



UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS

9709/31

Paper 3 Pure Mathematics 3 (P3)

May/June 2013

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
 Graph Paper
 List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.

Write your Centre number, candidate number and name on all the work you hand in.

Write in dark blue or black pen.

You may use a soft pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 75.

Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **3** printed pages and **1** blank page.



- 1 Find the quotient and remainder when $2x^2$ is divided by $x + 2$. [3]
- 2 Expand $\frac{1 + 3x}{\sqrt{1 + 2x}}$ in ascending powers of x up to and including the term in x^2 , simplifying the coefficients. [4]
- 3 Express $\frac{7x^2 - 3x + 2}{x(x^2 + 1)}$ in partial fractions. [5]
- 4 (i) Solve the equation $|4x - 1| = |x - 3|$. [3]
 (ii) Hence solve the equation $|4^{y+1} - 1| = |4^y - 3|$ correct to 3 significant figures. [3]
- 5 For each of the following curves, find the gradient at the point where the curve crosses the y -axis:
 (i) $y = \frac{1 + x^2}{1 + e^{2x}}$; [3]
 (ii) $2x^3 + 5xy + y^3 = 8$. [4]
- 6 The points P and Q have position vectors, relative to the origin O , given by

$$\overrightarrow{OP} = 7\mathbf{i} + 7\mathbf{j} - 5\mathbf{k} \quad \text{and} \quad \overrightarrow{OQ} = -5\mathbf{i} + \mathbf{j} + \mathbf{k}.$$
 The mid-point of PQ is the point A . The plane Π is perpendicular to the line PQ and passes through A .
 (i) Find the equation of Π , giving your answer in the form $ax + by + cz = d$. [4]
 (ii) The straight line through P parallel to the x -axis meets Π at the point B . Find the distance AB , correct to 3 significant figures. [5]
- 7 (a) Without using a calculator, solve the equation

$$3w + 2iw^* = 17 + 8i,$$
 where w^* denotes the complex conjugate of w . Give your answer in the form $a + bi$. [4]
 (b) In an Argand diagram, the loci

$$\arg(z - 2i) = \frac{1}{6}\pi \quad \text{and} \quad |z - 3| = |z - 3i|$$
 intersect at the point P . Express the complex number represented by P in the form $re^{i\theta}$, giving the exact value of θ and the value of r correct to 3 significant figures. [5]
- 8 (a) Show that $\int_2^4 4x \ln x \, dx = 56 \ln 2 - 12$. [5]
 (b) Use the substitution $u = \sin 4x$ to find the exact value of $\int_0^{\frac{1}{24}\pi} \cos^3 4x \, dx$. [5]

- 9 (i) Express $4 \cos \theta + 3 \sin \theta$ in the form $R \cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. Give the value of α correct to 4 decimal places. [3]

(ii) Hence

(a) solve the equation $4 \cos \theta + 3 \sin \theta = 2$ for $0 < \theta < 2\pi$, [4]

(b) find $\int \frac{50}{(4 \cos \theta + 3 \sin \theta)^2} d\theta$. [3]

- 10 Liquid is flowing into a small tank which has a leak. Initially the tank is empty and, t minutes later, the volume of liquid in the tank is $V \text{ cm}^3$. The liquid is flowing into the tank at a constant rate of 80 cm^3 per minute. Because of the leak, liquid is being lost from the tank at a rate which, at any instant, is equal to $kV \text{ cm}^3$ per minute where k is a positive constant.

(i) Write down a differential equation describing this situation and solve it to show that

$$V = \frac{1}{k}(80 - 80e^{-kt}). \quad [7]$$

(ii) It is observed that $V = 500$ when $t = 15$, so that k satisfies the equation

$$k = \frac{4 - 4e^{-15k}}{25}.$$

Use an iterative formula, based on this equation, to find the value of k correct to 2 significant figures. Use an initial value of $k = 0.1$ and show the result of each iteration to 4 significant figures. [3]

(iii) Determine how much liquid there is in the tank 20 minutes after the liquid started flowing, and state what happens to the volume of liquid in the tank after a long time. [2]

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1 Solve the equation $|x - 2| = \left|\frac{1}{3}x\right|$. [3]

2 The sequence of values given by the iterative formula

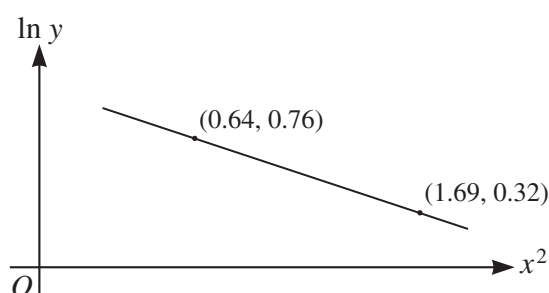
$$x_{n+1} = \frac{x_n(x_n^3 + 100)}{2(x_n^3 + 25)},$$

with initial value $x_1 = 3.5$, converges to α .

(i) Use this formula to calculate α correct to 4 decimal places, showing the result of each iteration to 6 decimal places. [3]

(ii) State an equation satisfied by α and hence find the exact value of α . [2]

3



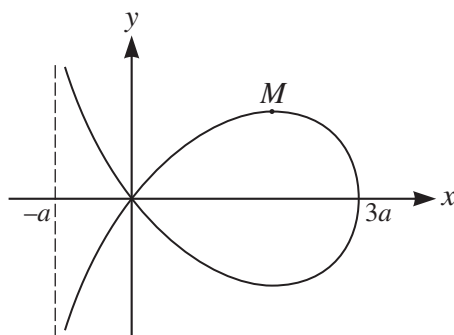
The variables x and y satisfy the equation $y = Ae^{-kx^2}$, where A and k are constants. The graph of $\ln y$ against x^2 is a straight line passing through the points $(0.64, 0.76)$ and $(1.69, 0.32)$, as shown in the diagram. Find the values of A and k correct to 2 decimal places. [5]

4 The polynomial $ax^3 - 20x^2 + x + 3$, where a is a constant, is denoted by $p(x)$. It is given that $(3x + 1)$ is a factor of $p(x)$.

(i) Find the value of a . [3]

(ii) When a has this value, factorise $p(x)$ completely. [3]

5



The diagram shows the curve with equation

$$x^3 + xy^2 + ay^2 - 3ax^2 = 0,$$

where a is a positive constant. The maximum point on the curve is M . Find the x -coordinate of M in terms of a . [6]

- 6 (i) By differentiating $\frac{1}{\cos x}$, show that the derivative of $\sec x$ is $\sec x \tan x$. Hence show that if $y = \ln(\sec x + \tan x)$ then $\frac{dy}{dx} = \sec x$. [4]

- (ii) Using the substitution $x = (\sqrt{3}) \tan \theta$, find the exact value of

$$\int_1^3 \frac{1}{\sqrt{3+x^2}} dx,$$

expressing your answer as a single logarithm. [4]

- 7 (i) By first expanding $\cos(x + 45^\circ)$, express $\cos(x + 45^\circ) - (\sqrt{2}) \sin x$ in the form $R \cos(x + \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. Give the value of R correct to 4 significant figures and the value of α correct to 2 decimal places. [5]

- (ii) Hence solve the equation

$$\cos(x + 45^\circ) - (\sqrt{2}) \sin x = 2,$$

for $0^\circ < x < 360^\circ$. [4]

- 8 (i) Express $\frac{1}{x^2(2x+1)}$ in the form $\frac{A}{x^2} + \frac{B}{x} + \frac{C}{2x+1}$. [4]

- (ii) The variables x and y satisfy the differential equation

$$y = x^2(2x+1) \frac{dy}{dx},$$

and $y = 1$ when $x = 1$. Solve the differential equation and find the exact value of y when $x = 2$. Give your value of y in a form not involving logarithms. [7]

- 9 (a) The complex number w is such that $\operatorname{Re} w > 0$ and $w + 3w^* = iw^2$, where w^* denotes the complex conjugate of w . Find w , giving your answer in the form $x + iy$, where x and y are real. [5]

- (b) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z which satisfy both the inequalities $|z - 2i| \leq 2$ and $0 \leq \arg(z + 2) \leq \frac{1}{4}\pi$. Calculate the greatest value of $|z|$ for points in this region, giving your answer correct to 2 decimal places. [6]

- 10 The points A and B have position vectors $2\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$ and $5\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ respectively. The plane p has equation $x + y = 5$.

- (i) Find the position vector of the point of intersection of the line through A and B and the plane p . [4]

- (ii) A second plane q has an equation of the form $x + by + cz = d$, where b , c and d are constants. The plane q contains the line AB , and the acute angle between the planes p and q is 60° . Find the equation of q . [7]

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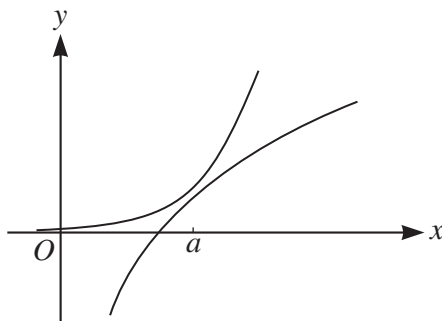
- 1 Solve the inequality $|4x + 3| > |x|$. [4]
- 2 It is given that $\ln(y + 1) - \ln y = 1 + 3 \ln x$. Express y in terms of x , in a form not involving logarithms. [4]
- 3 Solve the equation $\tan 2x = 5 \cot x$, for $0^\circ < x < 180^\circ$. [5]
- 4 (i) Express $(\sqrt{3}) \cos x + \sin x$ in the form $R \cos(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$, giving the exact values of R and α . [3]

(ii) Hence show that

$$\int_{\frac{1}{6}\pi}^{\frac{1}{2}\pi} \frac{1}{((\sqrt{3}) \cos x + \sin x)^2} dx = \frac{1}{4}\sqrt{3}. \quad [4]$$

- 5 The polynomial $8x^3 + ax^2 + bx + 3$, where a and b are constants, is denoted by $p(x)$. It is given that $(2x + 1)$ is a factor of $p(x)$ and that when $p(x)$ is divided by $(2x - 1)$ the remainder is 1. [5]
- (i) Find the values of a and b . [5]
- (ii) When a and b have these values, find the remainder when $p(x)$ is divided by $2x^2 - 1$. [3]

6



The diagram shows the curves $y = e^{2x-3}$ and $y = 2 \ln x$. When $x = a$ the tangents to the curves are parallel.

- (i) Show that a satisfies the equation $a = \frac{1}{2}(3 - \ln a)$. [3]
- (ii) Verify by calculation that this equation has a root between 1 and 2. [2]
- (iii) Use the iterative formula $a_{n+1} = \frac{1}{2}(3 - \ln a_n)$ to calculate a correct to 2 decimal places, showing the result of each iteration to 4 decimal places. [3]

- 7 The complex number z is defined by $z = a + ib$, where a and b are real. The complex conjugate of z is denoted by z^* .

(i) Show that $|z|^2 = zz^*$ and that $(z - ki)^* = z^* + ki$, where k is real. [2]

In an Argand diagram a set of points representing complex numbers z is defined by the equation $|z - 10i| = 2|z - 4i|$.

(ii) Show, by squaring both sides, that

$$zz^* - 2iz^* + 2iz - 12 = 0.$$

Hence show that $|z - 2i| = 4$. [5]

(iii) Describe the set of points geometrically. [1]

- 8 The variables x and t satisfy the differential equation

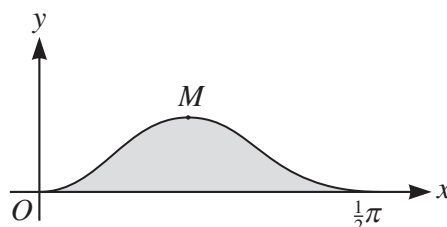
$$t \frac{dx}{dt} = \frac{k - x^3}{2x^2},$$

for $t > 0$, where k is a constant. When $t = 1$, $x = 1$ and when $t = 4$, $x = 2$.

(i) Solve the differential equation, finding the value of k and obtaining an expression for x in terms of t . [9]

(ii) State what happens to the value of x as t becomes large. [1]

9



The diagram shows the curve $y = \sin^2 2x \cos x$ for $0 \leq x \leq \frac{1}{2}\pi$, and its maximum point M .

(i) Find the x -coordinate of M . [6]

(ii) Using the substitution $u = \sin x$, find by integration the area of the shaded region bounded by the curve and the x -axis. [4]

- 10 The line l has equation $\mathbf{r} = \mathbf{i} + \mathbf{j} + \mathbf{k} + \lambda(a\mathbf{i} + 2\mathbf{j} + \mathbf{k})$, where a is a constant. The plane p has equation $x + 2y + 2z = 6$. Find the value or values of a in each of the following cases.

(i) The line l is parallel to the plane p . [2]

(ii) The line l intersects the line passing through the points with position vectors $3\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ and $\mathbf{i} + \mathbf{j} - \mathbf{k}$. [4]

(iii) The acute angle between the line l and the plane p is $\tan^{-1} 2$. [5]

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