Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME									
CENTRE NUMBER						CANDIDATE NUMBER				

4639031271

FURTHER MATHEMATICS **9231/11**

Paper 1 Further Pure Mathematics 1 **October/November 2023**

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

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1 **(a)** By considering $(r+1)^2 - r^2$, use the method of differences to prove that

$$\sum_{r=1}^{n} r = \tfrac{1}{2}n(n+1).$$ [4]

(b) Given that $\sum_{r=1}^{n}(r+a) = n$, find a in terms of n. [3]

2 Prove by mathematical induction that, for all positive integers n,

$$1+2x+3x^2+\ldots+nx^{n-1}=\frac{1-(n+1)x^n+nx^{n+1}}{(1-x)^2}.$$ [6]

3 The quartic equation $x^4 + bx^3 + cx^2 + dx - 2 = 0$ has roots α, β, γ, δ. It is given that

$$\alpha + \beta + \gamma + \delta = 3, \qquad \alpha^2 + \beta^2 + \gamma^2 + \delta^2 = 5, \qquad \alpha^{-1} + \beta^{-1} + \gamma^{-1} + \delta^{-1} = 6.$$

(a) Find the values of b, c and d. [6]

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(b) Given also that $\alpha^3 + \beta^3 + \gamma^3 + \delta^3 = -27$, find the value of $\alpha^4 + \beta^4 + \gamma^4 + \delta^4$. [2]

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4 The lines l_1 and l_2 have equations

$$\mathbf{r} = -2\mathbf{i} - 3\mathbf{j} - 5\mathbf{k} + \lambda(-4\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}) \quad \text{and} \quad \mathbf{r} = 2\mathbf{i} - 2\mathbf{j} + 3\mathbf{k} + \mu(2\mathbf{i} - 3\mathbf{j} + \mathbf{k})$$

respectively.

(a) Find the shortest distance between l_1 and l_2. [5]

The plane Π contains l_1 and the point with position vector $-\mathbf{i}-3\mathbf{j}-4\mathbf{k}$.

(b) Find an equation of Π, giving your answer in the form $ax+by+cz=d$. [4]

5 Let k be a constant. The matrices **A**, **B** and **C** are given by

$$\mathbf{A} = \begin{pmatrix} 1 & k & 3 \\ 2 & 1 & 3 \\ 3 & 2 & 5 \end{pmatrix}, \qquad \mathbf{B} = \begin{pmatrix} 0 & -2 \\ -1 & 3 \\ 0 & 0 \end{pmatrix} \qquad \text{and} \qquad \mathbf{C} = \begin{pmatrix} -2 & -1 & 1 \\ 1 & 1 & 3 \end{pmatrix}.$$

It is given that **A** is singular.

(a) Show that $\mathbf{CAB} = \begin{pmatrix} 3 & -7 \\ -9 & 3 \end{pmatrix}$. [5]

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(b) Find the equations of the invariant lines, through the origin, of the transformation in the $x-y$ plane represented by **CAB**. [5]

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(c) The matrices **D**, **E** and **F** represent geometrical transformations in the $x-y$ plane.

- **D** represents an enlargement, centre the origin.
- **E** represents a stretch parallel to the x-axis.
- **F** represents a reflection in the line $y = x$.

Given that $\mathbf{CAB} = \mathbf{D} - 9\mathbf{EF}$, find **D**, **E** and **F**. [5]

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6 **(a)** Show that the curve with Cartesian equation

$$\left(x-\tfrac{1}{2}\right)^2+y^2=\tfrac{1}{4}$$

has polar equation $r=\cos\theta$. [3]

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The curves C_1 and C_2 have polar equations

$$r=\cos\theta \quad \text{and} \quad r=\sin 2\theta$$

respectively, where $0 \leqslant \theta \leqslant \frac{1}{2}\pi$. The curves C_1 and C_2 intersect at the pole and at another point P.

(b) Find the polar coordinates of P. [3]

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(c) In a single diagram sketch C_1 and C_2, clearly identifying each curve, and mark the point P. [3]

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(d) The region R is enclosed by C_1 and C_2 and includes the line OP.

Find, in exact form, the area of R. [6]

7 The curve C has equation $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \dfrac{x^2+2}{x^2-x-2}$.

(a) Find the equations of the asymptotes of C. [2]

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(b) Find the coordinates of any stationary points on C, giving your answers correct to 1 decimal place. [4]

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(c) Sketch C, stating the coordinates of any intersections with the axes. [3]

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(d) Sketch the curve with equation $y = \frac{1}{\mathrm{f}(x)}$. [2]

(e) Find the set of values for which $\frac{1}{\mathrm{f}(x)} < \mathrm{f}(x)$. [4]

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

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