Rewarding Learning

Centre Number

Candidate Number

General Certificate of Secondary Education
November 2024

Mathematics

StemUp Model Solution

Unit M4
(With calculator)
Higher Tier



[GMC41]

GMC41

TUESDAY 19 NOVEMBER, 9.15am–11.15am

TIME

2 hours.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number in the spaces provided at the top of this page.
You must answer the questions in the spaces provided.
Do not write outside the boxed area on each page or on blank pages.
Complete in black ink only. **Do not write with a gel pen.**
Questions which require drawing or sketching should be completed using an HB pencil.
All working **must** be clearly shown in the spaces provided. Marks may be awarded for partially correct solutions.
You **may** use a calculator for this paper.
Answer **all twenty-two** questions.

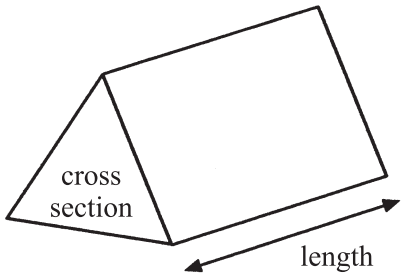
INFORMATION FOR CANDIDATES

The total mark for this paper is 100.
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
You should have a calculator, ruler, compasses and a protractor.
The Formula Sheet is on page 2.

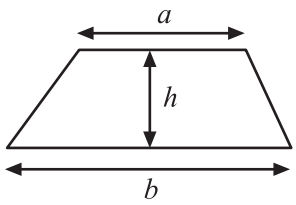
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Formula Sheet

Volume of prism = area of cross section × length

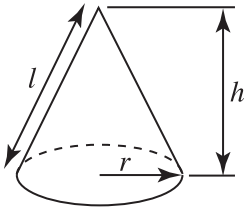


Area of trapezium = $\frac{1}{2}(a+b)h$



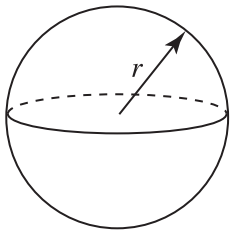
Volume of cone = $\frac{1}{3}\pi r^2 h$

Curved surface area of cone = $\pi r l$

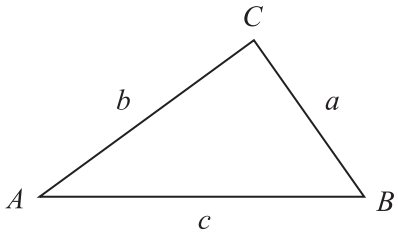


Volume of sphere = $\frac{4}{3}\pi r^3$

Surface area of sphere = $4\pi r^2$



In any triangle *ABC*



Quadratic Equation

The solutions of $ax^2 + bx + c = 0$ where $a \neq 0$, are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Sine Rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine Rule: $a^2 = b^2 + c^2 - 2bc\cos A$

Area of triangle = $\frac{1}{2}ab\sin C$

1 Anne invested £18 000 in an account paying 4% per annum compound interest.

After how many years will the value of her investment exceed £20 200?

Show your working out clearly.

Year 1: 4% of 18000 = 720, 18000 + 720 = £18720

Year 2: 4% of 18720 = 748.80, 18720 + 748.80 = £19468.80

Year 3: 4% of 19468.80 = 778.752, 19468.80 + 778.752 = £20247.55

(check: $18000 \times 1.04^2 = 19468.80 < 20200$)

($18000 \times 1.04^3 = 20247.552 > 20200$)

After 2 years £19468.80 is less than £20200,

after 3 years £20247.55 is more than £20200.

Answer 3 years [3]

[Turn over

2 x is a **two-digit** number.

The LCM (lowest common multiple) of x and 55 is 110

Find a possible value of x

$110 = 2 \times 5 \times 11$

$55 = 5 \times 11$

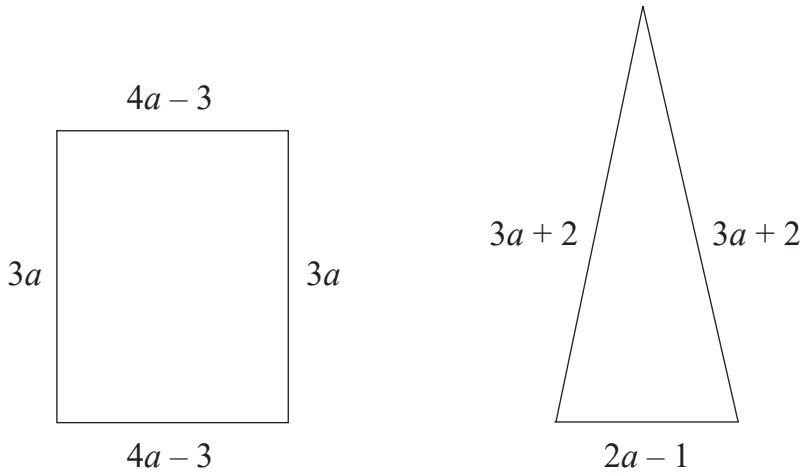
x must contain the factor 2:

$x = 2 \times 11 = 22$ (or $2 \times 5 = 10$)

LCM of 22 and 55 $= 2 \times 5 \times 11 = 110$ ✓

Answer 22 [2]

3 The rectangle and triangle drawn below have the **same perimeter**.



Work out the value of a, showing your method clearly.

Rectangle: $3a + 3a + (4a - 3) + (4a - 3) = 14a - 6$

Triangle: $(3a + 2) + (3a + 2) + (2a - 1) = 8a + 3$

$14a - 6 = 8a + 3$

$14a - 8a = 3 + 6$

$6a = 9$

$a = 9 \div 6 = 1.5$

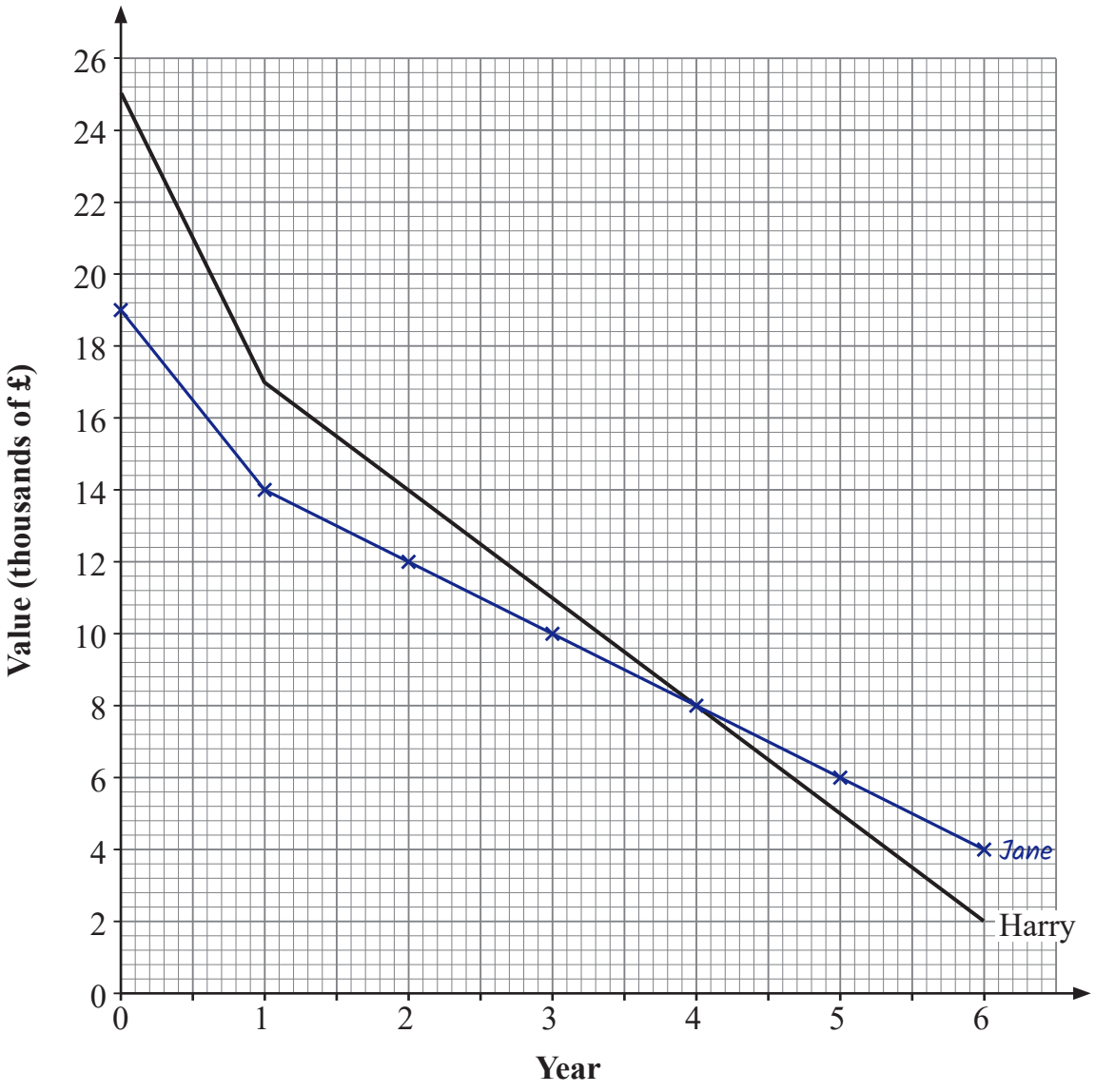
Answer $a =$ 1.5 [4]

[Turn over

4 Six years ago, Harry and Jane each bought a new car.

Harry bought his car for £25 000
In the first year, it decreased in value by £8 000
After that, it decreased in value by £3 000 each year.

A graph showing the value of Harry's car each year is shown below.



Jane bought her car for £19 000
In the first year, it decreased in value by £5 000
After that, it decreased in value by £2 000 each year.

(a) On the grid opposite, draw a graph showing the value of Jane's car each year. [2]

(b) Use the graph you have drawn to write down the year when Harry's car and Jane's car had the same value.

Answer Year 4 [1]

5 A doormat is in the shape of a semicircle with diameter 82 cm.

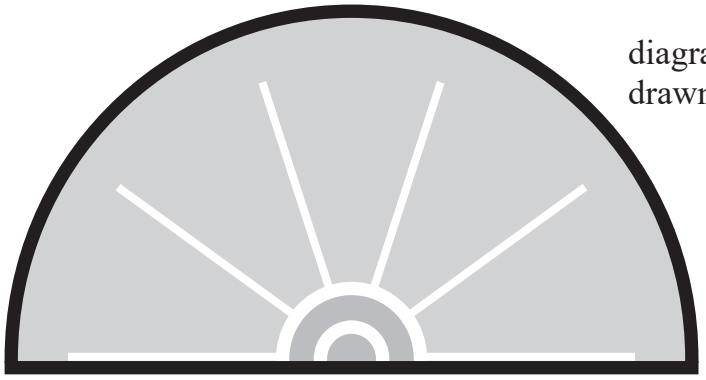


diagram not drawn accurately

Calculate the perimeter of the doormat.

Curved edge = half of circumference = (π × 82) ÷ 2

π × 82 = 257.6105976...

257.6105976... ÷ 2 = 128.8052988...

Perimeter = 128.8052988... + 82

= 210.8052988...

= 210.8 cm

Answer 210.8 cm [3]

[Turn over

6

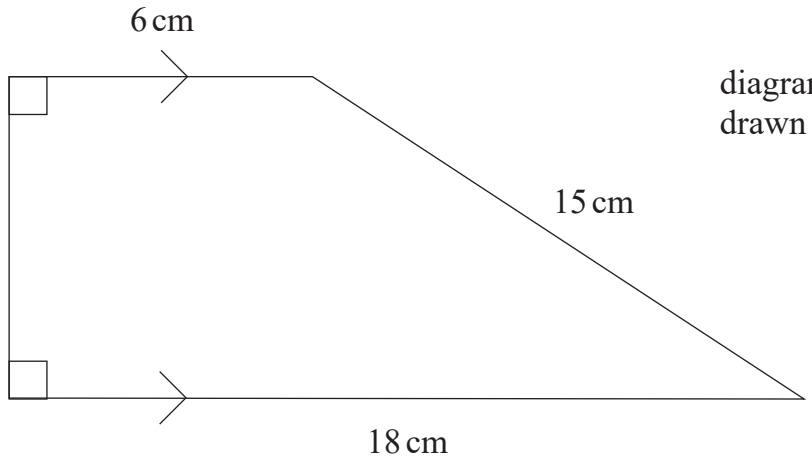


diagram not drawn accurately

Calculate the area of the trapezium.

The slanted side makes a right-angled triangle with base $18 - 6 = 12$ cm and hypotenuse 15 cm.

$h^2 + 12^2 = 15^2$

$h^2 = 225 - 144 = 81$

$h = \sqrt{81} = 9$ cm

$\text{Area} = \frac{1}{2}(a + b)h = \frac{1}{2} \times (6 + 18) \times 9$

$= 12 \times 9$

$= 108 \text{ cm}^2$

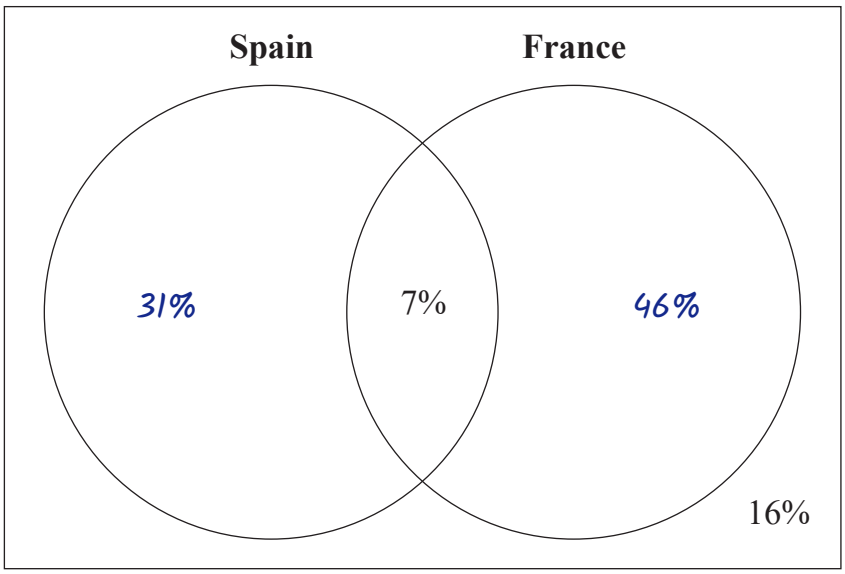
Answer 108 cm^2 [5]

7 Pupils in a year group were asked whether they had travelled to Spain or France.

Some information is shown in the Venn diagram.

53% had travelled to France.

Complete the Venn diagram below.



[2]

[Turn over

8 A group of people were surveyed about the number of driving lessons they had before passing their driving test.

The results are shown in the stem and leaf diagram.

1	2	3	5	5	8	KEY: 3 | 6 = 36 lessons
2	0	3	5	9		
3	1	6				
4	4					
5	0	1				

Alice passes her test.
When her number of driving lessons is included in the stem and leaf diagram, the range increases by 2 and the median decreases by 1

Work out the number of driving lessons Alice had.

Now: 14 values, range = 51 − 12 = 39, median = (23 + 25) ÷ 2 = 24

New range = 41, so Alice had 12 − 2 = 10 or 51 + 2 = 53 lessons.

New median must be 24 − 1 = 23 (8th of 15 values).

With 10: 10, 12, 13, 15, 15, 18, 20, 23, 25, ... 8th value = 23 ✓

With 53 the median would be 25 (increase), so Alice had 10.

Answer 10 lessons [2]

9 Solve the equation

$$\frac{18-x}{3} = 4 - 2x$$

$18 - x = 3(4 - 2x)$

$18 - x = 12 - 6x$

$6x - x = 12 - 18$

$5x = -6$

$x = -6 \div 5 = -1.2$

Answer $x =$ -1.2 [3]

[Turn over

10 In 2022 Rob sold his house for £227 700

This was a loss of 10%

He had bought the house from Lisa in 2019

She had made a profit of 15% in selling to Rob.

Work out how much Lisa originally paid for the house.

Rob sold for a 10% loss:

90% = £227700

1% = 227700 ÷ 90 = 2530

100% = 2530 × 100 = £253000 (Rob paid £253000)

Lisa sold to Rob at a 15% profit:

115% = £253000

1% = 253000 ÷ 115 = 2200

100% = 2200 × 100 = £220000

Answer £ 220000 [4]

11 A ladder is placed against a vertical wall.

To be safe, it must be inclined at an angle between 70° and 80° to the ground.

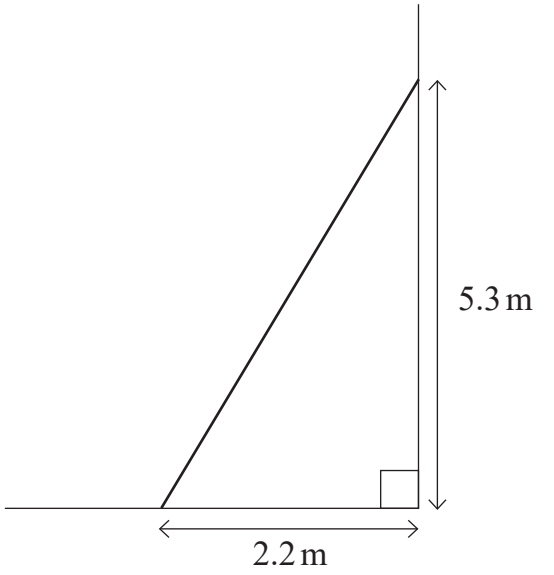


diagram not drawn accurately

Is the ladder positioned safely?

Show your working out clearly.

$\tan \alpha = \frac{opp}{adj} = \frac{5.3}{2.2}$

$\alpha = \tan^{-1}(5.3 \div 2.2)$

$= 67.45693899...°$

$= 67.5°$

67.5° is not between 70° and 80°,
so the ladder is not positioned safely.

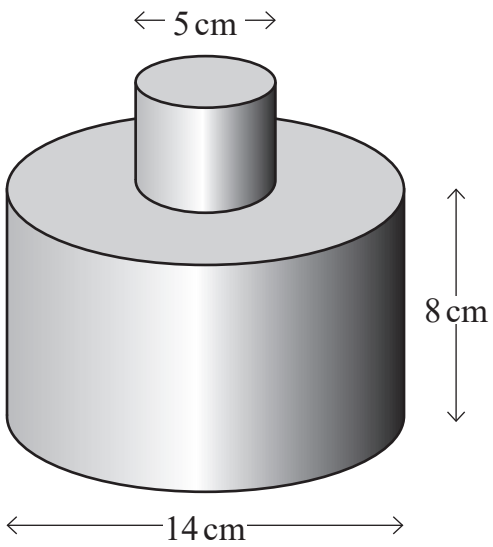
Answer No [3]

[Turn over

12 A solid is made from a cylinder of diameter 14 cm and height 8 cm, with a smaller cylinder of diameter 5 cm on top.

The total height of the solid is 12 cm.

Calculate the total surface area of the solid.



Base (bottom circle) = $\pi \times 7^2 = 153.93804...$

Top of small cylinder = $\pi \times 2.5^2 = 19.63495408...$

Top ring of large cylinder = $\pi \times 7^2 - \pi \times 2.5^2 = 134.3030859...$

Curved surface (large) = $2 \times \pi \times 7 \times 8 = 351.8583772...$

Curved surface (small) = $2 \times \pi \times 2.5 \times 4 = 62.83185307...$

(small cylinder height = $12 - 8 = 4$ cm)

Total = $153.93804 + 19.63495 + 134.30309$

$+ 351.85838 + 62.83185$

$= 722.5663103...$ $= 722.57$ cm^2

Answer 722.57 cm^2 [5]

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(Questions continue overleaf)

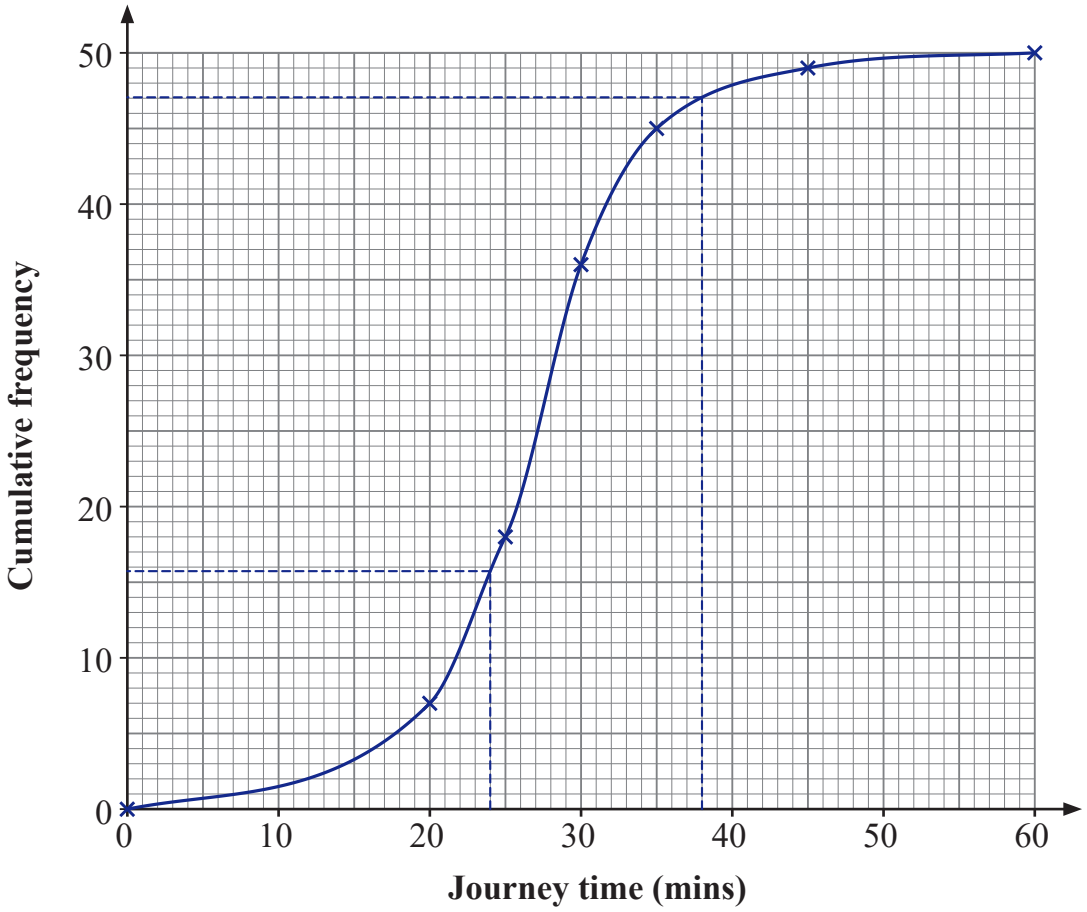
[Turn over

13 The grouped frequency table shows the time taken by 50 workers to travel to work.

Journey time (t mins)	Frequency	Journey time $\leq t$ mins	Cumulative frequency
$0 < t \leqslant 20$	7	20	7
$20 < t \leqslant 25$	11	25	18
$25 < t \leqslant 30$	18	30	36
$30 < t \leqslant 35$	9	35	45
$35 < t \leqslant 45$	4	45	49
$45 < t \leqslant 60$	1	60	50

(a) Complete the table above. [1]

(b) Draw the cumulative frequency graph on the grid below. [2]



(c) Use your graph to estimate the number of journeys that took between 24 and 38 minutes.

From graph: at 24 minutes cf ≈ 16, at 38 minutes cf ≈ 47

47 − 16 = 31

Answer *31* [2]

[Turn over

14 The light on a lighthouse flashes every 50 seconds.

The light on a different lighthouse flashes every 65 seconds.

After they flash together for the first time, how long will it be until they flash together again?

Give your answer in minutes and seconds.

$50 = 2 \times 5 \times 5$

$65 = 5 \times 13$

$LCM = 2 \times 5 \times 5 \times 13 = 650$ seconds

$650 \div 60 = 10$ remainder 50

650 s = 10 minutes 50 seconds

Answer 10 minutes 50 seconds [3]

15 The lines DBF and DAE are tangents to the circle as shown.

The lines BC and AC are equal.

Angle CAE = 61°

Use circle theorems to find the following angles, giving reasons for each answer.

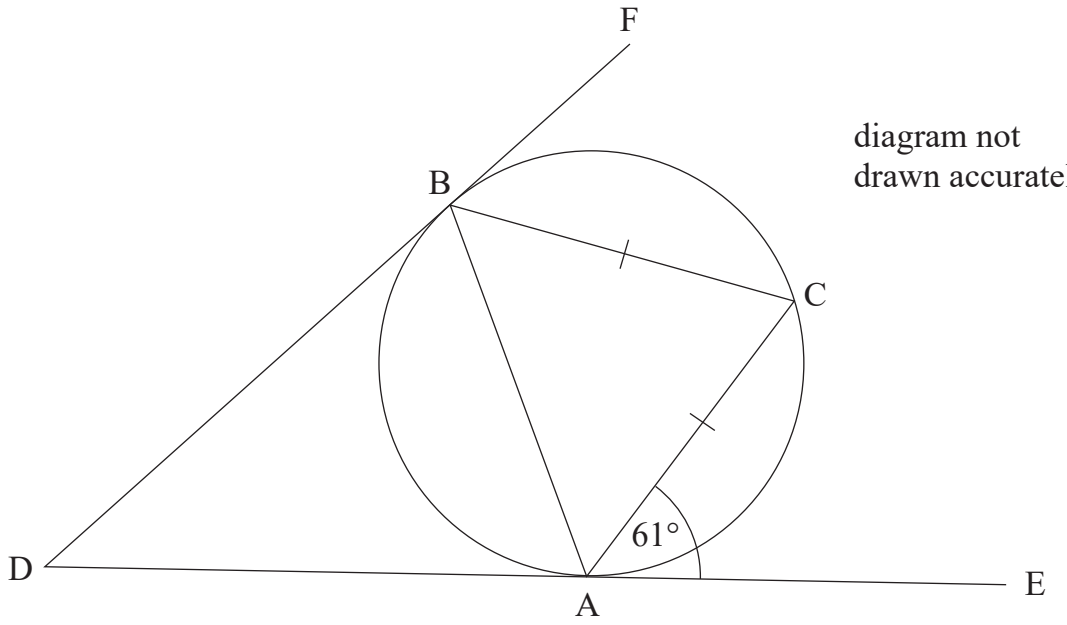


diagram not drawn accurately

(a) Angle ABC = 61 ° because of the alternate segment theorem (angle between tangent and chord = angle in the alternate segment). [2]

(b) Angle DAB = 58 ° because triangle ABC is isosceles so angle BAC = 61°, and angles on a straight line add up to 180° (180 − 61 − 61 = 58). [2]

(c) Angle BDA = 64 ° because tangents from the same point are equal, DA = DB, so triangle ABD is isosceles: 180 − 58 − 58 = 64. [2]

[Turn over

16 A stone is dropped from a bridge and falls at a speed of V m/s.

The distance it falls, S m, is calculated using the formula

$$S = \frac{V^2}{2g}$$

$V = 14.2$ m/s correct to 3 significant figures.

$g = 9.81$ m/s^2 correct to 3 significant figures.

Find the maximum possible distance the stone falls.

Upper bound of V = 14.25

Lower bound of g = 9.805

Maximum S uses the largest V and the smallest g:

$S = \frac{14.25^2}{2 \times 9.805} = \frac{203.0625}{19.61}$

= 10.35504844 m

Answer 10.35504844 m [3]

17 Find the equation of the line perpendicular to $y = 3x + 2$ which passes through the point (–3, 2).

Gradient of $y = 3x + 2$ is 3

Gradient of perpendicular $= -\frac{1}{3}$ $\left(3 \times -\frac{1}{3} = -1\right)$

$y = -\frac{1}{3}x + c$

At (–3, 2): $2 = -\frac{1}{3} \times (-3) + c$

$2 = 1 + c$

$c = 1$

Answer $y = -\frac{1}{3}x + 1$ [4]

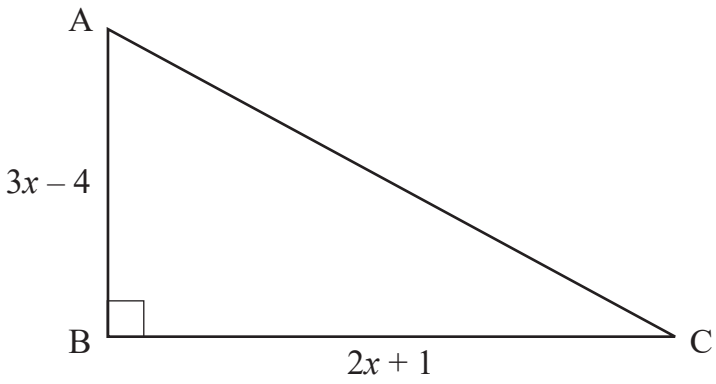
[Turn over

18 ABC is a right-angled triangle.

AB = $(3x - 4)$ cm.

BC = $(2x + 1)$ cm.

The area of ABC is $10.5\,\text{cm}^2$



(a) Show that $6x^2 - 5x - 25 = 0$

Area = $\frac{1}{2}$ × base × height = $\frac{1}{2}(2x + 1)(3x - 4) = 10.5$

$(2x + 1)(3x - 4) = 21$

$6x^2 - 8x + 3x - 4 = 21$

$6x^2 - 5x - 4 - 21 = 0$

$6x^2 - 5x - 25 = 0$

[3]

(b) Hence find the lengths of the lines AB and BC.

$6x^2 - 5x - 25 = 0$

$(2x - 5)(3x + 5) = 0$

$x = \frac{5}{2} = 2.5$ or $x = -\frac{5}{3}$ (reject, length < 0)

$AB = 3x - 4 = 3(2.5) - 4 = 3.5$

$BC = 2x + 1 = 2(2.5) + 1 = 6$

(check: ½ × 3.5 × 6 = 10.5 ✓)

Answer AB = 3.5 cm

Answer BC = 6 cm [3]

[Turn over

19 Simplify fully

$$\frac{2x^2 - 32}{3x^2 - 11x - 4}$$

Numerator: $2x^2 - 32 = 2(x^2 - 16) = 2(x + 4)(x - 4)$

Denominator: $3x^2 - 11x - 4 = (3x + 1)(x - 4)$

$$\frac{2x^2 - 32}{3x^2 - 11x - 4} = \frac{2(x + 4)(x - 4)}{(3x + 1)(x - 4)}$$

$$= \frac{2(x + 4)}{3x + 1}$$

Answer $\frac{2(x + 4)}{3x + 1}$ [5]



20 Solve the following equation, giving your answers to two decimal places.

$$\frac{2x}{3x+1} + \frac{4}{x-1} = \frac{5}{2}$$

$$\frac{2x(x-1) + 4(3x+1)}{(3x+1)(x-1)} = \frac{5}{2}$$

$$\frac{2x^2 - 2x + 12x + 4}{3x^2 - 3x + x - 1} = \frac{5}{2}$$

$$\frac{2x^2 + 10x + 4}{3x^2 - 2x - 1} = \frac{5}{2}$$

$$2(2x^2 + 10x + 4) = 5(3x^2 - 2x - 1)$$

$$4x^2 + 20x + 8 = 15x^2 - 10x - 5$$

$$0 = 11x^2 - 30x - 13$$

$$x = \frac{30 \pm \sqrt{((-30)^2 - 4 \times 11 \times (-13))}}{2 \times 11} = \frac{30 \pm \sqrt{1472}}{22}$$

$x = 3.107575099...$ or $x = -0.380302372...$

$x = 3.11$ or $x = -0.38$

Answer $x =$ 3.11 or −0.38 [8]

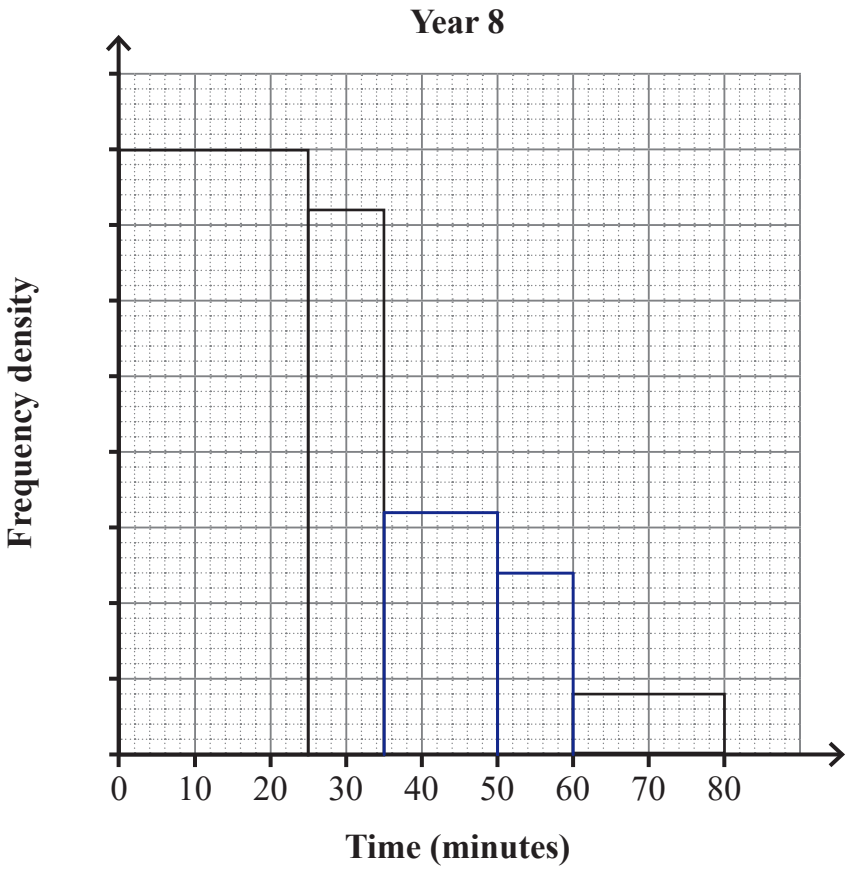
[Turn over

21 Year 8 pupils were surveyed on how much time they spent doing homework on one night.

The incomplete table and histogram show some information on this.

No student spent longer than 80 minutes on their homework.

Time, t (minutes)	Frequency
$0 < t \leqslant 25$	100
$25 < t \leqslant 35$	36
$35 < t \leqslant 50$	24
$50 < t \leqslant 60$	12
$60 < t \leqslant 80$	8



(a) Use the table to complete the histogram. [2]

(b) Use the histogram to complete the table. [2]

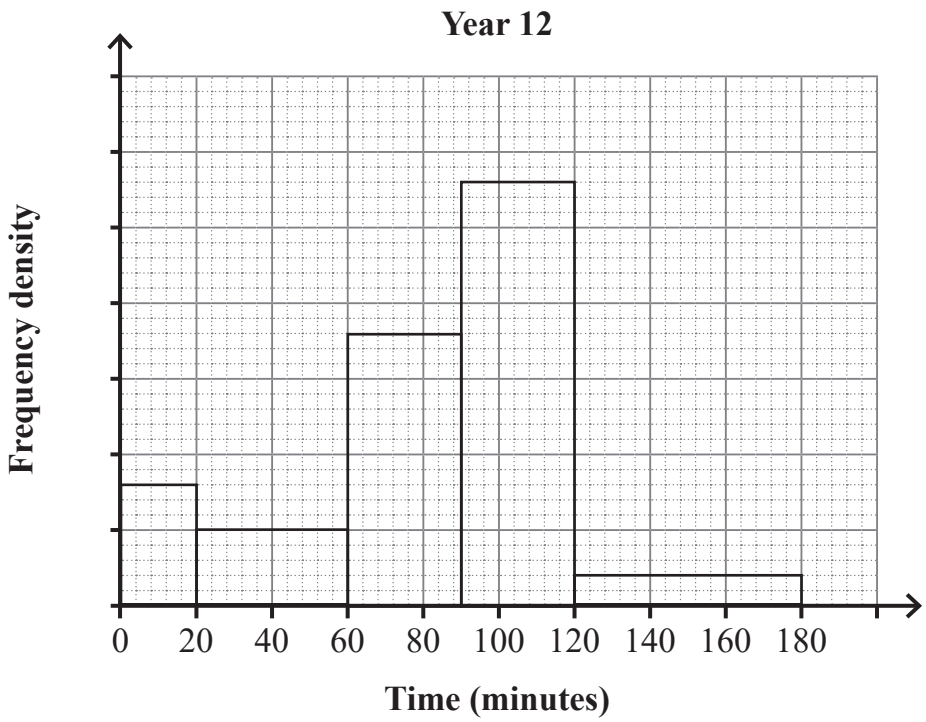
Frequency density = frequency ÷ class width: $0 < t \leq 25$: $100 \div 25 = 4$

$35 < t \leq 50$: $24 \div 15 = 1.6$ $50 < t \leq 60$: $12 \div 10 = 1.2$

From the histogram: $25 < t \leq 35$: $3.6 \times 10 = 36$ $60 < t \leq 80$: $0.4 \times 20 = 8$

[Turn over

The following histogram shows the time spent on homework by Year 12 students on the same night.



36 Year 12 students spent less than 60 minutes working on their homework.

(c) Find the mean time spent on homework by Year 12 students.

Less than 60 min: $0.8 \times 20 + 0.5 \times 40 = 16 + 20 = 36$, so these are the fds:

Frequencies: $0.8\times20 = 16$, $0.5\times40 = 20$, $1.8\times30 = 54$, $2.8\times30 = 84$, $0.2\times60 = 12$

Total $= 16 + 20 + 54 + 84 + 12 = 186$. Midpoints: 10, 40, 75, 105, 150

$$\text{Mean} = \frac{16\times10 + 20\times40 + 54\times75 + 84\times105 + 12\times150}{186}$$

$$= \frac{160 + 800 + 4050 + 8820 + 1800}{186} = \frac{15630}{186}$$

$$= 84.03225806\ldots = 84.03$$

Answer Mean = 84.03 minutes [6]

(d) A stratified sample is taken from the Year 12 students.

(i) Given that 14 students are sampled from the $90 < t \leqslant 120$ group, calculate the number of students in the sample.

$\frac{84}{186} \times n = 14$

$n = 14 \times \frac{186}{84} = 31$

Answer 31 [2]

(ii) Estimate the number of students **in the sample** who spent more than 150 minutes on homework.

More than 150 min: half of the $120 < t \leq 180$ group

$= 12 \div 2 = 6$ students

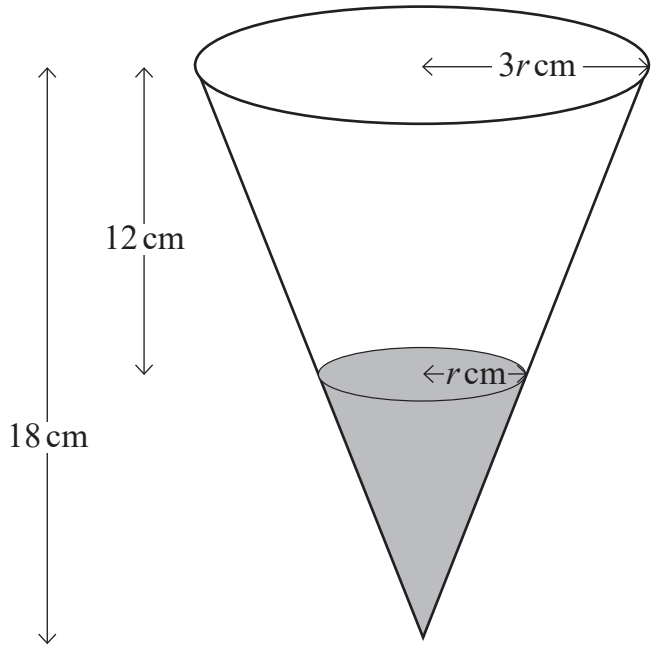
In the sample: $\frac{6}{186} \times 31 = 1$

Answer 1 [2]

[Turn over



22 An ice-cream cone is filled with chocolate at the bottom, as shown in the diagram.



The radius of the circle at the top of the chocolate is r cm and the radius of the circle at the top of the cone is $3r$ cm.

The height of the ice-cream cone is 18 cm and the distance between the top of the cone and the chocolate is 12 cm.

The volume of space left in the cone is 196 cm^3

Whole cone: radius 3r, height 18. Chocolate cone: radius r, height 18 − 12 = 6

Space left = volume of whole cone − volume of chocolate

Calculate the diameter of the cone.

$\frac{1}{3}\pi(3r)^2(18) - \frac{1}{3}\pi r^2(6) = 196$

$54\pi r^2 - 2\pi r^2 = 196$

$52\pi r^2 = 196$

$r^2 = \frac{196}{52\pi} = 1.199783417...$

$r = \sqrt{1.199783417...} = 1.095346254...$

Radius of top of cone $= 3r = 3.286038763...$

Diameter $d = 2 \times 3r = 6r = 6.572077527...$

$= 6.57$ cm

Answer $d =$ 6.57 cm [7]

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