



UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS

9709/31

Paper 3 Pure Mathematics 3 (P3)

May/June 2011

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
 Graph Paper
 List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.

Write your Centre number, candidate number and name on all the work you hand in.

Write in dark blue or black pen.

You may use a soft pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 75.

Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of 4 printed pages.



- 1 Expand $\sqrt[3]{(1 - 6x)}$ in ascending powers of x up to and including the term in x^3 , simplifying the coefficients. [4]

- 2 Find $\frac{dy}{dx}$ in each of the following cases:

(i) $y = \ln(1 + \sin 2x)$, [2]

(ii) $y = \frac{\tan x}{x}$. [2]

- 3 Points A and B have coordinates $(-1, 2, 5)$ and $(2, -2, 11)$ respectively. The plane p passes through B and is perpendicular to AB .

(i) Find an equation of p , giving your answer in the form $ax + by + cz = d$. [3]

(ii) Find the acute angle between p and the y -axis. [4]

- 4 The polynomial $f(x)$ is defined by

$$f(x) = 12x^3 + 25x^2 - 4x - 12.$$

(i) Show that $f(-2) = 0$ and factorise $f(x)$ completely. [4]

(ii) Given that

$$12 \times 27^y + 25 \times 9^y - 4 \times 3^y - 12 = 0,$$

state the value of 3^y and hence find y correct to 3 significant figures. [3]

- 5 The curve with equation

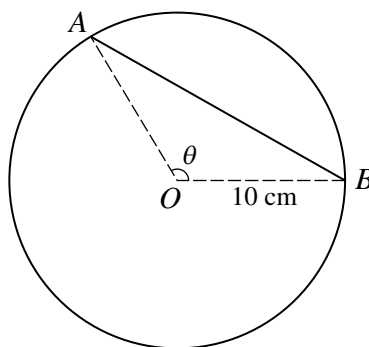
$$6e^{2x} + ke^y + e^{2y} = c,$$

where k and c are constants, passes through the point P with coordinates $(\ln 3, \ln 2)$.

(i) Show that $58 + 2k = c$. [2]

(ii) Given also that the gradient of the curve at P is -6 , find the values of k and c . [5]

6



The diagram shows a circle with centre O and radius 10 cm. The chord AB divides the circle into two regions whose areas are in the ratio 1 : 4 and it is required to find the length of AB . The angle AOB is θ radians.

(i) Show that $\theta = \frac{2}{5}\pi + \sin \theta$. [3]

(ii) Showing all your working, use an iterative formula, based on the equation in part (i), with an initial value of 2.1, to find θ correct to 2 decimal places. Hence find the length of AB in centimetres correct to 1 decimal place. [5]

7 The integral I is defined by $I = \int_0^2 4t^3 \ln(t^2 + 1) dt$.

(i) Use the substitution $x = t^2 + 1$ to show that $I = \int_1^5 (2x - 2) \ln x dx$. [3]

(ii) Hence find the exact value of I . [5]

8 The complex number u is defined by $u = \frac{6 - 3i}{1 + 2i}$.

(i) Showing all your working, find the modulus of u and show that the argument of u is $-\frac{1}{2}\pi$. [4]

(ii) For complex numbers z satisfying $\arg(z - u) = \frac{1}{4}\pi$, find the least possible value of $|z|$. [3]

(iii) For complex numbers z satisfying $|z - (1 + i)u| = 1$, find the greatest possible value of $|z|$. [3]

9 (i) Prove the identity $\cos 4\theta + 4 \cos 2\theta \equiv 8 \cos^4 \theta - 3$. [4]

(ii) Hence

(a) solve the equation $\cos 4\theta + 4 \cos 2\theta = 1$ for $-\frac{1}{2}\pi \leq \theta \leq \frac{1}{2}\pi$, [3]

(b) find the exact value of $\int_0^{\frac{1}{4}\pi} \cos^4 \theta d\theta$. [3]

[Question 10 is printed on the next page.]

- 10** The number of birds of a certain species in a forested region is recorded over several years. At time t years, the number of birds is N , where N is treated as a continuous variable. The variation in the number of birds is modelled by

$$\frac{dN}{dt} = \frac{N(1800 - N)}{3600}.$$

It is given that $N = 300$ when $t = 0$.

(i) Find an expression for N in terms of t . [9]

(ii) According to the model, how many birds will there be after a long time? [1]



UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS

9709/32

Paper 3 Pure Mathematics 3 (P3)

May/June 2011

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
 Graph Paper
 List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.

Write your Centre number, candidate number and name on all the work you hand in.

Write in dark blue or black pen.

You may use a soft pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 75.

Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **3** printed pages and **1** blank page.



- 1 Solve the inequality $|x| < |5 + 2x|$. [3]

- 2 (i) Show that the equation

$$\log_2(x + 5) = 5 - \log_2 x$$

can be written as a quadratic equation in x . [3]

- (ii) Hence solve the equation

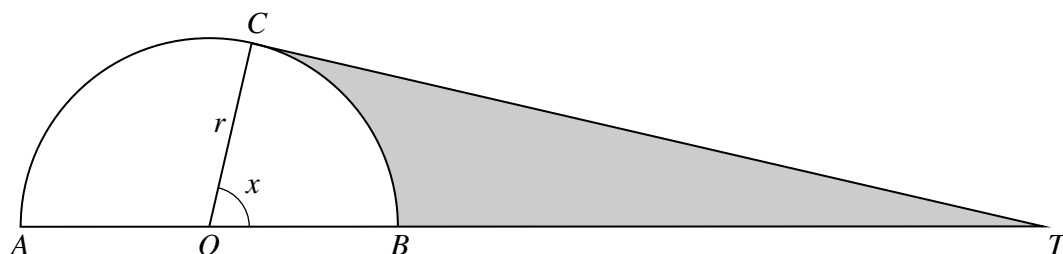
$$\log_2(x + 5) = 5 - \log_2 x. \quad [2]$$

- 3 Solve the equation

$$\cos \theta + 4 \cos 2\theta = 3,$$

giving all solutions in the interval $0^\circ \leq \theta \leq 180^\circ$. [5]

4



The diagram shows a semicircle ACB with centre O and radius r . The tangent at C meets AB produced at T . The angle BOC is x radians. The area of the shaded region is equal to the area of the semicircle.

- (i) Show that x satisfies the equation

$$\tan x = x + \pi. \quad [3]$$

- (ii) Use the iterative formula $x_{n+1} = \tan^{-1}(x_n + \pi)$ to determine x correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

- 5 The parametric equations of a curve are

$$x = \ln(\tan t), \quad y = \sin^2 t,$$

where $0 < t < \frac{1}{2}\pi$.

- (i) Express $\frac{dy}{dx}$ in terms of t . [4]

- (ii) Find the equation of the tangent to the curve at the point where $x = 0$. [3]

- 6 A certain curve is such that its gradient at a point (x, y) is proportional to xy . At the point $(1, 2)$ the gradient is 4.

(i) By setting up and solving a differential equation, show that the equation of the curve is $y = 2e^{x^2-1}$. [7]

(ii) State the gradient of the curve at the point $(-1, 2)$ and sketch the curve. [2]

- 7 (a) The complex number u is defined by $u = \frac{5}{a+2i}$, where the constant a is real.

(i) Express u in the form $x + iy$, where x and y are real. [2]

(ii) Find the value of a for which $\arg(u^*) = \frac{3}{4}\pi$, where u^* denotes the complex conjugate of u . [3]

- (b) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z which satisfy both the inequalities $|z| < 2$ and $|z| < |z - 2 - 2i|$. [4]

- 8 (i) Express $\frac{5x - x^2}{(1+x)(2+x^2)}$ in partial fractions. [5]

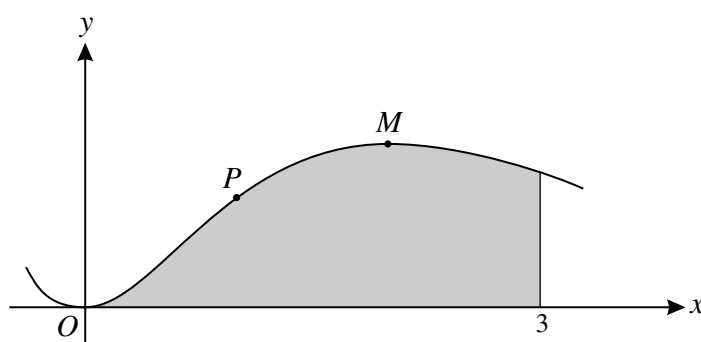
(ii) Hence obtain the expansion of $\frac{5x - x^2}{(1+x)(2+x^2)}$ in ascending powers of x , up to and including the term in x^3 . [5]

- 9 Two planes have equations $x + 2y - 2z = 7$ and $2x + y + 3z = 5$.

(i) Calculate the acute angle between the planes. [4]

(ii) Find a vector equation for the line of intersection of the planes. [6]

10



The diagram shows the curve $y = x^2 e^{-x}$.

(i) Show that the area of the shaded region bounded by the curve, the x -axis and the line $x = 3$ is equal to $2 - \frac{17}{e^3}$. [5]

(ii) Find the x -coordinate of the maximum point M on the curve. [4]

(iii) Find the x -coordinate of the point P at which the tangent to the curve passes through the origin. [2]

BLANK PAGE

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

University of Cambridge International Examinations is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.



UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS

9709/33

Paper 3 Pure Mathematics 3 (P3)

May/June 2011

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.

Write your Centre number, candidate number and name on all the work you hand in.

Write in dark blue or black pen.

You may use a soft pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 75.

Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **3** printed pages and **1** blank page.



- 1 Use logarithms to solve the equation $5^{2x-1} = 2(3^x)$, giving your answer correct to 3 significant figures. [4]

- 2 The curve $y = \frac{\ln x}{x^3}$ has one stationary point. Find the x -coordinate of this point. [4]

- 3 Show that $\int_0^1 (1-x)e^{-\frac{1}{2}x} dx = 4e^{-\frac{1}{2}} - 2$. [5]

- 4 (i) Show that the equation

$$\tan(60^\circ + \theta) + \tan(60^\circ - \theta) = k$$

can be written in the form

$$(2\sqrt{3})(1 + \tan^2 \theta) = k(1 - 3 \tan^2 \theta). \quad [4]$$

- (ii) Hence solve the equation

$$\tan(60^\circ + \theta) + \tan(60^\circ - \theta) = 3\sqrt{3},$$

giving all solutions in the interval $0^\circ \leq \theta \leq 180^\circ$. [3]

- 5 The polynomial $ax^3 + bx^2 + 5x - 2$, where a and b are constants, is denoted by $p(x)$. It is given that $(2x - 1)$ is a factor of $p(x)$ and that when $p(x)$ is divided by $(x - 2)$ the remainder is 12.

- (i) Find the values of a and b . [5]

- (ii) When a and b have these values, find the quadratic factor of $p(x)$. [2]

- 6 (i) By sketching a suitable pair of graphs, show that the equation

$$\cot x = 1 + x^2,$$

where x is in radians, has only one root in the interval $0 < x < \frac{1}{2}\pi$. [2]

- (ii) Verify by calculation that this root lies between 0.5 and 0.8. [2]

- (iii) Use the iterative formula

$$x_{n+1} = \tan^{-1} \left(\frac{1}{1 + x_n^2} \right)$$

to determine this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

- 7 (i) Find the roots of the equation

$$z^2 + (2\sqrt{3})z + 4 = 0,$$

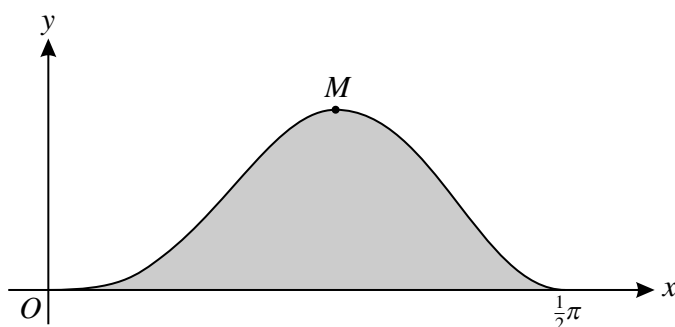
giving your answers in the form $x + iy$, where x and y are real. [2]

- (ii) State the modulus and argument of each root. [3]

- (iii) Showing all your working, verify that each root also satisfies the equation

$$z^6 = -64. \quad [3]$$

8



The diagram shows the curve $y = 5 \sin^3 x \cos^2 x$ for $0 \leq x \leq \frac{1}{2}\pi$, and its maximum point M .

- (i) Find the x -coordinate of M . [5]

- (ii) Using the substitution $u = \cos x$, find by integration the area of the shaded region bounded by the curve and the x -axis. [5]

- 9 In a chemical reaction, a compound X is formed from two compounds Y and Z . The masses in grams of X , Y and Z present at time t seconds after the start of the reaction are x , $10 - x$ and $20 - x$ respectively. At any time the rate of formation of X is proportional to the product of the masses of Y and Z present at the time. When $t = 0$, $x = 0$ and $\frac{dx}{dt} = 2$.

- (i) Show that x and t satisfy the differential equation

$$\frac{dx}{dt} = 0.01(10 - x)(20 - x). \quad [1]$$

- (ii) Solve this differential equation and obtain an expression for x in terms of t . [9]

- (iii) State what happens to the value of x when t becomes large. [1]

- 10 With respect to the origin O , the lines l and m have vector equations $\mathbf{r} = 2\mathbf{i} + \mathbf{k} + \lambda(\mathbf{i} - \mathbf{j} + 2\mathbf{k})$ and $\mathbf{r} = 2\mathbf{j} + 6\mathbf{k} + \mu(\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})$ respectively.

- (i) Prove that l and m do not intersect. [4]

- (ii) Calculate the acute angle between the directions of l and m . [3]

- (iii) Find the equation of the plane which is parallel to l and contains m , giving your answer in the form $ax + by + cz = d$. [5]

BLANK PAGE

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

University of Cambridge International Examinations is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.