



UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS

9709/31

Paper 3 Pure Mathematics 3 (P3)

May/June 2010

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
 Graph Paper
 List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.

Write your Centre number, candidate number and name on all the work you hand in.

Write in dark blue or black pen.

You may use a soft pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 75.

Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **4** printed pages.



- 1 Solve the inequality $|x + 3a| > 2|x - 2a|$, where a is a positive constant. [4]

- 2 Solve the equation

$$\sin \theta = 2 \cos 2\theta + 1,$$

giving all solutions in the interval $0^\circ \leq \theta \leq 360^\circ$. [6]

- 3 The variables x and y satisfy the equation $x^n y = C$, where n and C are constants. When $x = 1.10$, $y = 5.20$, and when $x = 3.20$, $y = 1.05$.

(i) Find the values of n and C . [5]

(ii) Explain why the graph of $\ln y$ against $\ln x$ is a straight line. [1]

- 4 (i) Using the expansions of $\cos(3x - x)$ and $\cos(3x + x)$, prove that

$$\frac{1}{2}(\cos 2x - \cos 4x) \equiv \sin 3x \sin x. [3]$$

(ii) Hence show that

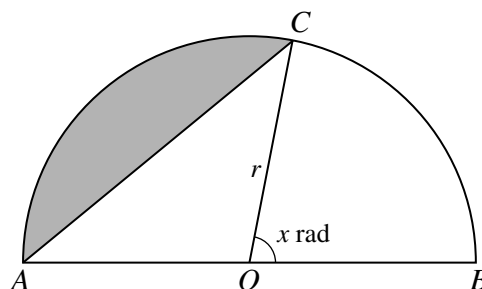
$$\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} \sin 3x \sin x \, dx = \frac{1}{8}\sqrt{3}. [3]$$

- 5 Given that $y = 0$ when $x = 1$, solve the differential equation

$$xy \frac{dy}{dx} = y^2 + 4,$$

obtaining an expression for y^2 in terms of x . [6]

6



The diagram shows a semicircle ACB with centre O and radius r . The angle BOC is x radians. The area of the shaded segment is a quarter of the area of the semicircle.

(i) Show that x satisfies the equation

$$x = \frac{3}{4}\pi - \sin x. \quad [3]$$

(ii) This equation has one root. Verify by calculation that the root lies between 1.3 and 1.5. [2]

(iii) Use the iterative formula

$$x_{n+1} = \frac{3}{4}\pi - \sin x_n$$

to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

7 The complex number $2 + 2i$ is denoted by u .

(i) Find the modulus and argument of u . [2]

(ii) Sketch an Argand diagram showing the points representing the complex numbers 1, i and u . Shade the region whose points represent the complex numbers z which satisfy both the inequalities $|z - 1| \leq |z - i|$ and $|z - u| \leq 1$. [4]

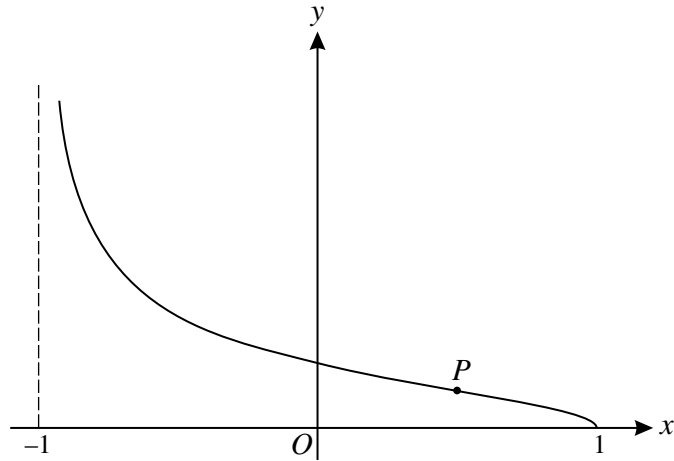
(iii) Using your diagram, calculate the value of $|z|$ for the point in this region for which $\arg z$ is least. [3]

8 (i) Express $\frac{2}{(x+1)(x+3)}$ in partial fractions. [2]

(ii) Using your answer to part (i), show that

$$\left(\frac{2}{(x+1)(x+3)} \right)^2 \equiv \frac{1}{(x+1)^2} - \frac{1}{x+1} + \frac{1}{x+3} + \frac{1}{(x+3)^2}. \quad [2]$$

(iii) Hence show that $\int_0^1 \frac{4}{(x+1)^2(x+3)^2} dx = \frac{7}{12} - \ln \frac{3}{2}$. [5]



The diagram shows the curve $y = \sqrt{\left(\frac{1-x}{1+x}\right)}$.

- (i) By first differentiating $\frac{1-x}{1+x}$, obtain an expression for $\frac{dy}{dx}$ in terms of x . Hence show that the gradient of the normal to the curve at the point (x, y) is $(1+x)\sqrt{(1-x^2)}$. [5]
- (ii) The gradient of the normal to the curve has its maximum value at the point P shown in the diagram. Find, by differentiation, the x -coordinate of P . [4]

10 The lines l and m have vector equations

$$\mathbf{r} = \mathbf{i} + \mathbf{j} + \mathbf{k} + s(\mathbf{i} - \mathbf{j} + 2\mathbf{k}) \quad \text{and} \quad \mathbf{r} = 4\mathbf{i} + 6\mathbf{j} + \mathbf{k} + t(2\mathbf{i} + 2\mathbf{j} + \mathbf{k})$$

respectively.

- (i) Show that l and m intersect. [4]
- (ii) Calculate the acute angle between the lines. [3]
- (iii) Find the equation of the plane containing l and m , giving your answer in the form $ax + by + cz = d$. [5]



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- 1 Solve the equation

$$\frac{2^x + 1}{2^x - 1} = 5,$$

giving your answer correct to 3 significant figures.

[4]

- 2 Show that $\int_0^\pi x^2 \sin x \, dx = \pi^2 - 4$.

[5]

- 3 It is given that $\cos a = \frac{3}{5}$, where $0^\circ < a < 90^\circ$. Showing your working and without using a calculator to evaluate a ,

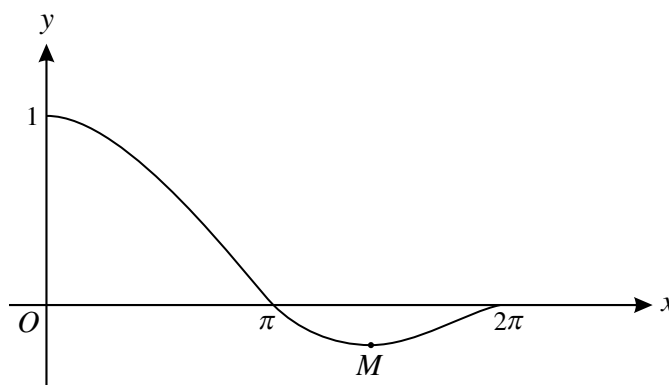
(i) find the exact value of $\sin(a - 30^\circ)$,

[3]

(ii) find the exact value of $\tan 2a$, and hence find the exact value of $\tan 3a$.

[4]

4



The diagram shows the curve $y = \frac{\sin x}{x}$ for $0 < x \leq 2\pi$, and its minimum point M .

(i) Show that the x -coordinate of M satisfies the equation

$$x = \tan x.$$

[4]

(ii) The iterative formula

$$x_{n+1} = \tan^{-1}(x_n) + \pi$$

can be used to determine the x -coordinate of M . Use this formula to determine the x -coordinate of M correct to 2 decimal places. Give the result of each iteration to 4 decimal places.

[3]

- 5 The polynomial $2x^3 + 5x^2 + ax + b$, where a and b are constants, is denoted by $p(x)$. It is given that $(2x + 1)$ is a factor of $p(x)$ and that when $p(x)$ is divided by $(x + 2)$ the remainder is 9.

(i) Find the values of a and b .

[5]

(ii) When a and b have these values, factorise $p(x)$ completely.

[3]

- 6 The equation of a curve is

$$x \ln y = 2x + 1.$$

(i) Show that $\frac{dy}{dx} = -\frac{y}{x^2}$. [4]

- (ii) Find the equation of the tangent to the curve at the point where $y = 1$, giving your answer in the form $ax + by + c = 0$. [4]

- 7 The variables x and t are related by the differential equation

$$e^{2t} \frac{dx}{dt} = \cos^2 x,$$

where $t \geq 0$. When $t = 0$, $x = 0$.

- (i) Solve the differential equation, obtaining an expression for x in terms of t . [6]

- (ii) State what happens to the value of x when t becomes very large. [1]

- (iii) Explain why x increases as t increases. [1]

- 8 The variable complex number z is given by

$$z = 1 + \cos 2\theta + i \sin 2\theta,$$

where θ takes all values in the interval $-\frac{1}{2}\pi < \theta < \frac{1}{2}\pi$.

- (i) Show that the modulus of z is $2 \cos \theta$ and the argument of z is θ . [6]

- (ii) Prove that the real part of $\frac{1}{z}$ is constant. [3]

- 9 The plane p has equation $3x + 2y + 4z = 13$. A second plane q is perpendicular to p and has equation $ax + y + z = 4$, where a is a constant.

- (i) Find the value of a . [3]

- (ii) The line with equation $\mathbf{r} = \mathbf{j} - \mathbf{k} + \lambda(\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$ meets the plane p at the point A and the plane q at the point B . Find the length of AB . [6]

- 10 (i) Find the values of the constants A , B , C and D such that

$$\frac{2x^3 - 1}{x^2(2x - 1)} \equiv A + \frac{B}{x} + \frac{C}{x^2} + \frac{D}{2x - 1}. \quad [5]$$

- (ii) Hence show that

$$\int_1^2 \frac{2x^3 - 1}{x^2(2x - 1)} dx = \frac{3}{2} + \frac{1}{2} \ln\left(\frac{16}{27}\right). \quad [5]$$

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1 Solve the inequality $|x - 3| > 2|x + 1|$. [4]

2 The variables x and y satisfy the equation $y^3 = Ae^{2x}$, where A is a constant. The graph of $\ln y$ against x is a straight line.

(i) Find the gradient of this line. [2]

(ii) Given that the line intersects the axis of $\ln y$ at the point where $\ln y = 0.5$, find the value of A correct to 2 decimal places. [2]

3 Solve the equation

$$\tan(45^\circ - x) = 2 \tan x,$$

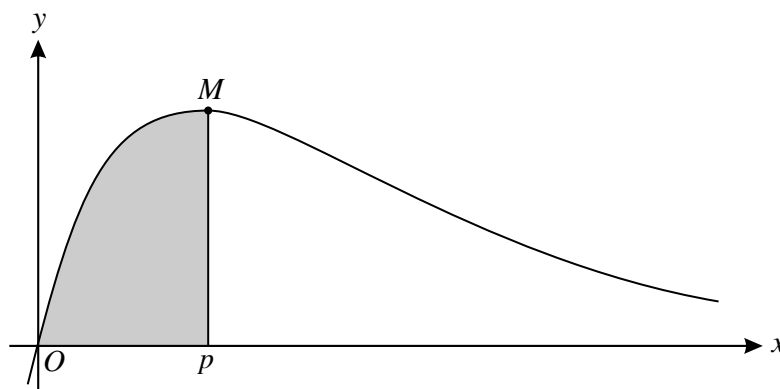
giving all solutions in the interval $0^\circ < x < 180^\circ$. [5]

4 Given that $x = 1$ when $t = 0$, solve the differential equation

$$\frac{dx}{dt} = \frac{1}{x} - \frac{x}{4},$$

obtaining an expression for x^2 in terms of t . [7]

5



The diagram shows the curve $y = e^{-x} - e^{-2x}$ and its maximum point M . The x -coordinate of M is denoted by p .

(i) Find the exact value of p . [4]

(ii) Show that the area of the shaded region bounded by the curve, the x -axis and the line $x = p$ is equal to $\frac{1}{8}$. [4]

- 6 The curve $y = \frac{\ln x}{x+1}$ has one stationary point.

(i) Show that the x -coordinate of this point satisfies the equation

$$x = \frac{x+1}{\ln x},$$

and that this x -coordinate lies between 3 and 4. [5]

(ii) Use the iterative formula

$$x_{n+1} = \frac{x_n + 1}{\ln x_n}$$

to determine the x -coordinate correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

- 7 (i) Prove the identity $\cos 3\theta \equiv 4 \cos^3 \theta - 3 \cos \theta$. [4]

(ii) Using this result, find the exact value of

$$\int_{\frac{1}{3}\pi}^{\frac{1}{2}\pi} \cos^3 \theta \, d\theta. [4]$$

- 8 (a) The equation $2x^3 - x^2 + 2x + 12 = 0$ has one real root and two complex roots. Showing your working, verify that $1 + i\sqrt{3}$ is one of the complex roots. State the other complex root. [4]

(b) On a sketch of an Argand diagram, show the point representing the complex number $1 + i\sqrt{3}$. On the same diagram, shade the region whose points represent the complex numbers z which satisfy both the inequalities $|z - 1 - i\sqrt{3}| \leq 1$ and $\arg z \leq \frac{1}{3}\pi$. [5]

- 9 (i) Express $\frac{4 + 5x - x^2}{(1 - 2x)(2 + x)^2}$ in partial fractions. [5]

(ii) Hence obtain the expansion of $\frac{4 + 5x - x^2}{(1 - 2x)(2 + x)^2}$ in ascending powers of x , up to and including the term in x^2 . [5]

- 10 The straight line l has equation $\mathbf{r} = 2\mathbf{i} - \mathbf{j} - 4\mathbf{k} + \lambda(\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$. The plane p has equation $3x - y + 2z = 9$. The line l intersects the plane p at the point A .

(i) Find the position vector of A . [3]

(ii) Find the acute angle between l and p . [4]

(iii) Find an equation for the plane which contains l and is perpendicular to p , giving your answer in the form $ax + by + cz = d$. [5]

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