

MARK SCHEME for the May/June 2012 question paper
for the guidance of teachers

9709 MATHEMATICS

9709/31

Paper 3, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- Cambridge will not enter into discussions or correspondence in connection with these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2012 question papers for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.



Page 2	Mark Scheme: Teachers' version	Syllabus	Paper
	GCE AS/A LEVEL – May/June 2012	9709	31

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- The symbol ∇ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously “correct” answers or results obtained from incorrect working.
- Note: B2 or A2 means that the candidate can earn 2 or 0.
B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.
- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

Page 3	Mark Scheme: Teachers' version	Syllabus	Paper
	GCE AS/A LEVEL – May/June 2012	9709	31

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)
CWO	Correct Working Only – often written by a ‘fortuitous’ answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through ✓” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Page 4	Mark Scheme: Teachers' version	Syllabus	Paper
	GCE AS/A LEVEL – May/June 2012	9709	31

- 1 State or imply $4 - 2^x = -10$ and 10 B1
Use correct method for solving equation of form $2^x = a$ M1
Obtain 3.81 A1 [3]
- 2 (i) Either Obtain correct (unsimplified) version of x or x^2 term from $(1 - 4x)^{\frac{1}{2}}$ M1
Obtain $1 + 2x$ A1
Obtain $+ 6x^2$ A1
Or Differentiate and evaluate $f(0)$ and $f'(0)$ where $f(x) = k(1 - 4x)^{-\frac{3}{2}}$ M1
Obtain $1 + 2x$ A1
Obtain $+ 6x^2$ A1 [3]
- (ii) Combine both x^2 terms from product of $1 + 2x$ and answer from part (i) M1
Obtain 5 A1 [2]
- 3 (i) Substitute $x = 2$ and equate to zero, or divide by $x - 2$ and equate constant remainder to zero, or equivalent M1
Obtain $a = 4$ A1 [2]
- (ii) (a) Find further (quadratic or linear) factor by division, inspection or factor theorem or equivalent M1
Obtain $x^2 + 2x - 8$ or $x + 4$ A1
State $(x - 2)^2(x + 4)$ or equivalent A1 [3]
- (b) State any two of the four (or six) roots B1✓
State all roots $(\pm\sqrt{2}, \pm 2i)$, provided two are purely imaginary B1✓ [2]
- 4 (i) Either Expand $(1 + 2i)^2$ to obtain $-3 + 4i$ or unsimplified equivalent B1
Multiply numerator and denominator by $2 - i$ M1
Obtain correct numerator $-2 + 11i$ or correct denominator 5 A1
Obtain $-\frac{2}{5} + \frac{11}{5}i$ or equivalent A1
Or Expand $(1 + 2i)^2$ to obtain $-3 + 4i$ or unsimplified equivalent B1
Obtain two equations in x and y and solve for x or y M1
Obtain final answer $x = -\frac{2}{5}$ A1
Obtain final answer $y = \frac{11}{5}$ A1 [4]
- (ii) Draw a circle M1
Show centre at relatively correct position, following their u A1✓
Draw circle passing through the origin A1 [3]

Page 5	Mark Scheme: Teachers' version	Syllabus	Paper
	GCE AS/A LEVEL – May/June 2012	9709	31

- 5 (i) Differentiate to obtain $4\cos\frac{1}{2}x - \frac{1}{2}\sec^2\frac{1}{2}x$ B1
- Equate to zero and find value of $\cos\frac{1}{2}x$ M1
- Obtain $\cos\frac{1}{2}x = \frac{1}{2}$ and confirm $\alpha = \frac{2}{3}\pi$ A1 [3]
- (ii) Integrate to obtain $-16\cos\frac{1}{2}x \dots$ B1
- $\dots + 2\ln\cos\frac{1}{2}x$ or equivalent B1
- Using limits 0 and $\frac{2}{3}\pi$ in $a\cos\frac{1}{2}x + b\ln\cos\frac{1}{2}x$ M1
- Obtain $8 + 2\ln\frac{1}{2}$ or exact equivalent A1 [4]
- 6 (i) Obtain $2y\frac{dy}{dx}$ as derivative of y^2 B1
- Obtain $-4y - 4x\frac{dy}{dx}$ as derivative of $-4xy$ B1
- Substitute $x = 2$ and $y = -3$ and find value of $\frac{dy}{dx}$
- (dependent on at least one B1 being earned and $\frac{d(45)}{dx} = 0$) M1
- Obtain $\frac{12}{7}$ or equivalent A1 [4]
- (ii) Substitute $\frac{dy}{dx} = 1$ in an expression involving $\frac{dy}{dx}$, x and y and obtain $ay = bx$ M1
- Obtain $y = x$ or equivalent A1
- Uses $y = x$ in original equation and demonstrate contradiction A1 [3]
- 7 Separate variables correctly and attempt integration on at least one side M1
- Obtain $\frac{1}{3}y^3$ or equivalent on left-hand side A1
- Use integration by parts on right-hand side (as far as $axe^{3x} + \int be^{3x} dx$) M1
- Obtain or imply $2xe^{3x} + \int 2e^{3x} dx$ or equivalent A1
- Obtain $2xe^{3x} - \frac{2}{3}e^{3x}$ A1
- Substitute $x = 0, y = 2$ in an expression containing terms Ay^3, Bxe^{3x}, Ce^{3x} , where $ABC \neq 0$, and find the value of c M1
- Obtain $\frac{1}{3}y^3 = 2xe^{3x} - \frac{2}{3}e^{3x} + \frac{10}{3}$ or equivalent A1
- Substitute $x = 0.5$ to obtain $y = 2.44$ A1 [8]

Page 6	Mark Scheme: Teachers' version	Syllabus	Paper
	GCE AS/A LEVEL – May/June 2012	9709	31

8	(i)	<u>Either</u>	Obtain $\pm \begin{pmatrix} 2 \\ -1 \\ -15 \end{pmatrix}$ for vector PA (where A is point on line) or equivalent	B1
			Use scalar product to find cosine of angle between PA and line	M1
			Obtain $\frac{42}{\sqrt{14 \times 230}}$ or equivalent	A1
			Use trigonometry to obtain $\sqrt{104}$ or 10.2 or equivalent	A1
		<u>Or 1</u>	Obtain $\pm \begin{pmatrix} 2n+2 \\ n-1 \\ 3n-15 \end{pmatrix}$ for PN (where N is foot of perpendicular)	B1
			Equate scalar product of PN and line direction to zero	
			<u>Or</u> equate derivative of PN^2 to zero	
			<u>Or</u> use Pythagoras' theorem in triangle PNA to form equation in n	M1
			Solve equation and obtain $n = 3$	A1
			Obtain $\sqrt{104}$ or 10.2 or equivalent	A1
		<u>Or 2</u>	Obtain $\pm \begin{pmatrix} 2 \\ -1 \\ -15 \end{pmatrix}$ for vector PA (where A is point on line)	B1
			Evaluate vector product of PA and line direction	M1
			Obtain $\pm \begin{pmatrix} 12 \\ -36 \\ -4 \end{pmatrix}$	A1
			Divide modulus of this by modulus of line direction and obtain $\sqrt{104}$ or 10.2 or equivalent	A1
		<u>Or 3</u>	Obtain $\pm \begin{pmatrix} 2 \\ -1 \\ -15 \end{pmatrix}$ for vector PA (where A is point on line)	B1
			Evaluate scalar product of PA and line direction to obtain distance AN	M1
			Obtain $3\sqrt{14}$ or equivalent	A1
			Use Pythagoras' theorem in triangle PNA and obtain $\sqrt{104}$ or 10.2 or equivalent	A1
		<u>Or 4</u>	Obtain $\pm \begin{pmatrix} 2 \\ -1 \\ -15 \end{pmatrix}$ for vector PA (where A is point on line)	B1
			Use a second point B on line and use cosine rule in triangle ABP to find angle A or angle B <u>or</u> use vector product to find area of triangle	M1
			Obtain correct answer (angle $A = 42.25\dots$)	A1
			Use trigonometry to obtain $\sqrt{104}$ or 10.2 or equivalent	A1

[4]

Page 7	Mark Scheme: Teachers' version	Syllabus	Paper
	GCE AS/A LEVEL – May/June 2012	9709	31

- (ii) Either Use scalar product to obtain a relevant equation in a, b, c , e.g. $2a + b + 3c = 0$ or $2a - b - 15c = 0$ M1
State two correct equations in a, b and c A1✓
Solve simultaneous equations to obtain one ratio M1
Obtain $a : b : c = -3 : 9 : -1$ or equivalent A1
Obtain equation $-3x + 9y - z = 28$ or equivalent A1
- Or 1 Calculate vector product of two of $\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}, \begin{pmatrix} 2 \\ -1 \\ -15 \end{pmatrix}$ and $\begin{pmatrix} 8 \\ 2 \\ -6 \end{pmatrix}$ or equiv M1
Obtain two correct components of the product A1✓
Obtain correct $\begin{pmatrix} -3 \\ 9 \\ -1 \end{pmatrix}$ or equivalent A1
Substitute in $-3x + 9y - z = d$ to find d or equivalent M1
Obtain equation $-3x + 9y - z = 28$ or equivalent A1
- Or 2 Form a two-parameter equation of the plane M1
Obtain $\mathbf{r} = \begin{pmatrix} 1 \\ 3 \\ -4 \end{pmatrix} + s \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} + t \begin{pmatrix} 2 \\ -1 \\ -15 \end{pmatrix}$ or equivalent A1✓
State three equations in x, y, z, s, t A1
Eliminate s and t M1
Obtain equation $3x - 9y + z = -28$ or equivalent A1 [5]

- 9 State or imply form $A + \frac{B}{2x+1} + \frac{C}{x+2}$ B1
State or obtain $A = 2$ B1
Use correct method for finding B or C M1
Obtain $B = 1$ A1
Obtain $C = -3$ A1
Obtain $2x + \frac{1}{2} \ln(2x+1) - 3 \ln(x+2)$ [Deduct B1✓ for each error or omission] B3✓
Substitute limits in expression containing $a \ln(2x+1) + b \ln(x+2)$ M1
Show full and exact working to confirm that $8 + \frac{1}{2} \ln 9 - 3 \ln 6 + 3 \ln 2$, or an equivalent expression, simplifies to given result $8 - \ln 9$ A1 [10]

[SR: If A omitted from the form of fractions, give B0B0M1A0A0 in (i); B0✓B1✓B1✓M1A0 in (ii).]

[SR: For a solution starting with $\frac{M}{2x+1} + \frac{Nx}{x+2}$ or $\frac{Px}{2x+1} + \frac{Q}{x+2}$, give B0B0M1A0A0 in (i); B1✓B1✓B1✓, if recover correct form, M1A0 in (ii).]

[SR: For a solution starting with $\frac{B}{2x+1} + \frac{Dx+E}{x+2}$, give M1A1 for one of $B = 1, D = 2, E = 1$ and A1 for the other two constants; then give B1B1 for $A = 2, C = -3$.]

[SR: For a solution starting with $\frac{Fx+G}{2x+1} + \frac{C}{x+2}$, give M1A1 for one of $C = -3, F = 4, G = 3$ and A1 for the other constants or constant; then give B1B1 for $A = 2, B = 1$.]

Page 8	Mark Scheme: Teachers' version	Syllabus	Paper
	GCE AS/A LEVEL – May/June 2012	9709	31

- 10 (i)** Use correct identity for $\tan 2x$ and obtains $at^4 + bt^3 + ct^2 + dt = 0$, where b may be zero M1
Obtain correct horizontal equation, e.g. $4t + 5t^2 - 5t^4 = 0$ A1
Obtain $kt(t^3 + et + f) = 0$ or equivalent M1
Confirm given results $t = 0$ and $t = \sqrt[3]{t + 0.8}$ A1 [4]
- (ii)** Consider sign of $t - \sqrt[3]{t + 0.8}$ at 1.2 and 1.3 or equivalent M1
Justify the given statement with correct calculations (-0.06 and 0.02) A1 [2]
- (iii)** Use the iterative formula correctly at least once with $1.2 < t_n < 1.3$ M1
Obtain final answer 1.276 A1
Show sufficient iterations to justify answer or show there is a change of sign in interval
(1.2755, 1.2765) A1 [3]
- (iv)** Evaluate $\tan^{-1}$ (answer from part (iii)) to obtain at least one value M1
Obtain -2.24 and 0.906 A1
State $-\pi$, 0 and π B1 [3]
[SR If A0, B0, allow B1 for any 3 roots]

MARK SCHEME for the May/June 2012 question paper
for the guidance of teachers

9709 MATHEMATICS

9709/32

Paper 3, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- Cambridge will not enter into discussions or correspondence in connection with these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2012 question papers for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.



Page 2	Mark Scheme: Teachers' version	Syllabus	Paper
	GCE AS/A LEVEL – May/June 2012	9709	32

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- The symbol ∇ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously “correct” answers or results obtained from incorrect working.
- Note: B2 or A2 means that the candidate can earn 2 or 0.
B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.
- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

Page 3	Mark Scheme: Teachers' version	Syllabus	Paper
	GCE AS/A LEVEL – May/June 2012	9709	32

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)
CWO	Correct Working Only – often written by a ‘fortuitous’ answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through ✓” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Page 4	Mark Scheme: Teachers' version	Syllabus	Paper
	GCE AS/A LEVEL – May/June 2012	9709	32

- 1 EITHER: Use law of the logarithm of a power or quotient and remove logarithms M1
Obtain a 3-term quadratic equation $x^2 - x - 3 = 0$, or equivalent A1
Solve 3-term quadratic obtaining 1 or 2 roots M1
Obtain answer 2.30 only A1
- OR1: Use an appropriate iterative formula, e.g. $x_{n+1} = \exp\left(\frac{1}{2} \ln(3x_n + 4)\right) - 1$ correctly at least once M1
Obtain answer 2.30 A1
Show sufficient iterations to at least 3 d.p. to justify 2.30 to 2 d.p., or show there is a sign change in the interval (2.295, 2.305) A1
Show there is no other root A1
- OR2: Use calculated values to obtain at least one interval containing the root M1
Obtain answer 2.30 A1
Show sufficient calculations to justify 2.30 to 3 s.f., e.g. show it lies in (2.295, 2.305) A1
Show there is no other root A1 [4]
- 2 (i) Using the formulae $\frac{1}{2}r^2\theta$ and $\frac{1}{2}bh$, form an equation in a and θ M1
Obtain given answer A1 [2]
- (ii) Use the iterative formula correctly at least once M1
Obtain answer $\theta = 1.32$ A1
Show sufficient iterations to 4 d.p. to justify 1.32 to 2 d.p., or show there is a sign change in the interval (1.315, 1.325) A1 [3]

Page 5	Mark Scheme: Teachers' version	Syllabus	Paper
	GCE AS/A LEVEL – May/June 2012	9709	32

- 3 *EITHER*: State a correct unsimplified term in x or x^2 of $(1-x)^{\frac{1}{2}}$ or $(1+x)^{-\frac{1}{2}}$ B1
- State correct unsimplified expansion of $(1-x)^{\frac{1}{2}}$ up to the term in x^2 B1
- State correct unsimplified expansion of $(1+x)^{-\frac{1}{2}}$ up to the term in x^2 B1
- Obtain sufficient terms of the product of the expansions of $(1-x)^{\frac{1}{2}}$ and $(1+x)^{-\frac{1}{2}}$ M1
- Obtain final answer $1-x+\frac{1}{2}x^2$ A1
- OR1*: State that the given expression equals $(1-x)(1-x^2)^{-\frac{1}{2}}$ and state that the first term of the expansion of $(1-x^2)^{-\frac{1}{2}}$ is 1 B1
- State correct unsimplified term in x^2 of $(1-x^2)^{-\frac{1}{2}}$ B1
- State correct unsimplified expansion of $(1-x^2)^{-\frac{1}{2}}$ up to the term in x^2 B1
- Obtain sufficient terms of the product of $(1-x)$ and the expansion M1
- Obtain final answer $1-x+\frac{1}{2}x^2$ A1
- OR2*: State correct unsimplified expansion of $(1+x)^{\frac{1}{2}}$ up to the term in x^2 B1
- Multiply expansion by $(1-x)$ and obtain $1-2x+2x^2$ B1
- Carry out correct method to obtain one non-constant term of the expansion of $(1-2x+2x^2)^{\frac{1}{2}}$ M1
- Obtain a correct unsimplified expansion with sufficient terms A1
- Obtain final answer $1-x+\frac{1}{2}x^2$ A1 [5]
- [Treat $(1+x)^{-1}(1-x^2)^{\frac{1}{2}}$ by the *EITHER* scheme.]
- [Symbolic coefficients, e.g. $\left(\frac{1}{2}\right)$, are not sufficient for the B marks.]
- 4 Use trig formulae to express equation in terms of $\cos \theta$ and $\sin \theta$ M1
- Use Pythagoras to obtain an equation in $\sin \theta$ M1
- Obtain 3-term quadratic $2\sin^2 \theta - 2\sin \theta - 1 = 0$, or equivalent A1
- Solve a 3-term quadratic and obtain a value of θ M1
- Obtain answer, e.g. 201.5° A1
- Obtain second answer, e.g. 338.5° , and no others in the given interval A1 [6]
- [Ignore answers outside the given interval. Treat answers in radians (3.52, 5.91) as a misread and deduct A1 from the marks for the angles.]
- 5 Separate variables correctly and attempt integration of both sides B1
- Obtain term $-e^{-y}$, or equivalent B1
- Obtain term $\frac{1}{2}e^{2x}$, or equivalent B1
- Evaluate a constant, or use limits $x=0, y=0$ in a solution containing terms ae^{-y} and be^{2x} M1
- Obtain correct solution in any form, e.g. $-e^{-y} = \frac{1}{2}e^{2x} - \frac{3}{2}$ A1
- Rearrange and obtain $y = \ln(2/(3 - e^{2x}))$, or equivalent A1 [6]

Page 6	Mark Scheme: Teachers' version	Syllabus	Paper
	GCE AS/A LEVEL – May/June 2012	9709	32

- 6 (i) State derivative in any correct form, e.g. $3 \cos x - 12 \cos^2 x \sin x$ B1 + B1
Equate derivative to zero and solve for $\sin 2x$, or $\sin x$ or $\cos x$ M1
Obtain answer $x = \frac{1}{12} \pi$ A1
Obtain answer $x = \frac{5}{12} \pi$ A1
Obtain answer $x = \frac{1}{2} \pi$ and no others in the given interval A1^{ft} [6]
- (ii) Carry out a method for determining the nature of the relevant stationary point M1
Obtain a maximum at $\frac{1}{12} \pi$ correctly A1 [2]
[Treat answers in degrees as a misread and deduct A1 from the marks for the angles.]
- 7 (i) *EITHER*: Multiply numerator and denominator by $1 + 3i$, or equivalent M1
Simplify numerator to $-5 + 5i$, or denominator to 10, or equivalent A1
Obtain final answer $-\frac{1}{2} + \frac{1}{2}i$, or equivalent A1
OR: Obtain two equations in x and y , and solve for x or for y M1
Obtain $x = -\frac{1}{2}$ or $y = \frac{1}{2}$, or equivalent A1
Obtain final answer $-\frac{1}{2} + \frac{1}{2}i$, or equivalent A1 [3]
- (ii) Show B and C in relatively correct positions in an Argand diagram B1
Show u in a relatively correct position B1^{ft} [2]
- (iii) Substitute exact arguments in the LHS $\arg(1 + 2i) - \arg(1 - 3i) = \arg u$, or equivalent M1
Obtain and use $\arg u = \frac{3}{4} \pi$ A1
Obtain the given result correctly A1 [3]

Page 7	Mark Scheme: Teachers' version	Syllabus	Paper
	GCE AS/A LEVEL – May/June 2012	9709	32

- 8 (i) State or imply $2u \, du = -dx$, or equivalent B1
Substitute for x and dx throughout M1
Obtain integrand $\frac{-10u}{6-u^2+u}$, or equivalent A1
Show correct working to justify the change in limits and obtain the given answer correctly A1 [4]
- (ii) State or imply the form of fractions $\frac{A}{3-u} + \frac{B}{2+u}$ and use a relevant method to find A or B M1
Obtain $A = 6$ and $B = -4$ A1
Integrate and obtain $-6 \ln(3-u) - 4 \ln(2+u)$, or equivalent A1^{ft} + A1^{ft}
Substitute limits correctly in an integral of the form $a \ln(3-u) + b \ln(2+u)$ M1
Obtain the given answer correctly having shown sufficient working A1 [6]
[The f.t. is on A and B .]
- 9 (i) Use correct product rule M1
Obtain derivative in any correct form, e.g. $\frac{\ln x}{2\sqrt{x}} + \frac{\sqrt{x}}{x}$ A1
Carry out a complete method to form an equation of the tangent at $x = 1$ M1
Obtain answer $y = x - 1$ A1 [4]
- (ii) State or imply that the indefinite integral for the volume is $\pi \int x(\ln x)^2 \, dx$ B1
Integrate by parts and reach $ax^2(\ln x)^2 + b \int x^2 \cdot \frac{\ln x}{x} \, dx$ M1*
Obtain $\frac{1}{2}x^2(\ln x)^2 - \int x \ln x \, dx$, or unsimplified equivalent A1
Attempt second integration by parts reaching $cx^2 \ln x + d \int x^2 \cdot \frac{1}{x} \, dx$ M1(dep*)
Complete the integration correctly, obtaining $\frac{1}{2}x^2(\ln x)^2 - \frac{1}{2}x^2 \ln x + \frac{1}{4}x^2$ A1
Substitute limits $x = 1$ and $x = e$, having integrated twice M1(dep*)
Obtain answer $\frac{1}{4}\pi(e^2 - 1)$, or exact equivalent A1 [7]
[If π omitted, or 2π or $\pi/2$ used, give B0 and then follow through.]
[Integration using parts $x \ln x$ and $\ln x$ is also viable.]

Page 8	Mark Scheme: Teachers' version	Syllabus	Paper
	GCE AS/A LEVEL – May/June 2012	9709	32

- 10 (i) *EITHER*: Substitute coordinates of a general point of l in given equation of plane m M1
Obtain equation in λ in any correct form A1
Verify that the equation is not satisfied for any value of λ A1
OR1: Substitute for $\mathbf{r}$ in the vector equation of plane m and expand scalar product M1
Obtain equation in λ in any correct form A1
Verify that the equation is not satisfied for any value of λ A1
OR2: Expand scalar product of a normal to m and a direction vector of l M1
Verify scalar product is zero A1
Verify that one point of l does not lie in the plane A1
OR3: Use correct method to find perpendicular distance of a general point of l from m M1
Obtain a correct unsimplified expression in terms of λ A1
Show that the perpendicular distance is $4/3$, or equivalent, for all λ A1
OR4: Use correct method to find the perpendicular distance of a particular point of l from m M1
Obtain answer $4/3$, or equivalent A1
Show that the perpendicular distance of a second point is also $4/3$, or equivalent A1 [3]
- (ii) *EITHER*: Express general point of l in component form, e.g. $(1 + 2\lambda, 1 + \lambda, -1 + 2\lambda)$ B1
Substitute in given equation of n and solve for λ M1
Obtain position vector $5\mathbf{i} + 3\mathbf{j} + 3\mathbf{k}$ from $\lambda = 2$ A1
OR: State or imply plane n has vector equation $\mathbf{r} \cdot (2\mathbf{i} - 2\mathbf{j} + \mathbf{k}) = 7$, or equivalent B1
Substitute for $\mathbf{r}$, expand scalar product and solve for λ M1
Obtain position vector $5\mathbf{i} + 3\mathbf{j} + 3\mathbf{k}$ from $\lambda = 2$ A1 [3]
- (iii) Form an equation in λ by equating perpendicular distances of a general point of l from m and n M1*
Obtain a correct modular or non-modular equation in λ in any form A1^h
Solve for λ and obtain a point, e.g. $7\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}$ from $\lambda = 3$ A1
Obtain a second point, e.g. $3\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ from $\lambda = 1$ A1
Use a correct method to find the distance between the two points M1(dep*)
Obtain answer 6 A1 [6]
[The f.t. is on the components of l .]

MARK SCHEME for the May/June 2012 question paper
for the guidance of teachers

9709 MATHEMATICS

9709/33

Paper 3, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- Cambridge will not enter into discussions or correspondence in connection with these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2012 question papers for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.



Page 2	Mark Scheme: Teachers' version	Syllabus	Paper
	GCE AS/A LEVEL – May/June 2012	9709	33

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- The symbol ∇ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously “correct” answers or results obtained from incorrect working.
- Note: B2 or A2 means that the candidate can earn 2 or 0.
B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.
- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

Page 3	Mark Scheme: Teachers' version	Syllabus	Paper
	GCE AS/A LEVEL – May/June 2012	9709	33

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)
CWO	Correct Working Only – often written by a ‘fortuitous’ answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through ✓” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Page 4	Mark Scheme: Teachers' version	Syllabus	Paper
	GCE AS/A LEVEL – May/June 2012	9709	33

- 1 EITHER: Obtain a correct unsimplified version of the x or x^2 term of the expansion of $(4 + 3x)^{-\frac{1}{2}}$ or $(1 + \frac{3}{4}x)^{-\frac{1}{2}}$ M1
State correct first term $\frac{1}{2}$ B1
Obtain the next two terms $-\frac{3}{16}x + \frac{27}{256}x^2$ A1 + A1
OR: Differentiate and evaluate $f(0)$ and $f'(0)$, where $f'(x) = k(4 + 3x)^{-\frac{3}{2}}$ M1
State correct first term $\frac{1}{2}$ B1
Obtain the next two terms $-\frac{3}{16}x + \frac{27}{256}x^2$ A1 + A1 [4]
[Symbolic coefficients, e.g. $\left(-\frac{1}{2}\right)$ are not sufficient for the M or B mark.]
- 2 Use law of the logarithm of a power and a product or quotient and remove logarithms M1
Obtain a correct equation in any form, e.g. $\frac{2x+3}{x^2} = 3$ A1
Solve 3-term quadratic obtaining at least one root M1
Obtain final answer 1.39 only A1 [4]
- 3 Obtain $\frac{dx}{d\theta} = 2 \cos 2\theta - 1$ or $\frac{dy}{d\theta} = -2 \sin 2\theta + 2 \cos \theta$, or equivalent B1
Use $\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta}$ M1
Obtain $\frac{dy}{dx} = \frac{-2 \sin 2\theta + 2 \cos \theta}{2 \cos 2\theta - 1}$, or equivalent A1
At any stage use correct double angle formulae throughout M1
Obtain the given answer following full and correct working A1 [5]
- 4 (i) Use correct quotient or product rule M1
Obtain correct derivative in any form, e.g. $\frac{2e^{2x}}{x^3} - \frac{3e^{2x}}{x^4}$ A1
Equate derivative to zero and solve a 2-term equation for non-zero x M1
Obtain $x = \frac{3}{2}$ correctly A1 [4]
- (ii) Carry out a method for determining the nature of a stationary point, e.g. test derivative either side M1
Show point is a minimum with no errors seen A1 [2]

Page 5	Mark Scheme: Teachers' version	Syllabus	Paper
	GCE AS/A LEVEL – May/June 2012	9709	33

- 5 (i) Substitute for x , separate variables correctly and attempt integration of both sides M1
Obtain term $\ln y$, or equivalent A1
Obtain term e^{-3t} , or equivalent A1
Evaluate a constant, or use $t = 0$, $y = 70$ as limits in a solution containing terms $a \ln y$ and $b e^{-3t}$ M1
Obtain correct solution in any form, e.g. $\ln y - \ln 70 = e^{-3t} - 1$ A1
Rearrange and obtain $y = 70 \exp(e^{-3t} - 1)$, or equivalent A1 [6]
- (ii) Using answer to part (i), either express p in terms of t or use $e^{-3t} \rightarrow 0$ to find the limiting value of y M1
Obtain answer $\frac{100}{e}$ from correct exact work A1 [2]
- 6 (i) Use $\tan(A + B)$ and $\tan 2A$ formulae to obtain an equation in $\tan x$ M1
Obtain a correct equation in $\tan x$ in any form A1
Obtain an expression of the form $a \tan^2 x = b$ M1
Obtain the given answer A1 [4]
- (ii) Substitute $k = 4$ in the given expression and solve for x M1
Obtain answer, e.g. $x = 16.8^\circ$ A1
Obtain second answer, e.g. $x = 163.2^\circ$, and no others in the given interval A1 [3]
[Ignore answers outside the given interval. Treat answers in radians as a misread and deduct A1 from the marks for the angles.]
- (iii) Substitute $k = 2$, show $\tan^2 x < 0$ and justify given statement correctly B1 [1]
- 7 (i) Substitute for x and dx throughout the integral M1
Obtain $\int 2u \cos u \, du$ A1
Integrate by parts and obtain answer of the form $au \sin u + b \cos u$, where $ab \neq 0$ M1
Obtain $2u \sin u + 2 \cos u$ A1
Use limits $u = 0$, $u = p$ correctly and equate result to 1 M1
Obtain the given answer A1 [6]
- (ii) Use the iterative formula correctly at least once M1
Obtain final answer $p = 1.25$ A1
Show sufficient iterations to 4 d.p. to justify its accuracy to 2 d.p., or show there is a sign change in the interval (1.245, 1.255) A1 [3]

Page 6	Mark Scheme: Teachers' version	Syllabus	Paper
	GCE AS/A LEVEL – May/June 2012	9709	33

- 8 (i) State or imply the form $A + \frac{B}{x+1} + \frac{C}{2x-3}$ B1
State or obtain $A = 2$ B1
Use a correct method for finding a constant M1
Obtain $B = -2$ A1
Obtain $C = -1$ A1 [5]
- (ii) Obtain integral $2x - 2\ln(x+1) - \frac{1}{2}\ln(2x-3)$ B3^h
(Deduct B1^h for each error or omission. The f.t. is on A, B, C .)
Substitute limits correctly in an expression containing terms $a\ln(x+1)$ and $b\ln(2x-3)$ M1
Obtain the given answer following full and exact working A1 [5]
[SR: If A omitted from the form of fractions, give B0B0M1A0A0 in (i); B1^hB1^hM1A0 in (ii).]
[SR: For a solution starting with $\frac{B}{x+1} + \frac{Dx+E}{2x-3}$, give M1A1 for one of $B = -2, D = 4, E = -7$ and A1 for the other two constants; then give B1B1 for $A = 2, C = -1$.]
[SR: For a solution starting with $\frac{Fx+G}{x+1} + \frac{C}{2x-3}$ or with $\frac{Fx}{x+1} + \frac{C}{2x-3}$, give M1A1 for one of $C = -1, F = 2, G = 0$ and A1 for the other constants or constant; then give B1B1 for $A = 2, B = -2$.]
- 9 (i) Express general point of l or m in component form, i.e. $(3-\lambda, -2+2\lambda, 1+\lambda)$ or $(4+a\mu, 4+b\mu, 2-\mu)$ B1
Equate components and eliminate either λ or μ from a pair of equations M1
Eliminate the other parameter and obtain an equation in a and b M1
Obtain the given answer A1 [4]
- (ii) Using the correct process equate the scalar product of the direction vectors to zero M1*
Obtain $-a + 2b - 1 = 0$, or equivalent A1
Solve simultaneous equations for a or for b M1(dep*)
Obtain $a = 3, b = 2$ A1 [4]
- (iii) Substitute found values in component equations and solve for λ or for μ M1
Obtain answer $\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ from either $\lambda = 2$ or from $\mu = -1$ A1 [2]

Page 7	Mark Scheme: Teachers' version	Syllabus	Paper
	GCE AS/A LEVEL – May/June 2012	9709	33

- 10 (a) EITHER: Eliminate u or w and obtain an equation in w or in u M1
- Obtain a quadratic in u or w , e.g. $u^2 - 4iu - 5 = 0$ or $w^2 + 4iw - 5 = 0$ A1
- Solve a 3-term quadratic for u or for w M1
- OR1: Having squared the first equation, eliminate u or w and obtain an equation in w or u M1
- Obtain a 2-term quadratic in u or w , e.g. $u^2 = -3 + 4i$ A1
- Solve a 2-term quadratic for u or for w M1
- OR2: Using $u = a + ib$, $w = c + id$, equate real and imaginary parts and obtain 4 equations in a, b, c and d M1
- Obtain 4 correct equations A1
- Solve for a and b , or for c and d M1
- Obtain answer $u = 1 + 2i$, $w = 1 - 2i$ A1
- Obtain answer $u = -1 + 2i$, $w = -1 - 2i$ and no other A1 [5]
- (b) (i) Show point representing $2 - 2i$ in relatively correct position B1
- Show a circle with centre $2 - 2i$ and radius 2 B1✓
- Show line for $\arg z = -\frac{1}{4}\pi$ B1
- Show line for $\operatorname{Re} z = 1$ B1
- Shade the relevant region B1 [5]
- (ii) State answer $2 + \sqrt{2}$, or equivalent (accept 3.41) B1 [1]