Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

3911748467

FURTHER MATHEMATICS **9231/31**

Paper 3 Further Mechanics **May/June 2024**

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use $10\,\text{m}\,\text{s}^{-2}$.

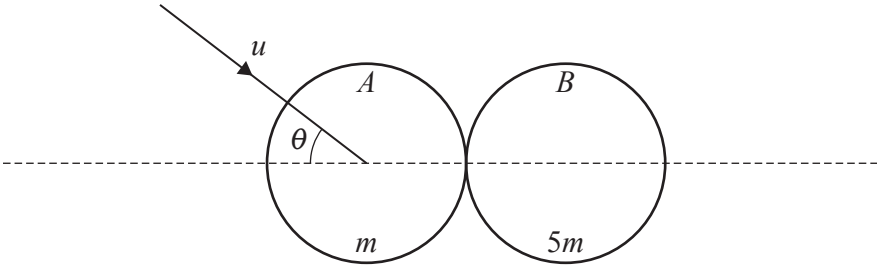
INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages.

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1



Two smooth uniform spheres A and B of equal radii have masses m and $5m$ respectively. Sphere A is moving on a smooth horizontal surface with speed u when it collides with sphere B which is at rest on the surface. Immediately before the collision, A's direction of motion makes an angle of θ with the line of centres. After the collision, the kinetic energies of A and B are equal. The coefficient of restitution between the spheres is $\frac{1}{2}$.

Find the value of $\tan\theta$. [6]

2 The points A and B are at the same horizontal level a distance $4a$ apart. The ends of a light elastic string, of natural length $4a$ and modulus of elasticity λ, are attached to A and B. A particle P of mass m is attached to the midpoint of the string. The system is in equilibrium with P at a distance $\frac{3}{2}a$ below M, the midpoint of AB.

(a) Find λ in terms of m and g. [3]

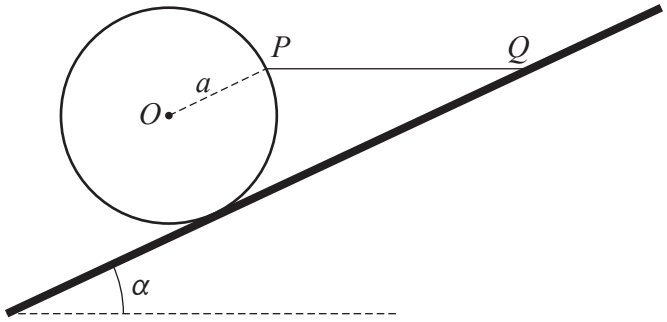
The particle P is pulled down vertically and released from rest at a distance $\frac{8}{3}a$ below M.

(b) Find, in terms of a and g, the speed of P as it passes through M in the subsequent motion. [4]

3 At time $t = 0$ seconds, a particle P is projected with speed $u\,\text{m s}^{-1}$ at an angle 60° above the horizontal from a point O. In the subsequent motion P moves freely under gravity. The direction of motion of P when $t = 5$ is perpendicular to its direction of motion when $t = 15$.

Find the value of u. [5]

4



A ring of weight W, with radius a and centre O, is at rest on a rough surface that is inclined to the horizontal at an angle α where $\tan\alpha = \frac{1}{2}$. The plane of the ring is perpendicular to the inclined surface and parallel to a line of greatest slope of the surface. The point P on the circumference of the ring is such that OP is parallel to the surface.

A light inextensible string is attached to P and to the point Q, which is on the surface, such that PQ is horizontal (see diagram). The points O, P and Q are in the same vertical plane. The system is in limiting equilibrium and the coefficient of friction between the ring and the surface is μ.

(a) Find, in terms of W, the tension in the string PQ. [4]

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(b) Find the value of μ. [3]

5 Two particles A and B of masses m and km respectively are connected by a light inextensible string of length a. The particles are placed on a rough horizontal circular turntable with the string taut and lying along a radius of the turntable. Particle A is at a distance a from the centre of the turntable and particle B is at a distance $2a$ from the centre of the turntable. The coefficient of friction between each particle and the turntable is $\frac{1}{5}$.

When the turntable is made to rotate with angular speed $\frac{2}{5}\sqrt{\frac{g}{a}}$, the system is in limiting equilibrium.

(a) Find the tension in the string, in terms of m and g. [4]

(b) Find the value of k. [3]

6 A particle P of mass 2 kg moving on a horizontal straight line has displacement x m from a fixed point O on the line and velocity $v\,\text{m s}^{-1}$ at time t s. The only horizontal force acting on P has magnitude $\frac{1}{10}(2v-1)^2\text{e}^{-t}$ N and acts towards O. When $t = 0$, $x = 1$ and $v = 3$.

(a) Find an expression for v in terms of t. [5]

(b) Find an expression for x in terms of t. [4]

7 A smooth sphere with centre O and of radius a is fixed to a horizontal plane. A particle P of mass m is projected horizontally from the highest point of the sphere with speed u, so that it begins to move along the surface of the sphere. The particle P loses contact with the sphere at the point Q on the sphere, where OQ makes an angle θ with the upward vertical through O.

(a) Show that $\cos\theta = \dfrac{u^2 + 2ag}{3ag}$. [4]

It is given that $\cos\theta = \frac{5}{6}$.

(b) Find, in terms of a and g, an expression for the vertical component of the velocity of P just before it hits the horizontal plane to which the sphere is fixed. [3]

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(c) Find an expression for the time taken by P to fall from Q to the plane. Give your answer in the form $k\sqrt{\frac{a}{g}}$, stating the value of k correct to 3 significant figures. [2]

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Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

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