Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER | | | | | |

CANDIDATE NUMBER | | | | |

9469423944

FURTHER MATHEMATICS **9231/41**

Paper 4 Further Probability & Statistics **October/November 2023**

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **12** pages. Any blank pages are indicated.

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1 Maya is an athlete who competes in 1500-metre races. Last summer her practice run times had mean 4.22 minutes. Over the winter she has done some intense training to try to improve her times. A random sample of 10 of her practice run times, x minutes, this summer are summarised as follows.

$$\sum x = 42.05 \qquad \sum x^2 = 176.83$$

Maya's new practice run times are normally distributed. She believes that on average her times have improved as a result of her training.

Test, at the 5% significance level, whether Maya's belief is supported by the data. [6]

2 A town council has published its plans for redeveloping the town centre and residents are being asked whether they approve or disapprove. A random sample of 250 responses has been selected from residents in the four main streets in the town: North, East, South and West Streets. The results are shown in the table.

	North Street	East Street	South Street	West Street
Approve	33	54	42	26
Disapprove	19	39	28	9

Test, at the 5% significance level, whether the opinions of the residents are independent of the streets on which they live. [7]

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3 Scientists are studying the effects of exercise on LDL blood cholesterol levels. Over a three-month period, a large group of people exercised for 20 minutes each day. For a randomly chosen sample of 10 of these people, the LDL blood cholesterol levels were measured at the beginning and the end of the three-month period. The results, measured in suitable units, are as follows.

	Person	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*
Cholesterol level	Beginning	72	84	120	90	102	135	64	75	80	88
	End	64	76	105	92	105	115	67	75	75	84

(a) Test, at the 2.5% significance level, whether there is evidence that the population mean LDL blood cholesterol level has reduced by more than 2 units after the three-month period. [7]

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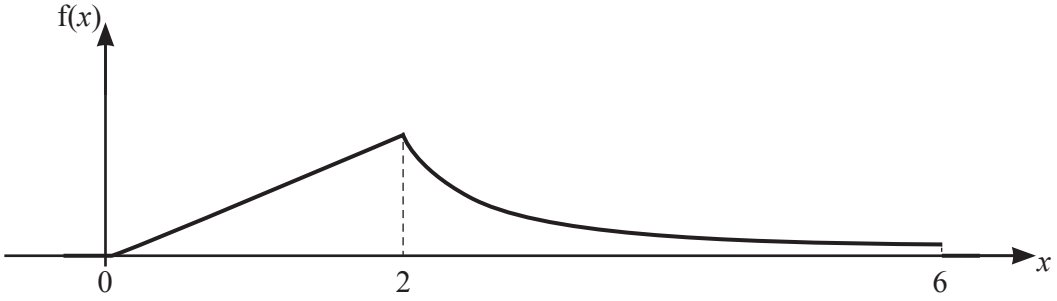
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(b) State any assumption that you have made in part **(a)**. [1]

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As shown in the diagram, the continuous random variable X has probability density function f given by

$$\mathrm{f}(x) = \begin{cases} mx & 0 \leqslant x \leqslant 2, \\ \dfrac{k}{x^2} + c & 2 \leqslant x \leqslant 6, \\ 0 & \text{otherwise,} \end{cases}$$

where m, k and c are constants.

(a) Given that $\mathrm{P}(X \leqslant 2) = \frac{1}{3}$, show that $m = \frac{1}{6}$ and find the values of k and c. [4]

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(b) Find the exact numerical value of the interquartile range of X. [5]

5 The random variable X has the geometric distribution Geo(p).

(a) Show that the probability generating function of X is $\frac{pt}{1-qt}$, where $q = 1-p$. [3]

(b) Use the probability generating function of X to show that $\mathrm{Var}(X) = \frac{q}{p^2}$. [5]

Kenny throws an ordinary fair 6-sided dice repeatedly. The random variable X is the number of throws that Kenny takes in order to obtain a 6. The random variable Z denotes the sum of two independent values of X.

(c) Find the probability generating function of Z. [2]

6 A school is conducting an experiment to see whether the distance that children can throw a ball increases in hot weather. On a cold day, all the children at the school were asked to throw a ball as far as possible. The distances thrown were measured and recorded. The median distance thrown by a random sample of 25 of the children was 22.0 m. The children were asked to throw the ball again on a hot day. The distances thrown by the same 25 children were measured and recorded and these distances, in m, are shown below.

21.2	23.5	22.9	18.6	19.4
22.1	26.5	20.2	25.7	20.6
22.3	17.4	22.2	27.0	23.9
28.2	22.6	27.2	23.0	23.7
19.8	22.7	23.3	21.5	24.3

The teacher claims that on average the distances thrown will be further when it is hot.

Carry out a Wilcoxon signed-rank test, at the 5% significance level, to test whether the data supports the teacher's claim. [10]

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Additional Page

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