



UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS

9709/51

Paper 5 Mechanics 2 (M2)

May/June 2011

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

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The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 50.

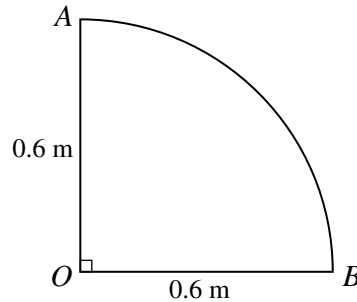
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- 1 A particle is projected with speed 15 m s^{-1} at an angle of 40° above the horizontal from a point on horizontal ground. Calculate the time taken for the particle to hit the ground. [2]

2



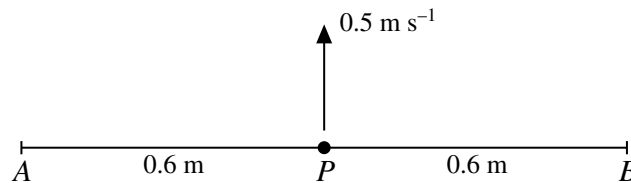
AOB is a uniform lamina in the shape of a quadrant of a circle with centre O and radius 0.6 m (see diagram).

- (i) Calculate the distance of the centre of mass of the lamina from A . [3]

The lamina is freely suspended at A and hangs in equilibrium.

- (ii) Find the angle between the vertical and the side AO of the lamina. [3]

3



A light elastic string of natural length 1.2 m and modulus of elasticity 24 N is attached to fixed points A and B on a smooth horizontal surface, where $AB = 1.2 \text{ m}$. A particle P is attached to the mid-point of the string. P is projected with speed 0.5 m s^{-1} along the surface in a direction perpendicular to AB (see diagram). P comes to instantaneous rest at a distance 0.25 m from AB .

- (i) Show that the mass of P is 0.8 kg . [3]

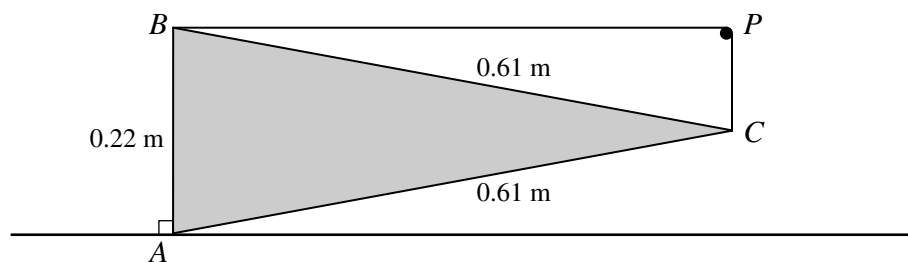
- (ii) Calculate the greatest deceleration of P . [3]

- 4 A particle P starts from rest at a point O and travels in a straight line. The acceleration of P is $(15 - 6x) \text{ m s}^{-2}$, where $x \text{ m}$ is the displacement of P from O .

- (i) Find the value of x for which P reaches its maximum velocity, and calculate this maximum velocity. [5]

- (ii) Calculate the acceleration of P when it is at instantaneous rest and $x > 0$. [2]

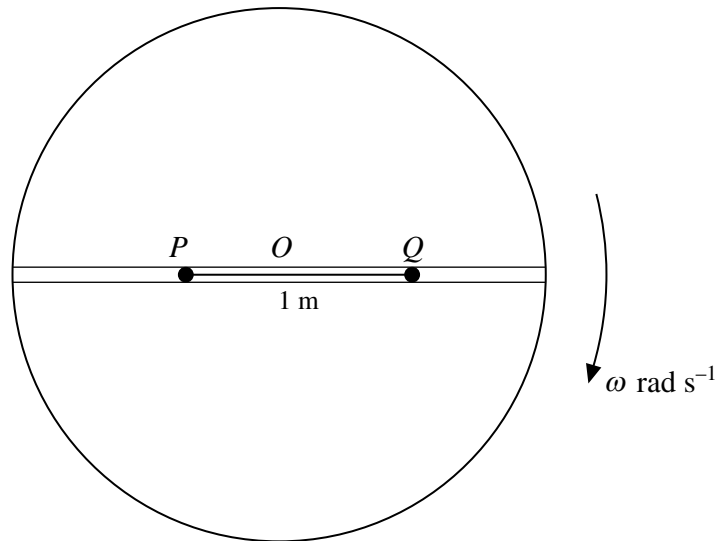
5



ABC is a uniform triangular lamina of weight 19 N , with $AB = 0.22\text{ m}$ and $AC = BC = 0.61\text{ m}$. The plane of the lamina is vertical. A rests on a rough horizontal surface, and AB is vertical. The equilibrium of the lamina is maintained by a light elastic string of natural length 0.7 m which passes over a small smooth peg P and is attached to B and C . The portion of the string attached to B is horizontal, and the portion of the string attached to C is vertical (see diagram).

- (i) Show that the tension in the string is 10 N . [3]
 - (ii) Calculate the modulus of elasticity of the string. [2]
 - (iii) Find the magnitude and direction of the force exerted by the surface on the lamina at A . [3]
- 6 A particle P is projected from a point O on horizontal ground. 0.4 s after the instant of projection, P is 5 m above the ground and a horizontal distance of 12 m from O .
- (i) Calculate the initial speed and the angle of projection of P . [6]
 - (ii) Find the direction of motion of the particle 0.4 s after the instant of projection. [3]

[Question 7 is printed on the next page.]



A narrow groove is cut along a diameter in the surface of a horizontal disc with centre O . Particles P and Q , of masses 0.2 kg and 0.3 kg respectively, lie in the groove, and the coefficient of friction between each of the particles and the groove is μ . The particles are attached to opposite ends of a light inextensible string of length 1 m . The disc rotates with angular velocity $\omega\text{ rad s}^{-1}$ about a vertical axis passing through O and the particles move in horizontal circles (see diagram).

- (i) Given that $\mu = 0.36$ and that both P and Q move in the same horizontal circle of radius 0.5 m , calculate the greatest possible value of ω and the corresponding tension in the string. [6]
- (ii) Given instead that $\mu = 0$ and that the tension in the string is 0.48 N , calculate
- the radius of the circle in which P moves and the radius of the circle in which Q moves, [3]
 - the speeds of the particles. [3]



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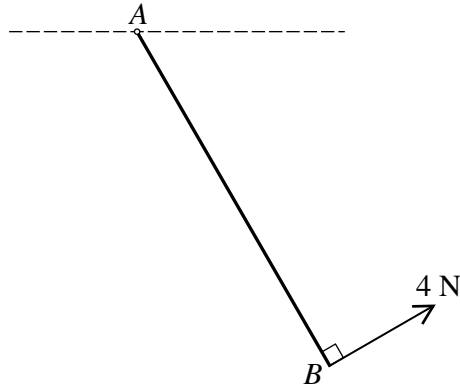
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1



A uniform rod AB of weight 16 N is freely hinged at A to a fixed point. A force of magnitude 4 N acting perpendicular to the rod is applied at B (see diagram). Given that the rod is in equilibrium,

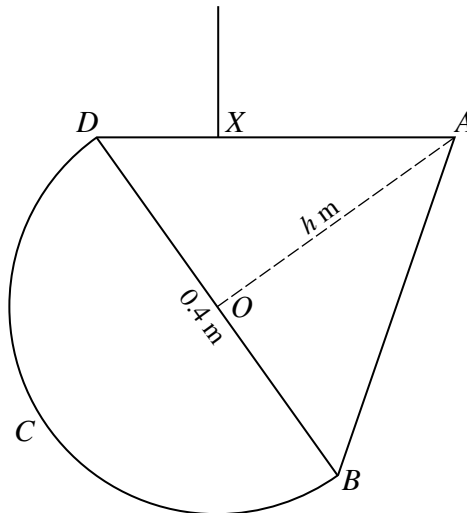
(i) calculate the angle the rod makes with the horizontal, [2]

(ii) find the magnitude and direction of the force exerted on the rod at A . [4]

- 2 A uniform lamina $ABCD$ consists of a semicircle BCD with centre O and diameter 0.4 m , and an isosceles triangle ABD with base $BD = 0.4\text{ m}$ and perpendicular height $h\text{ m}$. The centre of mass of the lamina is at O .

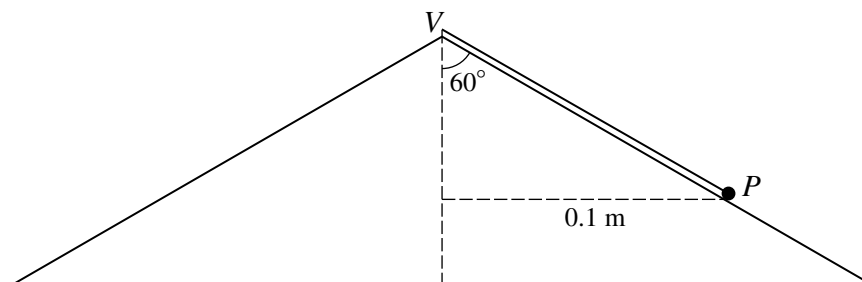
(i) Find the value of h . [4]

(ii)



The lamina is suspended from a vertical string attached to a point X on the side AD of the triangle (see diagram). Given the lamina is in equilibrium with AD horizontal, calculate XD . [3]

3



A particle P of mass 0.5 kg is attached to the vertex V of a fixed solid cone by a light inextensible string. P lies on the smooth curved surface of the cone and moves in a horizontal circle of radius 0.1 m with centre on the axis of the cone. The cone has semi-vertical angle 60° (see diagram).

- (i) Calculate the speed of P , given that the tension in the string and the contact force between the cone and P have the same magnitude. [4]
 - (ii) Calculate the greatest angular speed at which P can move on the surface of the cone. [4]
- 4 One end of a light elastic string of natural length 0.5 m and modulus of elasticity 12 N is attached to a fixed point O . The other end of the string is attached to a particle P of mass 0.24 kg . P is projected vertically upwards with speed 3 m s^{-1} from a position 0.8 m vertically below O .
- (i) Calculate the speed of the particle when it is moving upwards with zero acceleration. [5]
 - (ii) Show that the particle moves 0.6 m while it is moving upwards with constant acceleration. [4]
- 5 A particle P of mass 0.4 kg moves in a straight line on a horizontal surface and has velocity $v \text{ m s}^{-1}$ at time $t \text{ s}$. A horizontal force of magnitude $k\sqrt{v} \text{ N}$ opposes the motion of P . When $t = 0$, $v = 9$ and when $t = 2$, $v = 4$.
- (i) Express $\frac{dv}{dt}$ in terms of k and v , and hence show that $v = \frac{1}{4}(t - 6)^2$. [5]
 - (ii) Find the distance travelled by P in the first 3 seconds of its motion. [4]
- 6 A particle P is projected with speed 26 m s^{-1} at an angle of 30° below the horizontal, from a point O which is 80 m above horizontal ground.
- (i) Calculate the distance from O of the particle 2.3 s after projection. [4]
 - (ii) Find the horizontal distance travelled by P before it reaches the ground. [3]
 - (iii) Calculate the speed and direction of motion of P immediately before it reaches the ground. [4]

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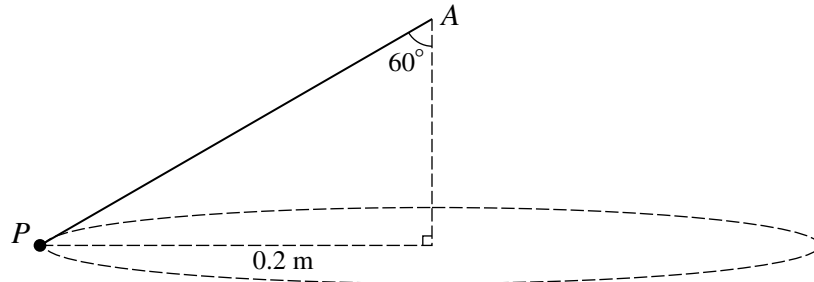
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A particle P of mass 0.4 kg is attached to a fixed point A by a light inextensible string. The string is inclined at 60° to the vertical. P moves with constant speed in a horizontal circle of radius 0.2 m . The centre of the circle is vertically below A (see diagram).

(i) Show that the tension in the string is 8 N . [2]

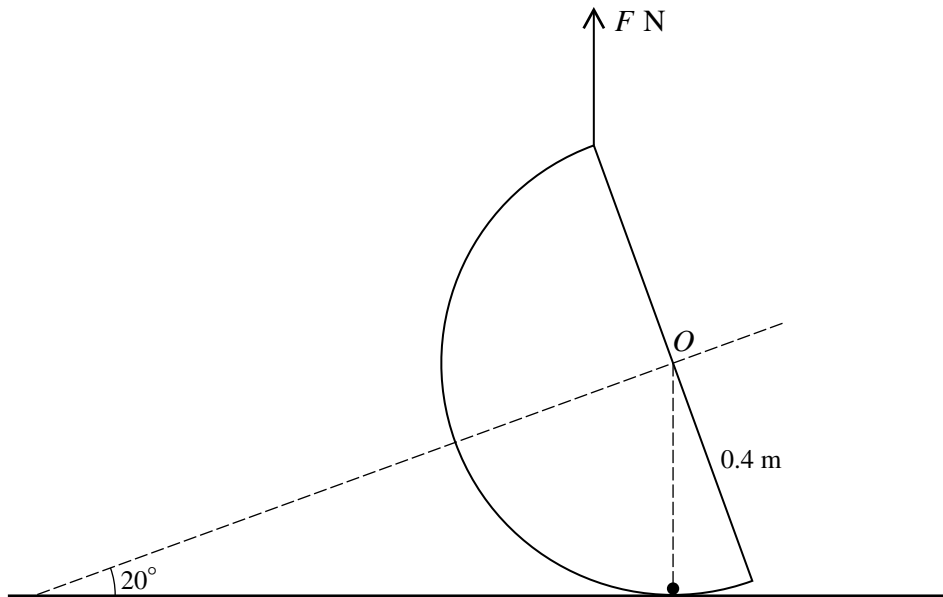
(ii) Calculate the speed of the particle. [2]

2 A stone is thrown with speed 15 m s^{-1} horizontally from the top of a vertical cliff 20 m above the sea. Calculate

(i) the distance from the foot of the cliff to the point where the stone enters the sea, [3]

(ii) the speed of the stone when it enters the sea. [3]

3



A smooth hemispherical shell, with centre O , weight 12 N and radius 0.4 m , rests on a horizontal plane. A particle of weight $W\text{ N}$ lies at rest on the inner surface of the hemisphere vertically below O . A force of magnitude $F\text{ N}$ acting vertically upwards is applied to the highest point of the hemisphere, which is in equilibrium with its axis of symmetry inclined at 20° to the horizontal (see diagram).

(i) Show, by taking moments about O , that $F = 16.48$ correct to 4 significant figures. [3]

(ii) Find the normal contact force exerted by the plane on the hemisphere in terms of W . Hence find the least possible value of W . [3]

- 4 The ends of a light elastic string of natural length 0.8 m and modulus of elasticity λ N are attached to fixed points A and B which are 1.2 m apart at the same horizontal level. A particle of mass 0.3 kg is attached to the centre of the string, and released from rest at the mid-point of AB . The particle descends 0.32 m vertically before coming to instantaneous rest.

(i) Calculate λ . [4]

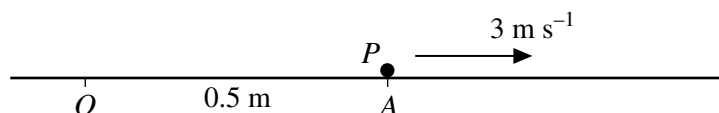
(ii) Calculate the speed of the particle when it is 0.25 m below AB . [4]

- 5 One end of a light elastic string of natural length 0.3 m and modulus of elasticity 6 N is attached to a fixed point O on a smooth horizontal plane. The other end of the string is attached to a particle P of mass 0.2 kg, which moves on the plane in a circular path with centre O . The angular speed of P is ω rad s⁻¹.

(i) For the case $\omega = 5$, calculate the extension of the string. [4]

(ii) Express the extension of the string in terms of ω , and hence find the set of possible value of ω . [4]

6

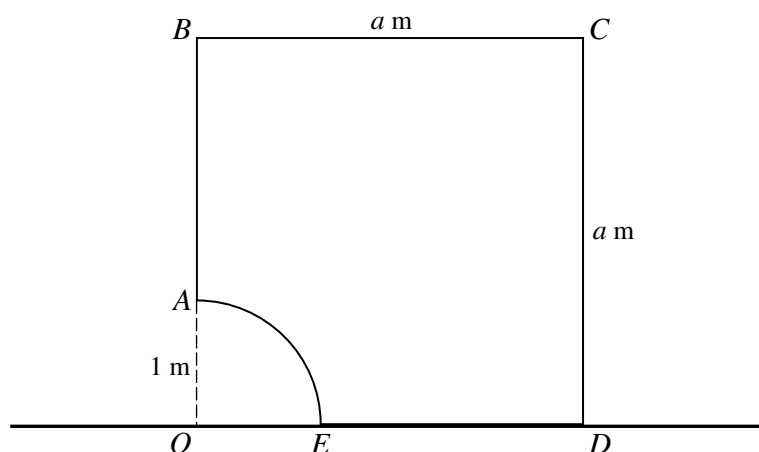


O and A are fixed points on a horizontal surface, with $OA = 0.5$ m. A particle P of mass 0.2 kg is projected horizontally with speed 3 m s^{-1} from A in the direction OA and moves in a straight line (see diagram). At time t s after projection, the velocity of P is $v \text{ m s}^{-1}$ and its displacement from O is x m. The coefficient of friction between the surface and P is 0.5, and a force of magnitude $\frac{0.4}{x^2}$ N acts on P in the direction PO .

(i) Show that, while the particle is in motion, $v \frac{dv}{dx} = -\left(5 + \frac{2}{x^2}\right)$. [2]

(ii) Calculate the distance travelled by P before it comes to rest, and show that P does not subsequently move. [7]

[Question 7 is printed on the next page.]



$ABCDE$ is the cross-section through the centre of mass of a uniform prism resting in equilibrium with DE on a horizontal surface. The cross-section has the shape of a square $OBCD$ with sides of length a m, from which a quadrant OAE of a circle of radius 1 m has been removed (see diagram).

- (i) Find the distance of the centre of mass of the prism from O , giving the answer in terms of a , π and $\sqrt{2}$. [5]

- (ii) Hence show that

$$3a^2(2 - a) < \frac{3}{2}\pi - 2,$$

and verify that this inequality is satisfied by $a = 1.68$ but not by $a = 1.67$. [4]