



UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS

9709/71

Paper 7 Probability & Statistics 2 (S2)

May/June 2012

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
 Graph Paper
 List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

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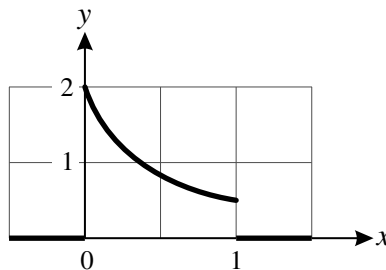


- 1 The weights, in grams, of packets of sugar are distributed with mean μ and standard deviation 23. A random sample of 150 packets is taken. The mean weight of this sample is found to be 494 g. Calculate a 98% confidence interval for μ . [3]
- 2 An examination consists of a written paper and a practical test. The written paper marks (M) have mean 54.8 and standard deviation 16.0. The practical test marks (P) are independent of the written paper marks and have mean 82.4 and standard deviation 4.8. The final mark is found by adding 75% of M to 25% of P . Find the mean and standard deviation of the final marks for the examination. [3]
- 3 When the council published a plan for a new road, only 15% of local residents approved the plan. The council then published a revised plan and, out of a random sample of 300 local residents, 60 approved the revised plan. Is there evidence, at the 2.5% significance level, that the proportion of local residents who approve the revised plan is greater than for the original plan? [5]
- 4 The random variable X has probability density function given by

$$f(x) = \begin{cases} \frac{k}{(x+1)^2} & 0 \leq x \leq 1, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

- (i) Show that $k = 2$. [2]
- (ii) Find a such that $P(X < a) = \frac{1}{5}$. [3]
- (iii)



The diagram shows the graph of $y = f(x)$. The median of X is denoted by m . Use the diagram to explain whether $m < 0.5$, $m = 0.5$ or $m > 0.5$. [2]

- 5 A random variable X has the distribution $\text{Po}(3.2)$.
- (i) A random value of X is found.
- (a) Find $P(X \geq 3)$. [2]
- (b) Find the probability that $X = 3$ given that $X \geq 3$. [3]
- (ii) Random samples of 120 values of X are taken.
- (a) Describe fully the distribution of the sample mean. [2]
- (b) Find the probability that the mean of a random sample of size 120 is less than 3.3. [3]

- 6** A survey taken last year showed that the mean number of computers per household in Branley was 1.66. This year a random sample of 50 households in Branley answered a questionnaire with the following results.

Number of computers	0	1	2	3	4	> 4
Number of households	5	12	18	10	5	0

- (i) Calculate unbiased estimates for the population mean and variance of the number of computers per household in Branley this year. [3]
 - (ii) Test at the 5% significance level whether the mean number of computers per household has changed since last year. [5]
 - (iii) Explain whether it is possible that a Type I error may have been made in the test in part (ii). [1]
 - (iv) State what is meant by a Type II error in the context of the test in part (ii), and give the set of values of the test statistic that could lead to a Type II error being made. [2]
- 7** At work Jerry receives emails randomly at a constant average rate of 15 emails per hour.
- (i) Find the probability that Jerry receives more than 2 emails during a 20-minute period at work. [3]
 - (ii) Jerry's working day is 8 hours long. Find the probability that Jerry receives fewer than 110 emails per day on each of 2 working days. [4]
 - (iii) At work Jerry also receives texts randomly and independently at a constant average rate of 1 text every 10 minutes. Find the probability that the total number of emails and texts that Jerry receives during a 5-minute period at work is more than 2 and less than 6. [4]

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- 1** The number of new enquiries per day at an office has a Poisson distribution. In the past the mean has been 3. Following a change of staff, the manager wishes to test, at the 5% significance level, whether the mean has increased.

(i) State the null and alternative hypotheses for this test. [1]

The manager notes the number, N , of new enquiries during a certain 6-day period. She finds that $N = 25$ and then, assuming that the null hypothesis is true, she calculates that $P(N \geq 25) = 0.0683$.

(ii) What conclusion should she draw? [2]

- 2** A population has mean 7 and standard deviation 3. A random sample of size n is chosen from this population.

(i) Write down the mean and standard deviation of the distribution of the sample mean. [2]

(ii) Under what circumstances does the sample mean have

(a) a normal distribution, [1]

(b) an approximately normal distribution? [1]

- 3** In a sample of 50 students at Batlin college, 18 support the football club Real Madrid.

(i) Calculate an approximate 98% confidence interval for the proportion of students at Batlin college who support Real Madrid. [4]

(ii) Give one condition for this to be a reliable result. [1]

- 4** Bacteria of a certain type are randomly distributed in the water in two ponds, A and B . The average numbers of bacteria per cm^3 in A and B are 0.32 and 0.45 respectively.

(i) Samples of 8 cm^3 of water from A and 12 cm^3 of water from B are taken at random. Find the probability that the total number of bacteria in these samples is at least 3. [3]

(ii) Find the probability that in a random sample of 155 cm^3 of water from A , the number of bacteria is less than 35. [5]

- 5** Fiona and Jhoti each take one shower per day. The times, in minutes, taken by Fiona and Jhoti to take a shower are represented by the independent variables $F \sim N(12.2, 2.8^2)$ and $J \sim N(11.8, 2.6^2)$ respectively. Find the probability that, on a randomly chosen day,

(i) the total time taken to shower by Fiona and Jhoti is less than 30 minutes, [4]

(ii) Fiona takes at least twice as long as Jhoti to take a shower. [4]

- 6** At a certain shop the weekly demand, in kilograms, for flour is modelled by the random variable X with probability density function given by

$$f(x) = \begin{cases} kx^{-\frac{1}{2}} & 4 \leq x \leq 25, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

- (i) Show that $k = \frac{1}{6}$. [2]
 - (ii) Calculate the mean weekly demand for flour at the shop. [3]
 - (iii) At the beginning of one week, the shop has 20 kg of flour in stock. Find the probability that this will not be enough to meet the demand for that week. [2]
 - (iv) Give a reason why the model may not be realistic. [1]
- 7** The weights, X kilograms, of bags of carrots are normally distributed. The mean of X is μ . An inspector wishes to test whether $\mu = 2.0$. He weighs a random sample of 200 bags and his results are summarised as follows.

$$\Sigma x = 430 \quad \Sigma x^2 = 1290$$

- (i) Carry out the test, at the 10% significance level. [6]
- (ii) You may now assume that the population variance of X is 1.85. The inspector weighs another random sample of 200 bags and carries out the same test at the 10% significance level.
 - (a) State the meaning of a Type II error in this context. [1]
 - (b) Given that $\mu = 2.12$, show that the probability of a Type II error is 0.652, correct to 3 significant figures. [7]

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- 1 Leaves from a certain type of tree have lengths that are distributed with standard deviation 3.2 cm. A random sample of 250 of these leaves is taken and the mean length of this sample is found to be 12.5 cm.
- (i) Calculate a 99% confidence interval for the population mean length. [3]
- (ii) Write down the probability that the whole of a 99% confidence interval will lie below the population mean. [1]
- 2 The independent random variables X and Y have the distributions $N(6.5, 14)$ and $N(7.4, 15)$ respectively. Find $P(3X - Y < 20)$. [5]
- 3 The lengths, x mm, of a random sample of 150 insects of a certain kind were found. The results are summarised by $\Sigma x = 7520$ and $\Sigma x^2 = 413\,540$.
- (i) Calculate unbiased estimates of the population mean and variance of the lengths of insects of this kind. [3]
- (ii) Using the values found in part (i), calculate an estimate of the probability that the mean length of a further random sample of 80 insects of this kind is greater than 53 mm. [3]
- 4 The number of lions seen per day during a standard safari has the distribution $Po(0.8)$. The number of lions seen per day during an off-road safari has the distribution $Po(2.7)$. The two distributions are independent.
- (i) Susan goes on a standard safari for one day. Find the probability that she sees at least 2 lions. [2]
- (ii) Deena goes on a standard safari for 3 days and then on an off-road safari for 2 days. Find the probability that she sees a total of fewer than 5 lions. [3]
- (iii) Khaled goes on a standard safari for n days, where n is an integer. He wants to ensure that his chance of not seeing any lions is less than 10%. Find the smallest possible value of n . [3]
- 5 (i) Deng wishes to test whether a certain coin is biased so that it is more likely to show Heads than Tails. He throws it 12 times. If it shows Heads more than 9 times, he will conclude that the coin is biased. Calculate the significance level of the test. [3]
- (ii) Deng throws another coin 100 times in order to test, at the 5% significance level, whether it is biased towards Heads. Find the rejection region for this test. [5]

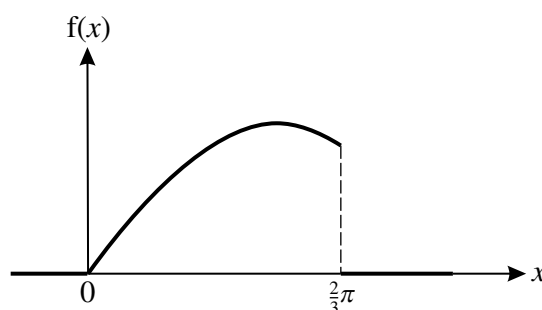
- 6 Last year Samir found that the time for his journey to work had mean 45.7 minutes and standard deviation 3.2 minutes. Samir wishes to test whether his journey times have increased this year. He notes the times, in minutes, for a random sample of 8 journeys this year with the following results.

46.2 41.7 49.2 47.1 47.2 48.4 53.7 45.5

It may be assumed that the population of this year's journey times is normally distributed with standard deviation 3.2 minutes.

- (i) State, with a reason, whether Samir should use a one-tail or a two-tail test. [2]
- (ii) Show that there is no evidence at the 5% significance level that Samir's mean journey time has increased. [5]
- (iii) State, with a reason, which one of the errors, Type I or Type II, might have been made in carrying out the test in part (ii). [2]

7



A random variable X has probability density function given by

$$f(x) = \begin{cases} k \sin x & 0 \leq x \leq \frac{2}{3}\pi, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant, as shown in the diagram.

- (i) Show that $k = \frac{2}{3}$. [2]
- (ii) Show that the median of X is 1.32, correct to 3 significant figures. [4]
- (iii) Find $E(X)$. [4]

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