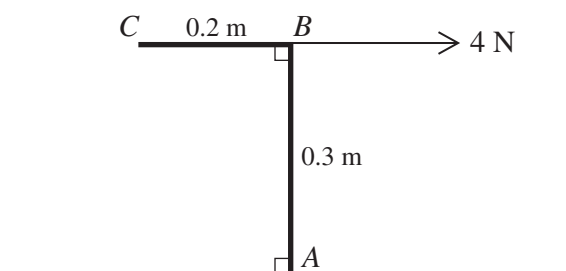
(iii) State the distance from O of the point at which the ball first strikes the ground. [1]

- 2 A particle P of mass 0.4 kg is attached to one end of a light elastic string of natural length 1.2 m and modulus of elasticity 19.2 N . The other end of the string is attached to a fixed point A . The particle P is released from rest at the point 2.7 m vertically above A . Calculate

(i) the initial acceleration of P , [3]

(ii) the speed of P when it reaches A . [4]

3



A uniform object ABC is formed from two rods AB and BC joined rigidly at right angles at B . The rod AB has length 0.3 m and the rod BC has length 0.2 m . The object rests with the end A on a rough horizontal surface and the rod AB vertical. The object is held in equilibrium by a horizontal force of magnitude 4 N applied at B and acting in the direction CB (see diagram).

(i) Find the distance of the centre of mass of the object from AB . [3]

(ii) Calculate the weight of the object. [2]

(iii) Find the least possible value of the coefficient of friction between the surface and the object. [2]

- 4 A particle of mass 0.2 kg is projected vertically downwards with initial speed 4 m s^{-1} . A resisting force of magnitude $0.09v \text{ N}$ acts vertically upwards on the particle during its descent, where $v \text{ m s}^{-1}$ is the downwards velocity of the particle at time t s after being set in motion.

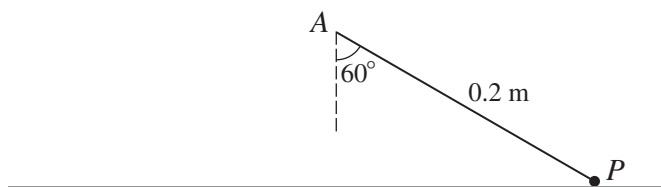
(i) Show that the acceleration of the particle is $(10 - 0.45v) \text{ m s}^{-2}$. [1]

(ii) Find v when $t = 1.5$. [5]

- 5 A particle P is projected with speed 50 m s^{-1} at an angle of 40° above the horizontal from a point O . For the instant 2.5 s after projection, calculate

- (i) the speed of P , [3]
 (ii) the angle between OP and the horizontal. [4]

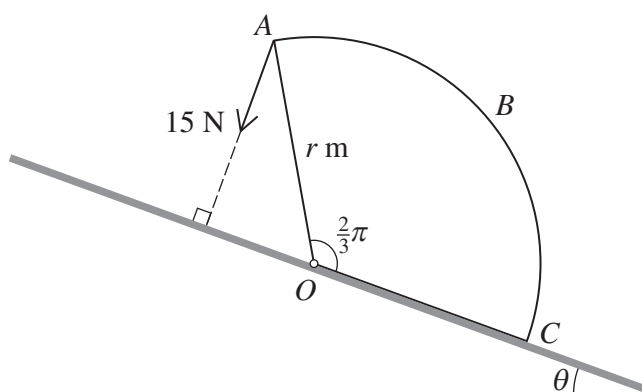
6



One end of a light inextensible string of length 0.2 m is attached to a fixed point A which is above a smooth horizontal table. A particle P of mass 0.3 kg is attached to the other end of the string. P moves on the table in a horizontal circle, with the string taut and making an angle of 60° with the downward vertical (see diagram).

- (i) Calculate the tension in the string if the speed of P is 1.2 m s^{-1} . [3]
 (ii) For the motion as described, show that the angular speed of P cannot exceed 10 rad s^{-1} , and hence find the greatest possible value for the kinetic energy of P . [6]

7



$OABC$ is the cross-section through the centre of mass of a uniform prism of weight 20 N . The cross-section is in the shape of a sector of a circle with centre O , radius $OA = r \text{ m}$ and angle $AOC = \frac{2}{3}\pi$ radians. The prism lies on a plane inclined at an angle θ radians to the horizontal, where $\theta < \frac{1}{3}\pi$. OC lies along a line of greatest slope with O higher than C . The prism is freely hinged to the plane at O . A force of magnitude 15 N acts at A , in a direction towards to the plane and at right angles to it (see diagram). Given that the prism remains in equilibrium, find the set of possible values of θ . [9]

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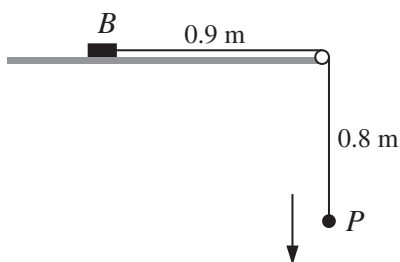
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- 1** A small sphere of mass 0.4 kg moves with constant speed 1.5 m s^{-1} in a horizontal circle inside a smooth fixed hollow cylinder of diameter 0.6 m . The axis of the cylinder is vertical, and the sphere is in contact with both the horizontal base and the vertical curved surface of the cylinder.
- (i) Calculate the magnitude of the force exerted on the sphere by the vertical curved surface of the cylinder. [2]
- (ii) Hence show that the magnitude of the total force exerted on the sphere by the cylinder is 5 N . [2]
- 2** A uniform semicircular lamina of radius 0.25 m has diameter AB . It is freely suspended at A from a fixed point and hangs in equilibrium.
- (i) Find the distance of the centre of mass of the lamina from the diameter AB . [1]
- (ii) Calculate the angle which the diameter AB makes with the vertical. [2]
- The lamina is now held in equilibrium with the diameter AB vertical by means of a force applied at B . This force has magnitude 6 N and acts at 45° to the upward vertical in the plane of the lamina.
- (iii) Calculate the weight of the lamina. [3]
- 3** A particle P of mass 0.2 kg is attached to one end of a light elastic string of natural length 1.6 m and modulus of elasticity 18 N . The other end of the string is attached to a fixed point O which is 1.6 m above a smooth horizontal surface. P is placed on the surface vertically below O and then projected horizontally. P moves with initial speed 1.5 m s^{-1} in a straight line on the surface. Show that, when $OP = 1.8\text{ m}$,
- (i) P is at instantaneous rest, [3]
- (ii) P is on the point of losing contact with the surface. [4]
- 4** A ball B is projected from a point O on horizontal ground at an angle of 40° above the horizontal. B hits the ground 1.8 s after the instant of projection. Calculate
- (i) the speed of projection of B , [2]
- (ii) the greatest height of B , [2]
- (iii) the distance from O of the point at which B hits the ground. [2]

5

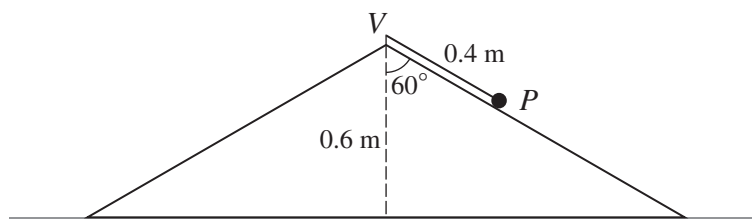


A block B of mass 3 kg is attached to one end of a light elastic string of modulus of elasticity 70 N and natural length 1.4 m . The other end of the string is attached to a particle P of mass 0.3 kg . B is at rest 0.9 m from the edge of a horizontal table and the string passes over a small smooth pulley at the edge of the table. P is released from rest at a point next to the pulley and falls vertically. At the first instant when P is 0.8 m below the pulley and descending, B is in limiting equilibrium with the part of the string attached to B horizontal (see diagram).

(i) Calculate the speed of P when B is first in limiting equilibrium. [5]

(ii) Find the coefficient of friction between B and the table. [3]

6



A uniform solid cone of height 0.6 m and mass 0.5 kg has its axis of symmetry vertical and its vertex V uppermost. The semi-vertical angle of the cone is 60° and the surface is smooth. The cone is fixed to a horizontal surface. A particle P of mass 0.2 kg is connected to V by a light inextensible string of length 0.4 m (see diagram).

(i) Calculate the height, above the horizontal surface, of the centre of mass of the cone with the particle. [3]

P is set in motion, and moves with angular speed 4 rad s^{-1} in a circular path on the surface of the cone.

(ii) Show that the tension in the string is 1.96 N , and calculate the magnitude of the force exerted on P by the cone. [5]

(iii) Find the speed of P . [1]

7 A particle P of mass 0.5 kg moves in a straight line on a smooth horizontal surface. The velocity of P is $v\text{ m s}^{-1}$ when the displacement of P from O is $x\text{ m}$. A single horizontal force of magnitude $0.16e^x\text{ N}$ acts on P in the direction OP . The velocity of P when it is at O is 0.8 m s^{-1} .

(i) Show that $v = 0.8e^{\frac{1}{2}x}$. [6]

(ii) Find the time taken by P to travel 1.4 m from O . [4]

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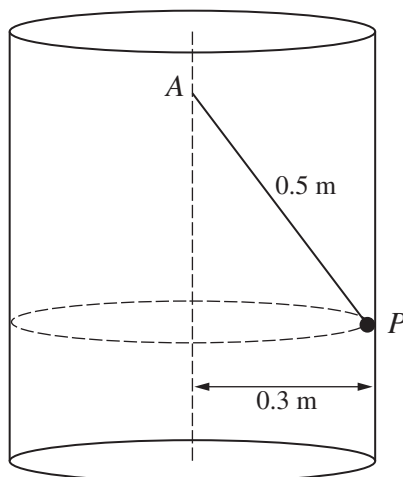
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- 1 A particle P is projected with speed 15 m s^{-1} at an angle of 60° above the horizontal. Find the direction of motion of P at the instant 0.9 s after projection. [4]
- 2 A particle P of mass 0.3 kg is attached to one end of a light elastic string of natural length 0.6 m and modulus of elasticity 45 N . The other end of the string is attached to a fixed point O . The particle P is released from rest at O and falls vertically. Find the extension of the string when P is at its lowest position. [4]
- 3 A ball is projected horizontally with speed 5 m s^{-1} from the top of a tower which is 30 m high. The tower stands on horizontal ground.
- (i) Find the speed and direction of motion of the ball when it reaches the ground. [3]
- (ii) Calculate the distance from the foot of the tower to the point where the ball reaches the ground. [3]

4



A smooth hollow cylinder of internal radius 0.3 m is fixed with its axis vertical. One end of a light inextensible string of length 0.5 m is fixed to a point A on the axis. The other end of the string is attached to a particle P of mass 0.2 kg which moves in a horizontal circle on the surface of the cylinder (see diagram).

- (i) Find the tension in the string. [3]
- (ii) Find the least angular speed of P for which the motion is possible. [2]
- (iii) Calculate the magnitude of the force exerted on P by the cylinder given that the speed of P is 1.8 m s^{-1} . [3]

- 5 One end of a light elastic string S_1 of modulus of elasticity 20 N and natural length 0.5 m is attached to a fixed point O . The other end of S_1 is attached to a particle P of mass 0.4 kg. P hangs in equilibrium vertically below O .

(i) Find the distance OP .

[2]

The opposite ends of a light inextensible string S_2 of length l m are now attached to O and P respectively. The elastic string S_1 remains attached to O and P . The particle P hangs in equilibrium vertically below O .

(ii) Find the tension in the inextensible string S_2 for each of the following cases:

(a) $l < 0.5$;

(b) $l > 0.6$;

(c) $l = 0.54$.

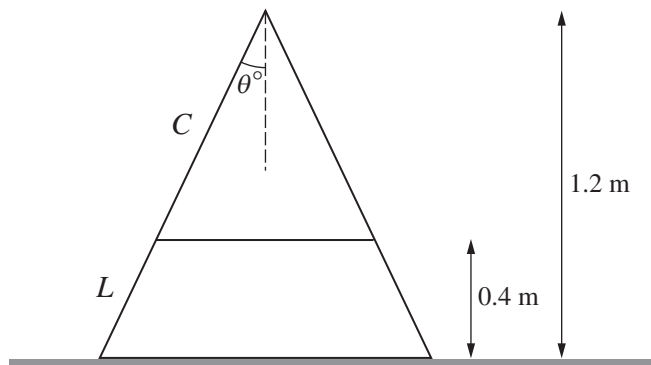
[4]

In the case $l = 0.54$, the inextensible string S_2 suddenly breaks and P begins to descend vertically.

(iii) Calculate the greatest speed of P in the subsequent motion.

[3]

6



A uniform solid cone of height 1.2 m and semi-vertical angle θ° is divided into two parts by a cut parallel to and 0.4 m from the circular base. The upper conical part, C , has weight 16 N, and the lower part, L , has weight 38 N. The two parts of the solid rest in equilibrium with the larger plane face of L on a horizontal surface and the smaller plane face of L covered by the base of C (see diagram).

(i) Calculate the distance of the centre of mass of L from its larger plane face.

[3]

An increasing horizontal force is applied to the vertex of C . Equilibrium is broken when the magnitude of this force first exceeds 4 N, and C begins to slide on L .

(ii) By considering the forces on C ,

(a) find the coefficient of friction between C and L ,

[1]

(b) show that $\theta > 14.0$, correct to 3 significant figures.

[2]

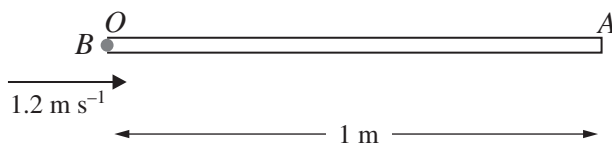
C is removed and L is placed with its curved surface on the horizontal surface.

(iii) Given that L is on the point of toppling, calculate θ .

[3]

- 7 A small ball B of mass 0.2 kg moves in a narrow fixed smooth cylindrical tube OA of length 1 m , closed at the end A . When the ball has displacement $x\text{ m}$ from O , it has velocity $v\text{ m s}^{-1}$ in the direction OA and experiences a resisting force of magnitude $\frac{k}{1-x}\text{ N}$.

(i)



The tube is fixed in a horizontal position and B is projected from O towards A with velocity 1.2 m s^{-1} (see diagram). Given that B comes to instantaneous rest after travelling 0.55 m , show that $k = 0.1803$, correct to 4 significant figures. [6]

- (ii) The tube is now fixed in a vertical position with O above A . The ball B is released from rest at O . Calculate the speed of B after it has descended 0.1 m . [4]