Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER | | | | | |

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8225278573

FURTHER MATHEMATICS **9231/31**

Paper 3 Further Mechanics **May/June 2023**

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use $10\,\text{m}\,\text{s}^{-2}$.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

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1 One end of a light elastic string, of natural length a and modulus of elasticity $3mg$, is attached to a fixed point O. The other end of the string is attached to a particle P of mass m. The string hangs with P vertically below O. The particle P is pulled vertically downwards so that the extension of the string is $2a$. The particle P is then released from rest.

(a) Find the speed of P when it is at a distance $\frac{3}{4}a$ below O. [3]

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(b) Find the initial acceleration of P when it is released from rest. [2]

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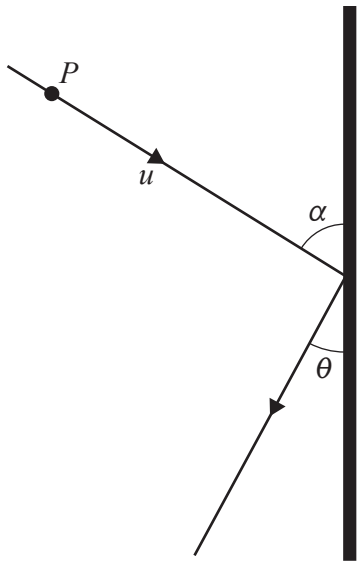
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A particle P of mass m is moving with speed u on a fixed smooth horizontal surface. It collides at an angle α with a fixed smooth vertical barrier. After the collision, P moves at an angle θ with the barrier, where $\tan\theta = \frac{1}{2}$ (see diagram). The coefficient of restitution between P and the barrier is e. The particle P loses 20% of its kinetic energy as a result of the collision.

Find the value of e. [5]

3 A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The particle P is held at the point A, where OA makes an angle θ with the downward vertical through O, and with the string taut. The particle P is projected perpendicular to OA in an upwards direction with speed u. It then starts to move along a circular path in a vertical plane. The string goes slack when P is at B, where angle AOB is 90° and the speed of P is $\sqrt{\frac{4}{5}ag}$.

(a) Find the value of $\sin\theta$. [2]

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(b) Find, in terms of m and g, the tension in the string when P is at A. [5]

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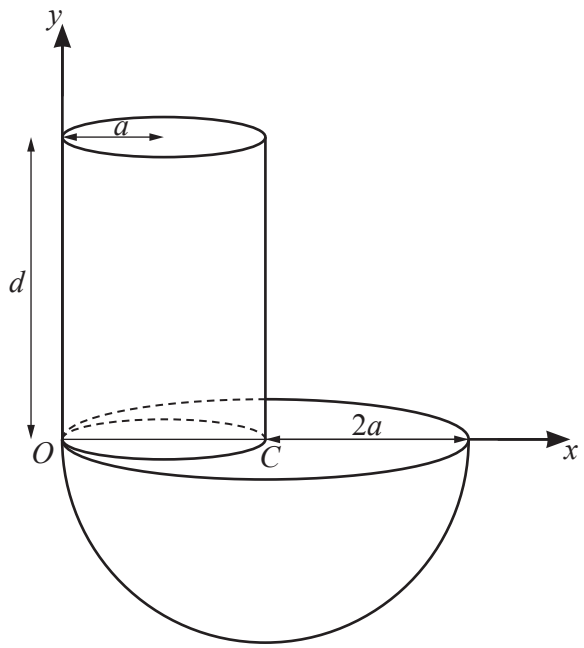
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An object is formed from a solid hemisphere, of radius $2a$, and a solid cylinder, of radius a and height d. The hemisphere and the cylinder are made of the same material. The cylinder is attached to the plane face of the hemisphere. The line OC forms a diameter of the base of the cylinder, where C is the centre of the plane face of the hemisphere and O is common to both circumferences (see diagram). Relative to axes through O, parallel and perpendicular to OC as shown, the centre of mass of the object is $(\overline{x},\overline{y})$.

(a) Show that $\overline{x} = \dfrac{32a^2 + 3ad}{16a + 3d}$ and find an expression, in terms of a and d, for $\overline{y}$. [5]

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The object is placed on a rough plane which is inclined to the horizontal at an angle θ where $\sin\theta = \frac{1}{6}$. The object is in equilibrium with CO horizontal, where CO lies in a vertical plane through a line of greatest slope.

(b) Find d in terms of a. [3]

5 A light elastic string of natural length a and modulus of elasticity λmg has one end attached to a fixed point O on a smooth horizontal surface. When a particle of mass m is attached to the free end of the string, it moves with speed v in a horizontal circle with centre O and radius x. When, instead, a particle of mass $2m$ is attached to the free end of the string, this particle moves with speed $\frac{1}{2}v$ in a horizontal circle with centre O and radius $\frac{3}{4}x$.

(a) Find x in terms of a. [5]

(b) Given that $v = \sqrt{12ag}$, find the value of λ. [2]

6 A particle P moving in a straight line has displacement x m from a fixed point O on the line and velocity $v\,\mathrm{m\,s^{-1}}$ at time t s. The acceleration of P, in $\mathrm{m\,s^{-2}}$, is given by $6v\sqrt{v+9}$. When $t = 0$, $x = 2$ and $v = 72$.

(a) Find an expression for v in terms of x. [4]

(b) Find an expression for x in terms of t. [5]

7 At time t s, a particle P is projected with speed $40\,\text{m}\,\text{s}^{-1}$ at an angle θ above the horizontal from a point O on a horizontal plane and moves freely under gravity. The greatest height achieved by P during its flight is H m and the corresponding time is T s.

(a) Obtain expressions for H and T in terms of θ. [2]

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During the time between $t = T$ and $t = 3$, P descends a distance $\frac{1}{4}H$.

(b) Find the value of θ. [4]

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(c) Find the speed of P when $t = 3$. [3]

Additional page

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