



UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS

9709/31

Paper 3 Pure Mathematics 3 (P3)

May/June 2012

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.

Write your Centre number, candidate number and name on all the work you hand in.

Write in dark blue or black pen.

You may use a soft pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 75.

Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **3** printed pages and **1** blank page.



- 1 Solve the equation $|4 - 2^x| = 10$, giving your answer correct to 3 significant figures. [3]

- 2 (i) Expand $\frac{1}{\sqrt{1-4x}}$ in ascending powers of x , up to and including the term in x^2 , simplifying the coefficients. [3]

- (ii) Hence find the coefficient of x^2 in the expansion of $\frac{1+2x}{\sqrt{4-16x}}$. [2]

- 3 The polynomial $p(x)$ is defined by

$$p(x) = x^3 - 3ax + 4a,$$

where a is a constant.

- (i) Given that $(x - 2)$ is a factor of $p(x)$, find the value of a . [2]

- (ii) When a has this value,

- (a) factorise $p(x)$ completely, [3]

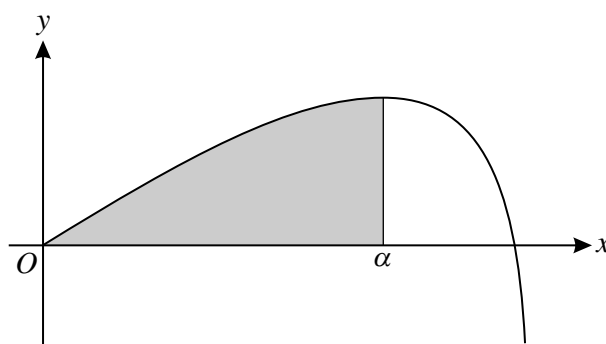
- (b) find all the roots of the equation $p(x^2) = 0$. [2]

- 4 The complex number u is defined by $u = \frac{(1+2i)^2}{2+i}$.

- (i) Without using a calculator and showing your working, express u in the form $x + iy$, where x and y are real. [4]

- (ii) Sketch an Argand diagram showing the locus of the complex number z such that $|z - u| = |u|$. [3]

5



The diagram shows the curve

$$y = 8 \sin \frac{1}{2}x - \tan \frac{1}{2}x$$

for $0 \leq x < \pi$. The x -coordinate of the maximum point is α and the shaded region is enclosed by the curve and the lines $x = \alpha$ and $y = 0$.

- (i) Show that $\alpha = \frac{2}{3}\pi$. [3]

- (ii) Find the exact value of the area of the shaded region. [4]

6 The equation of a curve is $3x^2 - 4xy + y^2 = 45$.

(i) Find the gradient of the curve at the point $(2, -3)$. [4]

(ii) Show that there are no points on the curve at which the gradient is 1. [3]

7 The variables x and y are related by the differential equation

$$\frac{dy}{dx} = \frac{6xe^{3x}}{y^2}.$$

It is given that $y = 2$ when $x = 0$. Solve the differential equation and hence find the value of y when $x = 0.5$, giving your answer correct to 2 decimal places. [8]

8 The point P has coordinates $(-1, 4, 11)$ and the line l has equation $\mathbf{r} = \begin{pmatrix} 1 \\ 3 \\ -4 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$.

(i) Find the perpendicular distance from P to l . [4]

(ii) Find the equation of the plane which contains P and l , giving your answer in the form $ax + by + cz = d$, where a, b, c and d are integers. [5]

9 By first expressing $\frac{4x^2 + 5x + 3}{2x^2 + 5x + 2}$ in partial fractions, show that

$$\int_0^4 \frac{4x^2 + 5x + 3}{2x^2 + 5x + 2} dx = 8 - \ln 9. \quad [10]$$

10 (i) It is given that $2 \tan 2x + 5 \tan^2 x = 0$. Denoting $\tan x$ by t , form an equation in t and hence show that either $t = 0$ or $t = \sqrt[3]{t + 0.8}$. [4]

(ii) It is given that there is exactly one real value of t satisfying the equation $t = \sqrt[3]{t + 0.8}$. Verify by calculation that this value lies between 1.2 and 1.3. [2]

(iii) Use the iterative formula $t_{n+1} = \sqrt[3]{t_n + 0.8}$ to find the value of t correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

(iv) Using the values of t found in previous parts of the question, solve the equation

$$2 \tan 2x + 5 \tan^2 x = 0$$

for $-\pi \leq x \leq \pi$. [3]

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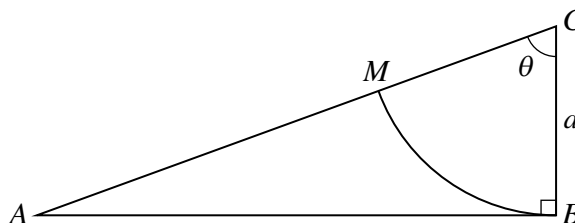
- 1 Solve the equation

$$\ln(3x + 4) = 2\ln(x + 1),$$

giving your answer correct to 3 significant figures.

[4]

2



In the diagram, ABC is a triangle in which angle ABC is a right angle and $BC = a$. A circular arc, with centre C and radius a , joins B and the point M on AC . The angle ACB is θ radians. The area of the sector CMB is equal to one third of the area of the triangle ABC .

- (i) Show that θ satisfies the equation

$$\tan \theta = 3\theta.$$

[2]

- (ii) This equation has one root in the interval $0 < \theta < \frac{1}{2}\pi$. Use the iterative formula

$$\theta_{n+1} = \tan^{-1}(3\theta_n)$$

to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places.

[3]

- 3 Expand $\sqrt{\left(\frac{1-x}{1+x}\right)}$ in ascending powers of x , up to and including the term in x^2 , simplifying the coefficients.

[5]

- 4 Solve the equation

$$\operatorname{cosec} 2\theta = \sec \theta + \cot \theta,$$

giving all solutions in the interval $0^\circ < \theta < 360^\circ$.

[6]

- 5 The variables x and y satisfy the differential equation

$$\frac{dy}{dx} = e^{2x+y},$$

and $y = 0$ when $x = 0$. Solve the differential equation, obtaining an expression for y in terms of x .

[6]

- 6 The equation of a curve is $y = 3 \sin x + 4 \cos^3 x$.

- (i) Find the x -coordinates of the stationary points of the curve in the interval $0 < x < \pi$.

[6]

- (ii) Determine the nature of the stationary point in this interval for which x is least.

[2]

7 Throughout this question the use of a calculator is not permitted.

The complex number u is defined by

$$u = \frac{1 + 2i}{1 - 3i}.$$

(i) Express u in the form $x + iy$, where x and y are real. [3]

(ii) Show on a sketch of an Argand diagram the points A , B and C representing the complex numbers u , $1 + 2i$ and $1 - 3i$ respectively. [2]

(iii) By considering the arguments of $1 + 2i$ and $1 - 3i$, show that

$$\tan^{-1} 2 + \tan^{-1} 3 = \frac{3}{4}\pi. \quad [3]$$

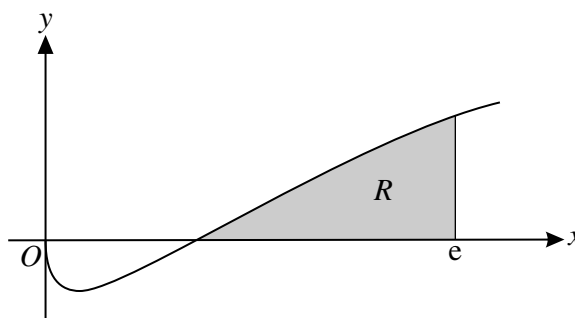
8 Let $I = \int_2^5 \frac{5}{x + \sqrt{6-x}} dx.$

(i) Using the substitution $u = \sqrt{6-x}$, show that

$$I = \int_1^2 \frac{10u}{(3-u)(2+u)} du. \quad [4]$$

(ii) Hence show that $I = 2 \ln\left(\frac{9}{2}\right).$ [6]

9



The diagram shows the curve $y = x^{\frac{1}{2}} \ln x$. The shaded region between the curve, the x -axis and the line $x = e$ is denoted by R .

(i) Find the equation of the tangent to the curve at the point where $x = 1$, giving your answer in the form $y = mx + c$. [4]

(ii) Find by integration the volume of the solid obtained when the region R is rotated completely about the x -axis. Give your answer in terms of π and e . [7]

[Question 10 is printed on the next page.]

- 10** Two planes, m and n , have equations $x + 2y - 2z = 1$ and $2x - 2y + z = 7$ respectively. The line l has equation $\mathbf{r} = \mathbf{i} + \mathbf{j} - \mathbf{k} + \lambda(2\mathbf{i} + \mathbf{j} + 2\mathbf{k})$.

(i) Show that l is parallel to m . [3]

(ii) Find the position vector of the point of intersection of l and n . [3]

(iii) A point P lying on l is such that its perpendicular distances from m and n are equal. Find the position vectors of the two possible positions for P and calculate the distance between them. [6]

[The perpendicular distance of a point with position vector $x_1\mathbf{i} + y_1\mathbf{j} + z_1\mathbf{k}$ from the plane $ax + by + cz = d$ is $\frac{|ax_1 + by_1 + cz_1 - d|}{\sqrt{a^2 + b^2 + c^2}}$.]



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- 1 Expand $\frac{1}{\sqrt{4+3x}}$ in ascending powers of x , up to and including the term in x^2 , simplifying the coefficients. [4]

- 2 Solve the equation $\ln(2x+3) = 2\ln x + \ln 3$, giving your answer correct to 3 significant figures. [4]

- 3 The parametric equations of a curve are

$$x = \sin 2\theta - \theta, \quad y = \cos 2\theta + 2 \sin \theta.$$

Show that $\frac{dy}{dx} = \frac{2 \cos \theta}{1 + 2 \sin \theta}$. [5]

- 4 The curve with equation $y = \frac{e^{2x}}{x^3}$ has one stationary point.

(i) Find the x -coordinate of this point. [4]

(ii) Determine whether this point is a maximum or a minimum point. [2]

- 5 In a certain chemical process a substance A reacts with another substance B . The masses in grams of A and B present at time t seconds after the start of the process are x and y respectively. It is given that $\frac{dy}{dt} = -0.6xy$ and $x = 5e^{-3t}$. When $t = 0$, $y = 70$.

(i) Form a differential equation in y and t . Solve this differential equation and obtain an expression for y in terms of t . [6]

(ii) The percentage of the initial mass of B remaining at time t is denoted by p . Find the exact value approached by p as t becomes large. [2]

- 6 It is given that $\tan 3x = k \tan x$, where k is a constant and $\tan x \neq 0$.

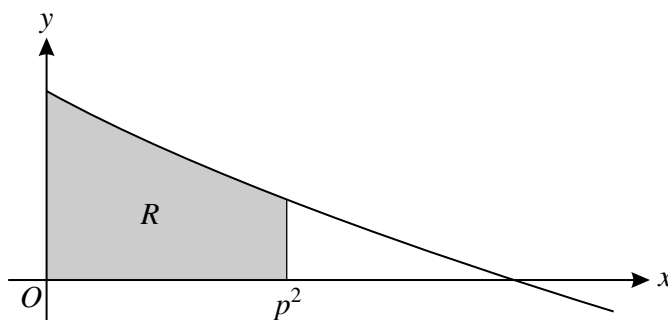
(i) By first expanding $\tan(2x+x)$, show that

$$(3k-1)\tan^2 x = k-3. \quad [4]$$

(ii) Hence solve the equation $\tan 3x = k \tan x$ when $k = 4$, giving all solutions in the interval $0^\circ < x < 180^\circ$. [3]

(iii) Show that the equation $\tan 3x = k \tan x$ has no root in the interval $0^\circ < x < 180^\circ$ when $k = 2$. [1]

7



The diagram shows part of the curve $y = \cos(\sqrt{x})$ for $x \geq 0$, where x is in radians. The shaded region between the curve, the axes and the line $x = p^2$, where $p > 0$, is denoted by R . The area of R is equal to 1.

(i) Use the substitution $x = u^2$ to find $\int_0^{p^2} \cos(\sqrt{x}) \, dx$. Hence show that $\sin p = \frac{3 - 2 \cos p}{2p}$. [6]

(ii) Use the iterative formula $p_{n+1} = \sin^{-1}\left(\frac{3 - 2 \cos p_n}{2p_n}\right)$, with initial value $p_1 = 1$, to find the value of p correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

8 Let $f(x) = \frac{4x^2 - 7x - 1}{(x+1)(2x-3)}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Show that $\int_2^6 f(x) \, dx = 8 - \ln\left(\frac{49}{3}\right)$. [5]

9 The lines l and m have equations $\mathbf{r} = 3\mathbf{i} - 2\mathbf{j} + \mathbf{k} + \lambda(-\mathbf{i} + 2\mathbf{j} + \mathbf{k})$ and $\mathbf{r} = 4\mathbf{i} + 4\mathbf{j} + 2\mathbf{k} + \mu(a\mathbf{i} + b\mathbf{j} - \mathbf{k})$ respectively, where a and b are constants.

(i) Given that l and m intersect, show that

$$2a - b = 4. \quad [4]$$

(ii) Given also that l and m are perpendicular, find the values of a and b . [4]

(iii) When a and b have these values, find the position vector of the point of intersection of l and m . [2]

10 (a) The complex numbers u and w satisfy the equations

$$u - w = 4i \quad \text{and} \quad uw = 5.$$

Solve the equations for u and w , giving all answers in the form $x + iy$, where x and y are real. [5]

(b) (i) On a sketch of an Argand diagram, shade the region whose points represent complex numbers satisfying the inequalities $|z - 2 + 2i| \leq 2$, $\arg z \leq -\frac{1}{4}\pi$ and $\operatorname{Re} z \geq 1$, where $\operatorname{Re} z$ denotes the real part of z . [5]

(ii) Calculate the greatest possible value of $\operatorname{Re} z$ for points lying in the shaded region. [1]

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