

MARK SCHEME for the May/June 2013 series

9709 MATHEMATICS

9709/31

Paper 3, maximum raw mark 75

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Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- The symbol ∇ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously “correct” answers or results obtained from incorrect working.
- Note: B2 or A2 means that the candidate can earn 2 or 0.
B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.
- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

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The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)
CWO	Correct Working Only – often written by a ‘fortuitous’ answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through ✓” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

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- 1 Carry out division or equivalent at least as far as two terms of quotient
Obtain quotient $2x - 4$
Obtain remainder 8
M1
A1
A1 [3]
- 2 Obtain $1 - x$ as first two terms of $(1 + 2x)^{-\frac{1}{2}}$
Obtain $+\frac{3}{2}x^2$ or unsimplified equivalent as third term of $(1 + 2x)^{-\frac{1}{2}}$
Multiply $1 + 3x$ by attempt at $(1 + 2x)^{-\frac{1}{2}}$, obtaining sufficient terms
Obtain final answer $1 + 2x - \frac{3}{2}x^2$
B1
B1
M1
A1 [4]
- 3 State or imply correct form $\frac{A}{x} + \frac{Bx + C}{x^2 + 1}$
Use any relevant method to find at least one constant
Obtain $A = 2$
Obtain $B = 5$
Obtain $C = -3$
B1
M1
A1
A1
A1 [5]
- 4 (i) Either State or imply non-modular equation $(4x - 1)^2 = (x - 3)^2$ or pair of linear equations $4x - 1 = \pm(x - 3)$
Solve a three-term quadratic equation or two linear equations
Obtain $-\frac{2}{3}$ and $\frac{4}{5}$
B1
M1
A1
- Or Obtain value $-\frac{2}{3}$ from inspection or solving linear equation
Obtain value $\frac{4}{5}$ similarly
B1
B2 [3]
- (ii) State or imply at least $4^y = \frac{4}{5}$, following a positive answer from part (i)
Apply logarithms and use $\log a^b = b \log a$ property
Obtain -0.161 and no other answer
B1✓
M1
A1 [3]
- 5 (i) Use correct quotient rule or equivalent
Obtain $\frac{(1 + e^{2x})2x - (1 + x^2)2e^{2x}}{(1 + e^{2x})^2}$ or equivalent
Substitute $x = 0$ and obtain $-\frac{1}{2}$ or equivalent
M1
A1
A1 [3]
- (ii) Differentiate y^3 and obtain $3y^2 \frac{dy}{dx}$
Differentiate $5xy$ and obtain $5y + 5x \frac{dy}{dx}$
Obtain $6x^2 + 5y + 5x \frac{dy}{dx} + 3y^2 \frac{dy}{dx} = 0$
B1
B1
B1

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- Substitute $x = 0, y = 2$ to obtain $-\frac{5}{6}$ or equivalent following correct work B1 [4]
- 6 (i) State or imply A is $(1, 4, -2)$ B1
State or imply $\overrightarrow{QP} = 12\mathbf{i} + 6\mathbf{j} - 6\mathbf{k}$ or equivalent B1
Use QP as normal and A as mid-point to find equation of plane M1
Obtain $12x + 6y - 6z = 48$ or equivalent A1 [4]
- (ii) Either State equation of PB is $\mathbf{r} = 7\mathbf{i} + 7\mathbf{j} - 5\mathbf{k} + \lambda \mathbf{i}$ B1
Set up and solve a relevant equation for λ . M1
Obtain $\lambda = -9$ and hence B is $(-2, 7, -5)$ A1
Use correct method to find distance between A and B . M1
Obtain 5.20 A1
- Or Obtain 12 for result of scalar product of QP and $\mathbf{i}$ or equivalent B1
Use correct method involving moduli, scalar product and cosine to find angle APB M1
Obtain 35.26° or equivalent A1
Use relevant trigonometry to find AB M1
Obtain 5.20 A1 [5]
- 7 (a) State or imply $3a + 3bi + 2i(a - bi) = 17 + 8i$ B1
Consider real and imaginary parts to obtain two linear equations in a and b M1*
Solve two simultaneous linear equations for a or b M1 (dep*)
Obtain $7 - 2i$ A1 [4]
- (b) Either Show or imply a triangle with side 2 B1
State at least two of the angles $\frac{1}{4}\pi, \frac{2}{3}\pi$ and $\frac{1}{12}\pi$ B1
State or imply argument is $\frac{1}{4}\pi$ B1
Use sine rule or equivalent to find r M1
Obtain $6.69e^{\frac{1}{4}\pi i}$ A1
- Or State $y = x$. B1
State $y = \frac{1}{\sqrt{3}}x + 2$ or $\frac{\sqrt{3}}{2} = \frac{x}{\sqrt{x^2 + (y-2)^2}}$ or $\frac{1}{2} = \frac{y-2}{\sqrt{x^2 + (y-2)^2}}$ B1
State or imply argument is $\frac{\pi}{4}$ B1
Solve for x or y . M1
Obtain $6.69e^{\frac{1}{4}\pi i}$ A1 [5]
- 8 (a) Carry out integration by parts and reach $ax^2 \ln x + b \int \frac{1}{2} x^2 dx$ M1*
Obtain $2x^2 \ln x - \int \frac{1}{x} \cdot 2x^2 dx$ A1
Obtain $2x^2 \ln x - x^2$ A1
Use limits, having integrated twice M1 (dep*)
Confirm given result $56 \ln 2 - 12$ A1 [5]

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	(b) State or imply $\frac{du}{dx} = 4 \cos 4x$	B1	
	Carry out complete substitution except limits	M1	
	Obtain $\int (\frac{1}{4} - \frac{1}{4}u^2) du$ or equivalent	A1	
	Integrate to obtain form $k_1u + k_2u^3$ with non-zero constants k_1, k_2	M1	
	Use appropriate limits to obtain $\frac{11}{96}$	A1	[5]
9	(i) State or imply $R = 5$	B1	
	Use relevant trigonometry to find α	M1	
	Obtain $\alpha = 0.6435$	A1	[3]
	(ii) (a) Carry out appropriate method to find one value in given range	M1	
	Obtain 1.80	A1	
	Carry out appropriate method to find second value in given range	M1	
	Obtain 5.77 and no other value	A1	[4]
	(b) Express integrand as $k \sec^2(\theta - \text{their } \alpha)$ for any constant k	M1	
	Integrate to obtain result $k \tan(\theta - \text{their } \alpha)$	A1	
	Obtain correct answer $2 \tan(\theta - 0.6435)$	A1	[3]
10	(i) State $\frac{dV}{dt} = 80 - kV$	B1	
	Correctly separate variables and attempt integration of one side	M1	
	Obtain $a \ln(80 - kV) = t$ or equivalent	M1*	
	Obtain $-\frac{1}{k} \ln(80 - kV) = t$ or equivalent	A1	
	Use $t = 0$ and $V = 0$ to find constant of integration or as limits	M1 (dep*)	
	Obtain $-\frac{1}{k} \ln(80 - kV) = t - \frac{1}{k} \ln 80$ or equivalent	A1	
	Obtain given answer $V = \frac{1}{k}(80 - 80e^{-kt})$ correctly	A1	[7]
	(ii) Use iterative formula correctly at least once	M1	
	Obtain final answer 0.14	A1	
	Show sufficient iterations to 4 s.f. to justify answer to 2 s.f. or show a sign change in the interval (0.135, 0.145)	A1	[3]
	(iii) State a value between 530 and 540 cm ³ inclusive	B1	
	State or imply that volume approaches 569 cm ³ (allowing any value between 567 and 571 inclusive)	B1	[2]

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- 1 *EITHER*: State or imply non-modular equation $(x-2)^2 = \left(\frac{1}{3}x\right)^2$,
or pair of equations $x-2 = \pm \frac{1}{3}x$ M1
Obtain answer $x = 3$ A1
Obtain answer $x = \frac{3}{2}$, or equivalent A1
OR: Obtain answer $x = 3$ by solving an equation or by inspection B1
State or imply the equation $x-2 = -\frac{1}{3}$, or equivalent M1
Obtain answer $x = \frac{3}{2}$, or equivalent A1 [3]
- 2 (i) Use the iterative formula correctly at least once M1
Obtain final answer 3.6840 A1
Show sufficient iterations to at least 6 d.p. to justify 3.6840, or show there is a sign change in the interval (3.68395, 3.68405) A1 [3]
- (ii) State a suitable equation, e.g. $x = \frac{x(x^3+100)}{2(x^3+25)}$ B1
State that the value of α is $3\sqrt{50}$, or exact equivalent B1 [2]
- 3 *EITHER*: State or imply $\ln y = \ln A - kx^2$ B1
Substitute values of $\ln y$ and x^2 , and solve for k or $\ln A$ M1
Obtain $k = 0.42$ or $A = 2.80$ A1
Solve for $\ln A$ or k M1
Obtain $A = 2.80$ or $k = 0.42$ A1
OR1: State or imply $\ln y = \ln A - kx^2$ B1
Using values of $\ln y$ and x^2 , equate gradient of line to $-k$ and solve for k M1
Obtain $k = 0.42$ A1
Solve for $\ln A$ M1
Obtain $A = 2.80$ A1
OR2: Obtain two correct equations in k and A and substituting y - and x^2 - values in
 $y = Ae^{-kx^2}$ B1
Solve for k M1
Obtain $k = 0.42$ A1
Solve for A M1
Obtain $A = 2.80$ A1 [5]
[SR: If unsound substitutions are made, e.g. using $x = 0.64$ and $y = 0.76$, give B1M0A0M1A0 in the *EITHER* and *OR1* schemes, and B0M1A0M1A0 in the *OR2* scheme.]

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- 4 (i) Substitute $x = -\frac{1}{3}$, or divide by $3x + 1$, and obtain a correct equation,
e.g. $-\frac{1}{27}a - \frac{20}{9} - \frac{1}{3} + 3 = 0$ B1
Solve for a an equation obtained by a valid method M1
Obtain $a = 12$ A1 [3]
- (ii) Commence division by $3x + 1$ reaching a partial quotient $\frac{1}{3}ax^2 + kx$ M1
Obtain quadratic factor $4x^2 - 8x + 3$ A1
Obtain factorisation $(3x + 1)(2x - 1)(2x - 3)$ A1 [3]
[The M1 is earned if inspection reaches an unknown factor $\frac{1}{3}ax^2 + Bx + C$ and an equation in B and/or C , or an unknown factor $Ax^2 + Bx + 3$ and an equation in A and/or B , or if two coefficients with the correct moduli are stated without working.]
[If linear factors are found by the factor theorem, give B1B1 for $(2x - 1)$ and $(2x - 3)$, and B1 for the complete factorisation.]
[Synthetic division giving $12x^2 - 24x + 9$ as quadratic factor earns M1A1, but the final factorisation needs $(x + \frac{1}{3})$, or equivalent, in order to earn the second A1.]
[SR: If $x = \frac{1}{3}$ is used in substitution or synthetic division, give the M1 in part (i) but give M0 in part (ii).]
- 5 EITHER: State $2ay \frac{dy}{dx}$ as derivative of ay^2 B1
State $y^2 + 2xy \frac{dy}{dx}$ as derivative of xy^2 B1
Equate derivative of LHS to zero and set $\frac{dy}{dx}$ equal to zero M1
Obtain $3x^2 + y^2 - 6ax = 0$, or horizontal equivalent A1
Eliminate y and obtain an equation in x M1
Solve for x and obtain answer $x = \sqrt{3}a$ A1
- OR1: Rearrange equation in the form $y^2 = \frac{3ax^2 - x^3}{x + a}$ and attempt differentiation of one side B1
Use correct quotient or product rule to differentiate RHS M1
Obtain correct derivative of RHS in any form A1
Set $\frac{dy}{dx}$ equal to zero and obtain an equation in x M1
Obtain a correct horizontal equation free of surds A1
Solve for x and obtain answer $x = \sqrt{3}a$ A1
- OR2: Rearrange equation in the form $y = \left(\frac{3ax^2 - x^3}{x + a} \right)^{\frac{1}{2}}$ and differentiation of RHS B1
Use correct quotient or product rule and chain rule M1
Obtain correct derivative in any form A1

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	Equate derivative to zero and obtain an equation in x	M1	
	Obtain a correct horizontal equation free of surds	A1	
	Solve for x and obtain answer $x = \sqrt{3a}$	A1	[6]
6	(i)		
	Use correct quotient or chain rule to differentiate $\sec x$	M1	
	Obtain given derivative, $\sec x \tan x$, correctly	A1	
	Use chain rule to differentiate y	M1	
	Obtain the given answer	A1	[4]
	(ii)		
	Using $\frac{dx}{d\theta} \sqrt{3} \sec^2 \theta$, or equivalent, express integral in terms of θ and $d\theta$	M1	
	Obtain $\int \sec \theta d\theta$	A1	
	Use limits $\frac{1}{6}\pi$ and $\frac{1}{3}\pi$ correctly in an integral form of the form $k \ln(\sec \theta + \tan \theta)$	M1	
	Obtain a correct exact final answer in the given form, e.g. $\ln\left(\frac{2+\sqrt{3}}{\sqrt{3}}\right)$	A1	[4]
7	(i)		
	Use $\cos(A+B)$ formula to express the given expression in terms of $\cos x$ and $\sin x$	M1	
	Collect terms and reach $\frac{\cos x}{\sqrt{2}} - \frac{3}{\sqrt{2}} \sin x$, or equivalent	A1	
	Obtain $R = 2.236$	A1	
	Use trig formula to find α	M1	
	Obtain $\alpha = 71.57^\circ$ with no errors seen	A1	[5]
	(ii)		
	Evaluate $\cos^{-1}(2/2.236)$ to at least 1 d.p. (26.56° to 2 d.p., use of $R = \sqrt{5}$ gives 26.57°)	B1✓	
	Carry out an appropriate method to find a value of x in the interval $0^\circ < x < 360^\circ$	M1	
	Obtain answer, e.g. $x = 315^\circ$ (315.0°)	A1	
	Obtain second answer, e.g. 261.9° and no others in the given interval	A1	[4]
	[Ignore answers outside the given range.]		
	[Treat answers in radians as a misread and deduct A1 from the answers for the angles.]		
	[SR: Conversion of the equation to a correct quadratic in $\sin x$, $\cos x$, or $\tan x$ earns B1, then M1 for solving a 3-term quadratic and obtaining a value of x in the given interval, and A1 + A1 for the two correct answers (candidates must reject spurious roots to earn the final A1).]		
8	(i)		
	Use any relevant method to determine a constant	M1	
	Obtain one of the values $A = 1$, $B = -2$, $C = 4$	A1	
	Obtain a second value	A1	
	Obtain the third value	A1	[4]
	[If A and C are found by the cover up rule, give B1 + B1 then M1A1 for finding B . If only one is found by the rule, give B1M1A1A1.]		
	(ii)		
	Separate variables and obtain one term by integrating $\frac{1}{y}$ or a partial fraction	M1	
	Obtain $\ln y = -\frac{1}{2} - 2 \ln(2x+1) + c$, or equivalent	A3✓	

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	Evaluate a constant, or use limits $x = 1, y = 1$, in a solution containing at least three terms of the form $k \ln y, l/x, m \ln x$ and $n \ln(2x + 1)$, or equivalent	M1	
	Obtain solution $\ln y = -\frac{1}{2} - 2 \ln x + 2 \ln(2x + 1) + c$, or equivalent	A1	
	Substitute $x = 2$ and obtain $y = \frac{25}{36}e^{\frac{1}{2}}$, or exact equivalent free of logarithms	A1	[7]
	(The f.t. is on A, B, C . Give A2 [✓] if there is only one error or omission in the integration; A1 [✓] if two.)		
9	(a)		
	Substitute $w = x + iy$ and state a correct equation in x and y	B1	
	Use $i^2 = -1$ and equate real parts	M1	
	Obtain $y = -2$	A1	
	Equate imaginary parts and solve for x	M1	
	Obtain $x = 2\sqrt{2}$, or equivalent, only	A1	[5]
	(b)		
	Show a circle with centre $2i$	B1	
	Show a circle with radius 2	B1	
	Show half line from -2 at $\frac{1}{4}\pi$ to real axis	B1	
	Shade the correct region	B1	
	Carry out a complete method for calculating the greatest value of $ z $	M1	
	Obtain answer 3.70	A1	[6]
10	(i)		
	Carry out a correct method for finding a vector equation for AB	M1	
	Obtain $\mathbf{r} = 2\mathbf{i} - 3\mathbf{j} + 2\mathbf{k} + \lambda(3\mathbf{i} + \mathbf{j} - \mathbf{k})$ or		
	$\mathbf{r} = \mu(2\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}) + (1 - \mu)(5\mathbf{i} - 2\mathbf{j} + \mathbf{k})$, or equivalent	A1	
	Substitute components in equation of p and solve for λ or for μ	M1	
	Obtain $\lambda = \frac{3}{2}$ or $\mu = -\frac{1}{2}$ and final answer $\frac{13}{2}\mathbf{i} - \frac{3}{2}\mathbf{j} + \frac{1}{2}\mathbf{k}$, or equivalent	A1	[4]
	(ii)		
	Either equate scalar product of direction vector of AB and normal to q to zero or substitute for A and B in the equation of q and subtract expressions	M1*	
	Obtain $3 + b - c = 0$, or equivalent	A1	
	Using the correct method for the moduli, divide the scalar product of the normals to p and q by the product of their moduli and equate to $\pm\frac{1}{2}$, or form horizontal equivalent	M1*	
	Obtain correct equation in any form, e.g. $\frac{1+b}{\sqrt{(1+b^2+c^2)}\sqrt{(1+1)}} = \pm\frac{1}{2}$	A1	
	Solve simultaneous equations for b or for c	M1 (dep*)	
	Obtain $b = -4$ and $c = -1$	A1	
	Use a relevant point and obtain final answer $x - 4y - z = 12$, or equivalent	A1 [✓]	[7]
	(The f.t. is on b and c .)		

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Paper 3, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2013 series for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level components and some Ordinary Level components.

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- The symbol ∇ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously “correct” answers or results obtained from incorrect working.
- Note: B2 or A2 means that the candidate can earn 2 or 0.
B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.
- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

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The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)
CWO	Correct Working Only – often written by a “fortuitous” answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through ✓” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

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- 1 EITHER: State or imply non-modular inequality $(4x + 3)^2 > x^2$, or corresponding equation or pair of equations $4x + 3 = \pm x$ M1
Obtain a critical value, e.g. -1 A1
Obtain a second critical value, e.g. $-\frac{3}{5}$ A1
State final answer $x < -1, x > -\frac{3}{5}$ A1
- OR: Obtain critical value $x = -1$, by solving a linear equation or inequality, or from a graphical method or by inspection B1
Obtain the critical value $-\frac{3}{5}$ similarly B2
State final answer $x < -1, x > -\frac{3}{5}$ B1 [4]
[Do not condone $\leq$ or $\geq$.]
- 2 Use law for the logarithm of a product, quotient or power M1
Use $\ln e = 1$ or $\exp(1) = e$ M1
Obtain correct equation free of logarithms in any form, e.g. $\frac{y+1}{y} = ex^3$ A1
Rearrange as $y = (ex^3 - 1)^{-1}$, or equivalent A1 [4]
- 3 Use correct $\tan 2A$ formula and $\cot x = 1/\tan x$ to form an equation in $\tan x$ M1
Obtain a correct horizontal equation in any form A1
Solve an equation in $\tan^2 x$ for x M1
Obtain answer, e.g. 40.2° A1
Obtain second answer, e.g. 139.8° , and no other in the given interval A1✓ [5]
[Ignore answers outside the given interval.]
[Treat answers in radians as a misread and deduct A1 from the marks for the angles.]
[SR: For the answer $x = 90^\circ$ give B1 and A1 for one of the other angles.]
- 4 (i) State $R = 2$ B1
Use trig formula to find α M1
Obtain $\alpha = \frac{1}{6}\pi$ with no errors seen A1 [3]
- (ii) Substitute denominator of integrand and state integral $k \tan(x - \alpha)$ M1*
State correct indefinite integral $\frac{1}{4} \tan\left(x - \frac{1}{6}\pi\right)$ A1✓
Substitute limits M1 (dep*)
Obtain the given answer correctly A1 [4]

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- 5 (i) Substitute $x = -\frac{1}{2}$, or divide by $(2x + 1)$, and obtain a correct equation, e.g. $a - 2b + 8 = 0$ B1
Substitute $x = \frac{1}{2}$ and equate to 1, or divide by $(2x - 1)$ and equate constant remainder to 1 M1
Obtain a correct equation, e.g. $a + 2b + 12 = 0$ A1
Solve for a or for b M1
Obtain $a = -10$ and $b = -1$ A1 [5]
- (ii) Divide by $2x^2 - 1$ and reach a quotient of the form $4x + k$ M1
Obtain quotient $4x - 5$ A1
Obtain remainder $3x - 2$ A1 [3]
- 6 (i) State the correct derivatives $2e^{2x-3}$ and $2/x$ B1
Equate derivatives and use a law of logarithms on an equation equivalent to $ke^{2x-3} = m/x$ M1
Obtain the given result correctly (or work *vice versa*) A1 [3]
- (ii) Consider the sign of $a - \frac{1}{2}(3 - \ln a)$ when $a = 1$ and $a = 2$, or equivalent M1
Complete the argument with correct calculated values A1 [2]
- (iii) Use the iterative formula correctly at least once M1
Obtain final answer 1.35 A1
Show sufficient iterations to 4 d.p. to justify 1.35 to 2 d.p., or show there is a sign change in the interval (1.345, 1.355) A1 [3]
- 7 (i) Show that $a^2 + b^2 = (a + ib)(a - ib)$ B1
Show that $(a + ib - ki)^* = a - ib + ki$ B1 [2]
- (ii) Square both sides and express the given equation in terms of z and z^* M1
Obtain a correct equation in any form, e.g. $(z - 10i)(z^* + 10i) = 4(z - 4i)(z^* + 4i)$ A1
Obtain the given equation A1
Either express $|z - 2i| = 4$ in terms of z and z^* or reduce the given equation to the form
 $|z - u| = r$ M1
Obtain the given answer correctly A1 [5]
- (iii) State that the locus is a circle with centre $2i$ and radius 5 B1 [1]

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- 8 (i) Separate variables correctly and integrate at least one side M1
Obtain term $\ln t$, or equivalent B1
Obtain term of the form $a \ln(k - x^3)$ M1
Obtain term $-\frac{2}{3} \ln(k - x^3)$, or equivalent A1
EITHER: Evaluate a constant or use limits $t = 1, x = 1$ in a solution containing $a \ln t$ and $b \ln(k - x^3)$ M1*
Obtain correct answer in any form e.g. $\ln t = -\frac{2}{3} \ln(k - x^3) + \frac{2}{3} \ln(k - 1)$ A1
Use limits $t = 4, x = 2$, and solve for k M1(dep*)
Obtain $k = 9$ A1
OR: Using limits $t = 1, x = 1$ and $t = 4, x = 2$ in a solution containing $a \ln t$ and $b \ln(k - x^3)$ obtain an equation in k M1*
Obtain a correct equation in any form, e.g. $\ln 4 = -\frac{2}{3} \ln(k - 8) + \frac{2}{3} \ln(k - 1)$ A1
Solve for k M1(dep*)
Obtain $k = 9$ A1
Substitute $k = 9$ and obtain $x = (9 - 8t^{\frac{3}{2}})^{\frac{1}{3}}$ A1 [9]
- (ii) State that x approaches $9^{\frac{1}{3}}$, or equivalent B1✓ [1]
- 9 (i) Use product rule M1
Obtain correct derivative in any form, e.g. $4\sin 2x \cos 2x \cos x - \sin^2 2x \sin x$ A1
Equate derivative to zero and use a double angle formula M1*
Reduce equation to one in a single trig function M1(dep*)
Obtain a correct equation in any form,
e.g. $10 \cos^3 x = 6 \cos x, 4 = 6 \tan^2 x$ or $4 = 10 \sin^2 x$ A1
Solve and obtain $x = 0.685$ A1 [6]
- (ii) Using $du = \pm \cos x \, dx$, or equivalent, express integral in terms of u and du M1
Obtain $\int 4u^2(1 - u^2) \, du$, or equivalent A1
Use limits $u = 0$ and $u = 1$ in an integral of the form $au^3 + bu^5$ M1
Obtain answer $\frac{8}{15}$ (or 0.533) A1 [4]
- 10 (i) Equate scalar product of direction vector of l and p to zero M1
Solve for a and obtain $a = -6$ A1 [2]
- (ii) Express general point of l correctly in parametric form, e.g. $3\mathbf{i} + 2\mathbf{j} + \mathbf{k} + \mu(2\mathbf{i} + \mathbf{j} + 2\mathbf{k})$
or $(1 - \mu)(3\mathbf{i} + 2\mathbf{j} + \mathbf{k}) + \mu(\mathbf{i} + \mathbf{j} - \mathbf{k})$ B1
Equate at least two pairs of corresponding components of l and the second line and solve for λ or for μ M1
Obtain either $\lambda = \frac{2}{3}$ or $\mu = \frac{1}{3}$; or $\lambda = \frac{2}{a-1}$ or $\mu = \frac{1}{a-1}$; or reach $\lambda(a-4) = 0$
or $(1 + \mu)(a-4) = 0$ A1
Obtain $a = 4$ having ensured (if necessary) that all three component equations are satisfied A1 [4]

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- (iii) Using the correct process for the moduli, divide scalar product of direction vector if l and normal to p by the product of their moduli and equate to the sine of the given angle, or form an equivalent horizontal equation M1*
- Use $\frac{2}{\sqrt{5}}$ as sine of the angle A1
- State equation in any form, e.g. $\frac{a+6}{\sqrt{(a^2+4+1)}\sqrt{(1+4+4)}} = \frac{2}{\sqrt{5}}$ A1
- Solve for a M1 (dep*)
- Obtain answers for $a = 0$ and $a = \frac{60}{31}$, or equivalent A1 [5]
- [Allow use of the cosine of the angle to score M1M1.]