Rewarding Learning

Centre Number

Candidate Number

General Certificate of Secondary Education
November 2023

Mathematics

StemUp Model Solution

Unit M4
(With calculator)
Higher Tier



[GMC41]

GMC41

TUESDAY 21 NOVEMBER, 9.15 am – 11.15 am

TIME

2 hours.

INSTRUCTIONS TO CANDIDATES

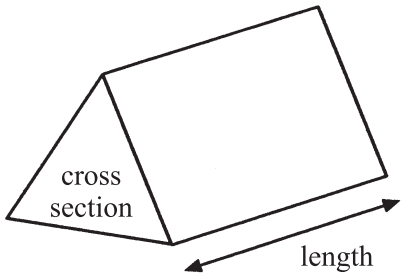
Write your Centre Number and Candidate Number in the spaces provided at the top of this page.
You must answer the questions in the spaces provided.
Do not write outside the boxed area on each page or on blank pages.
Complete in black ink only. **Do not write with a gel pen.**
Answer **all twenty-six** questions.
All working should be clearly shown in the spaces provided. Marks may be awarded for partially correct solutions.
You **may** use a calculator for this paper.

INFORMATION FOR CANDIDATES

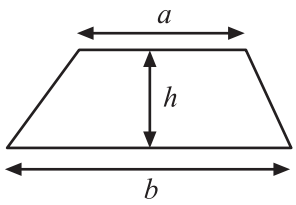
The total mark for this paper is 100.
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
You should have a calculator, ruler, compasses and a protractor.
The Formula Sheet is on page 2.

Formula Sheet

Volume of prism = area of cross section × length

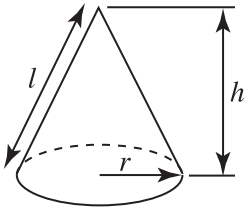


Area of trapezium = $\frac{1}{2}(a+b)h$



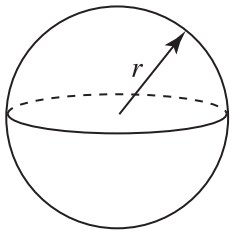
Volume of cone = $\frac{1}{3}\pi r^2 h$

Curved surface area of cone = $\pi r l$

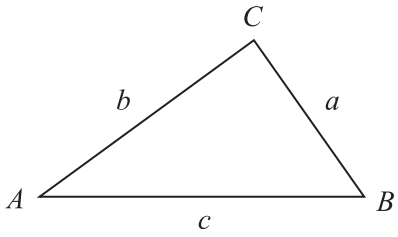


Volume of sphere = $\frac{4}{3}\pi r^3$

Surface area of sphere = $4\pi r^2$



In any triangle *ABC*



Quadratic Equation

The solutions of $ax^2 + bx + c = 0$ where $a \neq 0$, are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Sine Rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine Rule: $a^2 = b^2 + c^2 - 2bc\cos A$

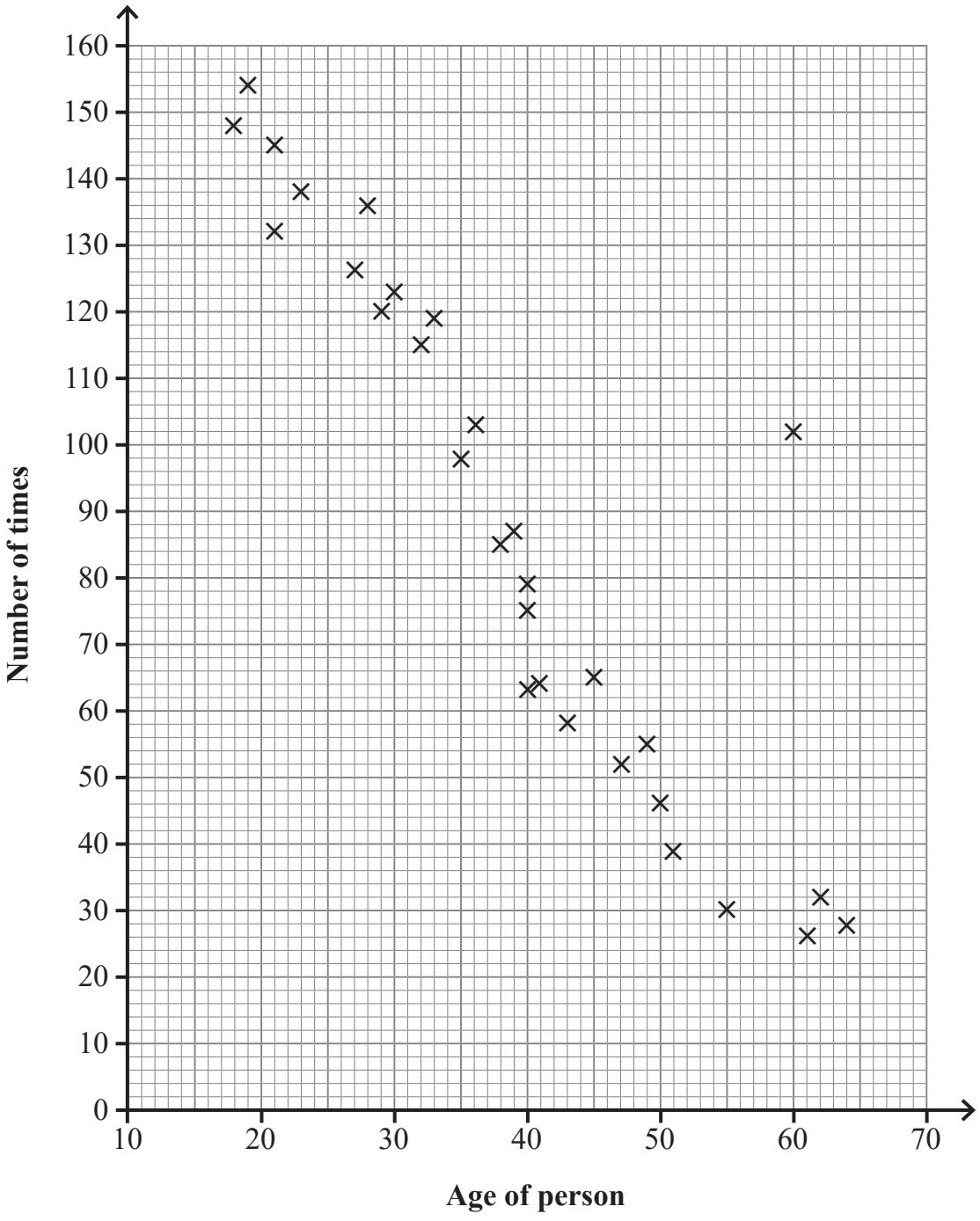
Area of triangle = $\frac{1}{2}ab\sin C$

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(Questions begin overleaf)

1 The scatter graph shows information about how many times a day people checked their phones.



14116

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(a) Describe the correlation between the age of a person and the number of times they checked their phone.

Answer _____ *negative* _____ [1]

(b) Insert the correct word to complete the following sentence.

The data point at (60, 102) is an _____ *outlier* _____ . [1]

[Turn over

2 A sports referee trains by running diagonally across a rectangular pitch, as shown in the diagram below.

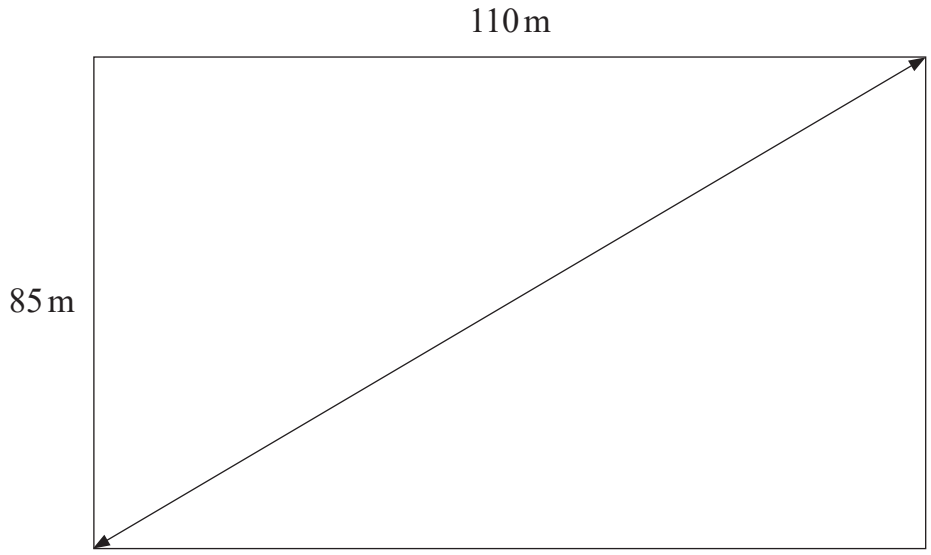


diagram not drawn to scale

The referee wants to run a distance of **at least** 1 km.

How many diagonal runs are needed?

$d^2 = 85^2 + 110^2$

$d^2 = 7225 + 12\,100 = 19\,325$

$d = \sqrt{19\,325} = 139.0143877...$ m

1 km = 1000 m

$1000 \div 139.0143877... = 7.193500...$

7 runs is less than 1 km, so 8 runs are needed

Answer 8 [5]

3 Find the area of the kite below.

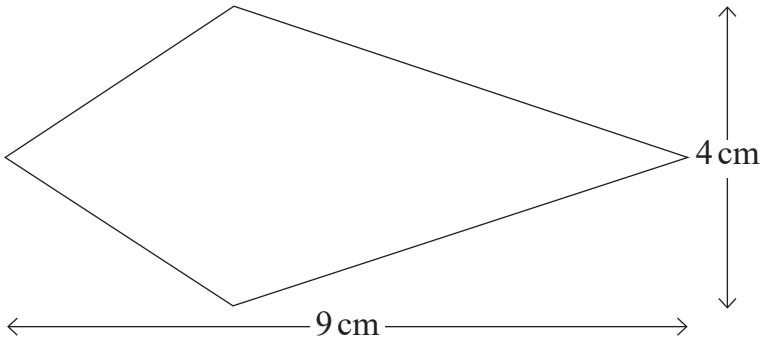


diagram not drawn to scale

Area = $\frac{1}{2}$ × 9 × 4

= 18

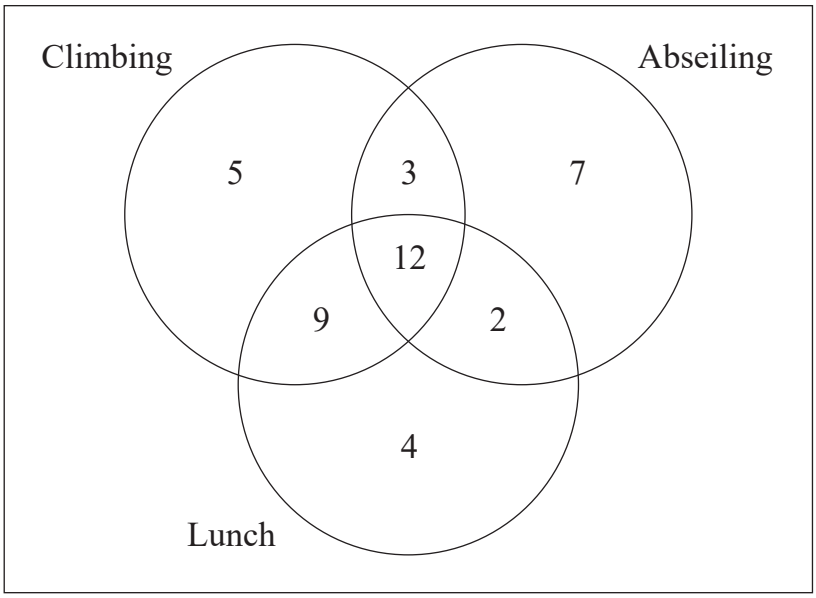
Answer 18 cm^2 [2]

[Turn over

4 A company took 42 employees on a team-building day.

Each person was able to choose which sporting activities to do and whether or not to have lunch.

The Venn diagram shows what choices they made.



(a) How many people did not take part in either of the sporting activities?

only in Lunch: 4

Answer 4 [1]

(b) How many people did not have lunch?

5 + 3 + 7 = 15

Answer 15 [1]

(c) The cost for someone to take part in one sporting activity and have lunch was £16

How much did the company pay for people who took part in one sporting activity and had lunch?

one activity and lunch: 9 + 2 = 11 people

11 × £16 = £176

Answer £ 176 [2]

5 C has the coordinates (–2, 4) and D has the coordinates (8, 7).

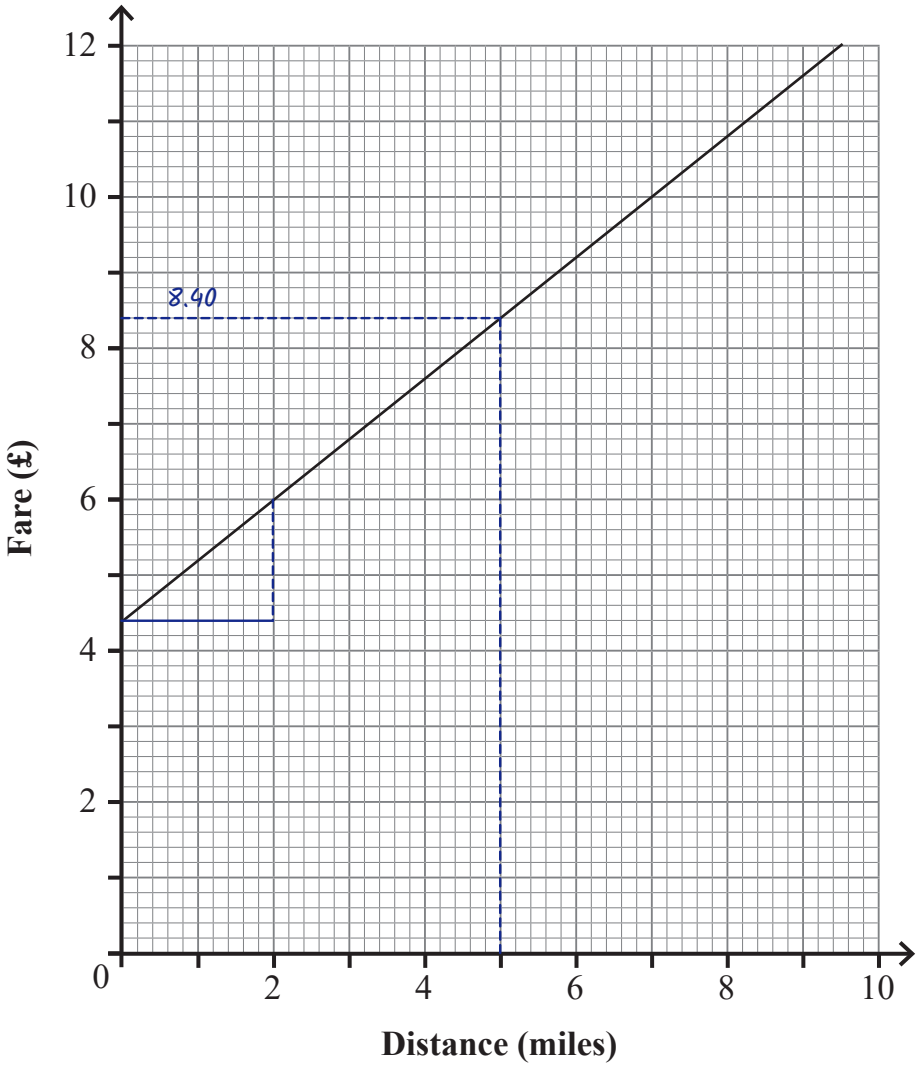
Find the coordinates of the midpoint of the line CD.

x: $\frac{-2 + 8}{2} = \frac{6}{2} = 3$ y: $\frac{4 + 7}{2} = \frac{11}{2} = 5.5$

Answer (3 , 5.5) [2]

[Turn over

6 The graph represents the fare (in £) for a taxi journey of a distance in miles.



(a) Paula was charged £8.40 for her journey to work.

How many miles did Paula travel?

Answer 5 miles [1]

(b) What is the cost per mile charged by the taxi firm?

2 miles: £6.00 − £4.40 = £1.60

1 mile: £1.60 ÷ 2 = £0.80

Answer £0.80 per mile [2]

7 **(a)** Raymond's car insurance quote is £1200 for a year.

He is entitled to 30% off for his no claims discount.

If he pays online, he will get a 5% discount off this reduced price.

How much will the online price be?

30% of £1200 = 0.3 × 1200 = £360

£1200 − £360 = £840

5% of £840 = 0.05 × 840 = £42

£840 − £42 = £798

Answer £ 798 [3]

(b) Raymond's father says the total discount was 35%

Is he correct?

You must show work to justify your answer.

Total saving = £1200 − £798 = £402

$\frac{402}{1200} \times 100 = 33.5\%$

33.5% is not 35%, so he is not correct

(35% of £1200 = £420, not £402)

Answer No [2]

[Turn over

8 Solve $\frac{x}{3} - 5 = 2$

$\frac{x}{3} = 2 + 5 = 7, \quad x = 7 \times 3$

Answer $x =$ 21 [2]

9 Find the lowest common multiple (LCM) of 60 and 36

$60 = 2 \times 2 \times 3 \times 5 = 2^2 \times 3 \times 5$

$36 = 2 \times 2 \times 3 \times 3 = 2^2 \times 3^2$

$LCM = 2^2 \times 3^2 \times 5$

$= 2 \times 2 \times 3 \times 3 \times 5 = 180$

Answer 180 [3]

10 A rectangle has length $(x + 5)$ cm and breadth $(x + 2)$ cm.

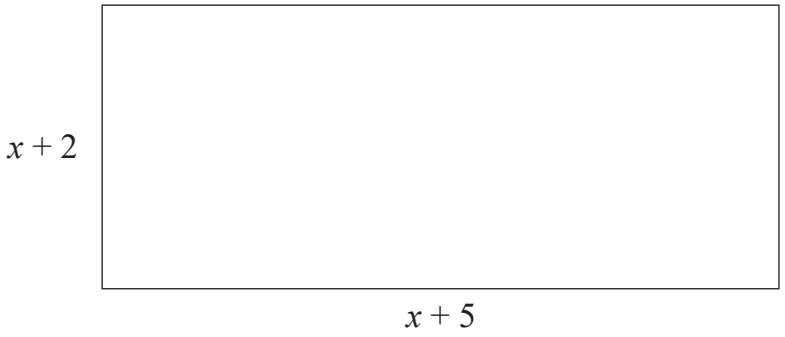


diagram not drawn to scale

The area of the rectangle is 54 cm^2

Jacob is asked to find the value of x.

He starts his solution as shown.

$$(x + 5)(x + 2) = 54$$

$$x^2 + 5x + 2x + 10 = 54$$

$$x^2 + 7x + 10 = 54$$

Continue the solution of this equation to find the value of x for this rectangle.

$x^2 + 7x + 10 - 54 = 0$

$x^2 + 7x - 44 = 0$

$(x + 11)(x - 4) = 0$

$x = -11$ or $x = 4$

a length cannot be negative, so $x = 4$

Answer $x =$ 4 [3]

[Turn over

11 Show that $\frac{3x-1}{2}+\frac{11-7x}{6}$ simplifies to the single fraction $\frac{x+4}{3}$

Show each stage of your working clearly.

$$\frac{3x-1}{2}+\frac{11-7x}{6}=\frac{3(3x-1)+11-7x}{6}$$

$$=\frac{9x-3+11-7x}{6}$$

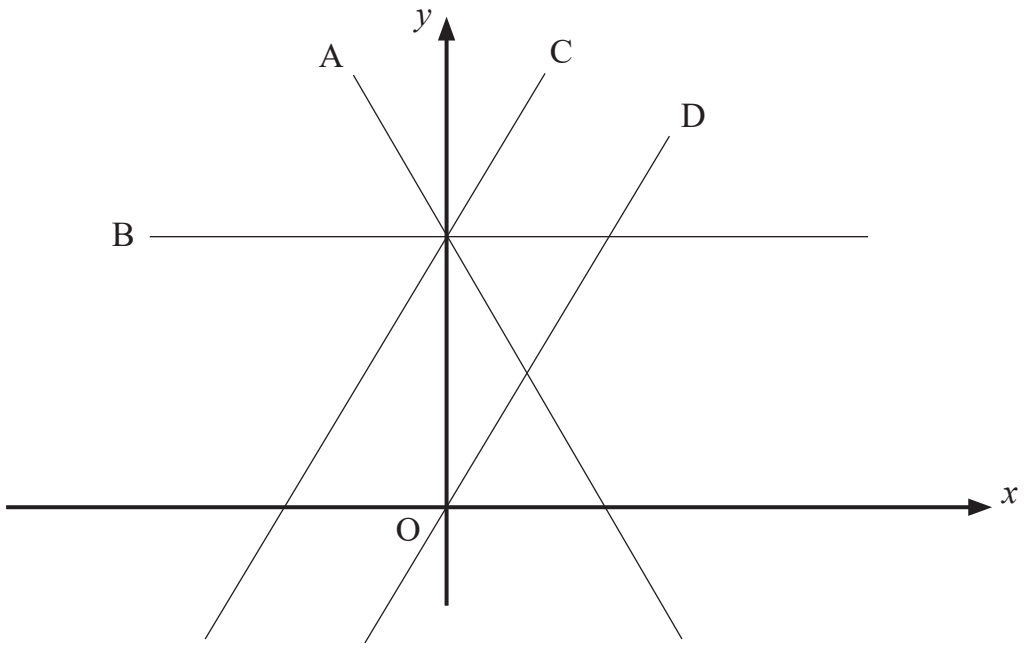
$$=\frac{2x+8}{6}$$

$$=\frac{2(x+4)}{6}$$

$$=\frac{x+4}{3}$$

[3]

12



A, B, C and D are four straight lines.

C and D are parallel.

The equations of three of these lines are

$$y = 4x \qquad y = 5 - 4x \qquad y = 5$$

Use this information to find the equation of the fourth line.

D passes through O, so D is y = 4x

A has a negative gradient, so A is y = 5 − 4x

B is horizontal, so B is y = 5

C is parallel to D, so gradient of C = 4

C crosses the y-axis at (0, 5), so c = 5

Answer C: y = 4x + 5 [4]

[Turn over

13 A force of 240 N is applied to a brake pad with an area of 0.32 m^2

Calculate the pressure produced by the brake pad.

Give units with your answer.

Pressure = Force ÷ Area

$= \frac{240}{0.32}$

$= 750$

Answer *750 N/m^2* [3]

14 A circular pizza is cut into six equal slices.

The pizza has a diameter of 10 inches.

If two slices of pizza are eaten, what area of pizza remains?

radius = 10 ÷ 2 = 5 inches
Area of pizza = $\pi \times 5^2 = 78.53981634$

4 slices remain: $\frac{4}{6} = \frac{2}{3}$ of the pizza

$\frac{2}{3} \times 78.53981634 = 52.35987756$

Answer 52.36 square inches [3]

[Turn over

15 **(a)** Calculate the interquartile range (IQR) for the following data.

24, 25, 25, 27, 28, 30, 30, 31, 34, 92

n = 10: LQ = 3rd value = 25
UQ = 8th value = 31
IQR = 31 − 25 = 6

Answer *6* [2]

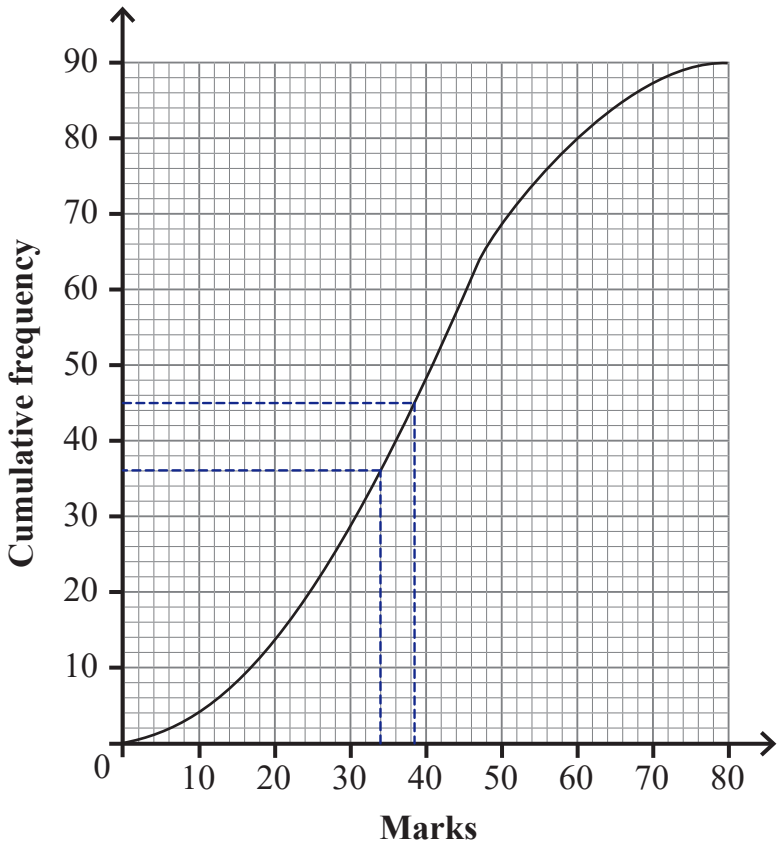
(b) For the data in **(a)** which average would be most representative?

Give a reason for your answer.

Answer *Median* because *it is not affected by the extreme value 92 (outlier), but the mean is pulled up by it* [2]

16 90 apprentices sat an engineering examination.

The cumulative frequency graph for the results is shown.



(a) Use the graph to estimate the median mark.

90 ÷ 2 = 45th value

Answer 39 [1]

(b) The pass mark was 34

Use the graph to estimate how many apprentices passed.

36 got below 34, so 90 − 36 = 54

Answer 54 [2]

[Turn over

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17 The table below shows information about the ages of a company's workforce.

Age (years)	Number of workers
21–30	340
31–40	712
41–50	405
Over 50	127

The owner of the company wants to carry out a survey on health and well-being.

He decides to take a stratified sample of 100 workers.

(a) Explain why it is appropriate to take a stratified sample.

The age groups have very different numbers of workers,

so each age group is represented fairly (in proportion). [1]

(b) Calculate the number of workers he should sample from each age group.

Total = 340 + 712 + 405 + 127 = 1584

21–30: $\frac{340}{1584} \times 100 = 21.46... \approx 21$

31–40: $\frac{712}{1584} \times 100 = 44.94... \approx 45$

41–50: $\frac{405}{1584} \times 100 = 25.56... \approx 26$

Over 50: $\frac{127}{1584} \times 100 = 8.01... \approx 8$

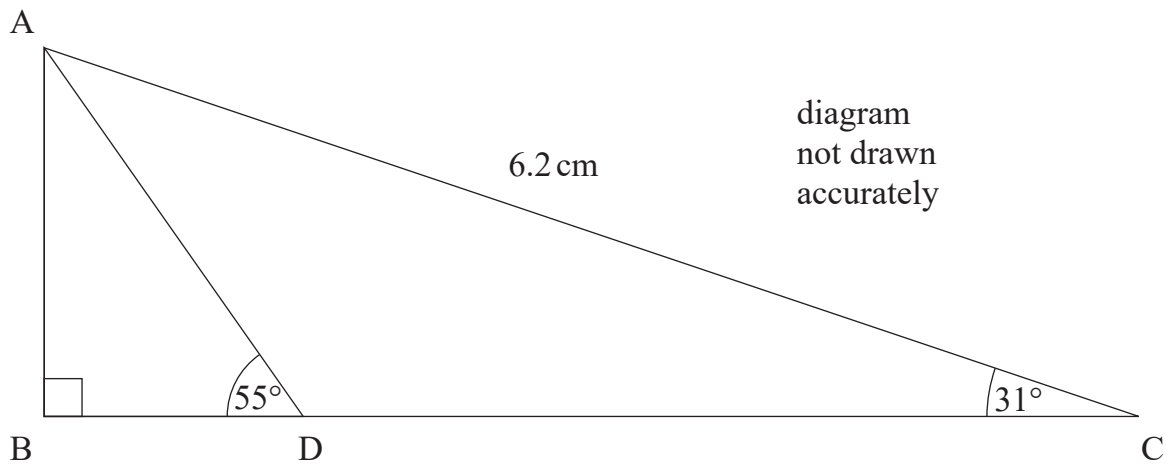
Answer 21–30 *21*

31–40 *45*

41–50 *26*

Over 50 *8* [3]

18



ABC is a right-angled triangle with angle BCA = 31°

AC = 6.2 cm

D is a point on the line BC such that angle ADB = 55°

Calculate the length of the line BD.

Give your answer to 3 significant figures.

$\sin 31° = \frac{AB}{6.2}$

$AB = 6.2 \times \sin 31° = 3.193236064$

$\tan 55° = \frac{AB}{BD} = \frac{3.193236064}{BD}$

$BD = \frac{3.193236064}{\tan 55°} = 2.235927964$

Answer 2.24 cm [4]

[Turn over

19 A box, open at the top, has a total external surface area of 140 cm^2

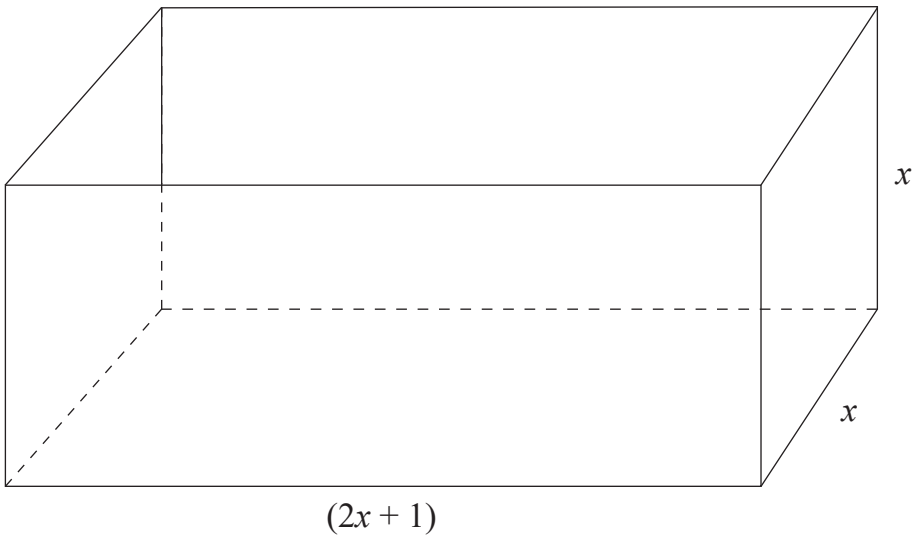


diagram not drawn accurately

The length of the box is $(2x + 1)$ cm.

The width and height of the box are both x cm.

(a) Show that $8x^2 + 3x - 140 = 0$

base: $x(2x + 1)$, front and back: $2 \times x(2x + 1)$

two ends: $2 \times x^2$ (no top)

$3x(2x + 1) + 2x^2 = 140$

$6x^2 + 3x + 2x^2 = 140$

$8x^2 + 3x - 140 = 0$

[3]

(b) Calculate the volume of the box.

$8x^2 + 3x - 140 = 0$

$(x - 4)(8x + 35) = 0$

$x = 4$ or $x = -4.375$ (not possible, length)

$x = 4$: length $= 2 \times 4 + 1 = 9$ cm

Volume $= 9 \times 4 \times 4$

$= 144$

Answer 144 cm^3 [4]

[Turn over

20 The length of a rectangle is 5.4 cm correct to 1 decimal place.

The area of the rectangle is 23.21 cm^2 correct to 4 significant figures.

Calculate the minimum value of the width of the rectangle.

Write down all the figures on your calculator display.

upper bound of length = 5.45 cm
lower bound of area = 23.205 cm^2
min width = min area ÷ max length

$= \frac{23.205}{5.45} = 4.257798165$

Answer 4.257798165 cm [3]

21

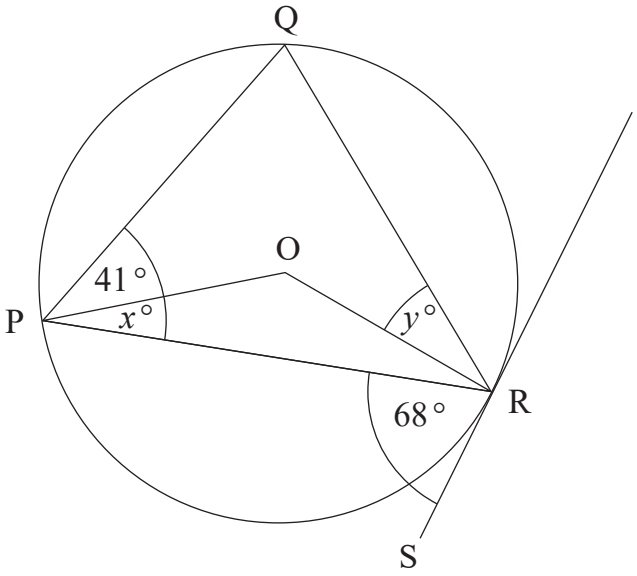


diagram not drawn accurately

The diagram shows a circle with centre O.

SR is a tangent to the circle.

Find the size of angles x and y giving reasons for each stage of your working.

Angle PQR = 68° (alternate segment theorem)
Angle POR = 2 × 68 = 136° (angle at the centre is twice the angle at the circumference)
OP = OR (radii), so triangle OPR is isosceles
x = (180 − 136) ÷ 2 = 22° (base angles of an isosceles triangle are equal)
reflex angle POR = 360 − 136 = 224°
y = 360 − 41 − 68 − 224 = 27° (angles in a quadrilateral add up to 360°)

Answer $x =$ 22 °

$y =$ 27 ° [6]

[Turn over

22 Factorise $8ax^2 - 6axy - 5ay^2$

$= a(8x^2 - 6xy - 5y^2)$

$= a(4x - 5y)(2x + y)$

check: $(4x - 5y)(2x + y) = 8x^2 + 4xy - 10xy - 5y^2$

Answer $a(4x - 5y)(2x + y)$ [3]



23 Solve the following equation, giving your answers to 2 decimal places.

$$\frac{4}{3x+1} + \frac{5}{2x-3} = \frac{1}{3}$$

$\frac{4(2x-3) + 5(3x+1)}{(3x+1)(2x-3)} = \frac{1}{3}$

$\frac{8x - 12 + 15x + 5}{6x^2 - 9x + 2x - 3} = \frac{1}{3}$

$\frac{23x - 7}{6x^2 - 7x - 3} = \frac{1}{3}$

$6x^2 - 7x - 3 = 3(23x - 7)$

$6x^2 - 7x - 3 = 69x - 21$

$6x^2 - 76x + 18 = 0$

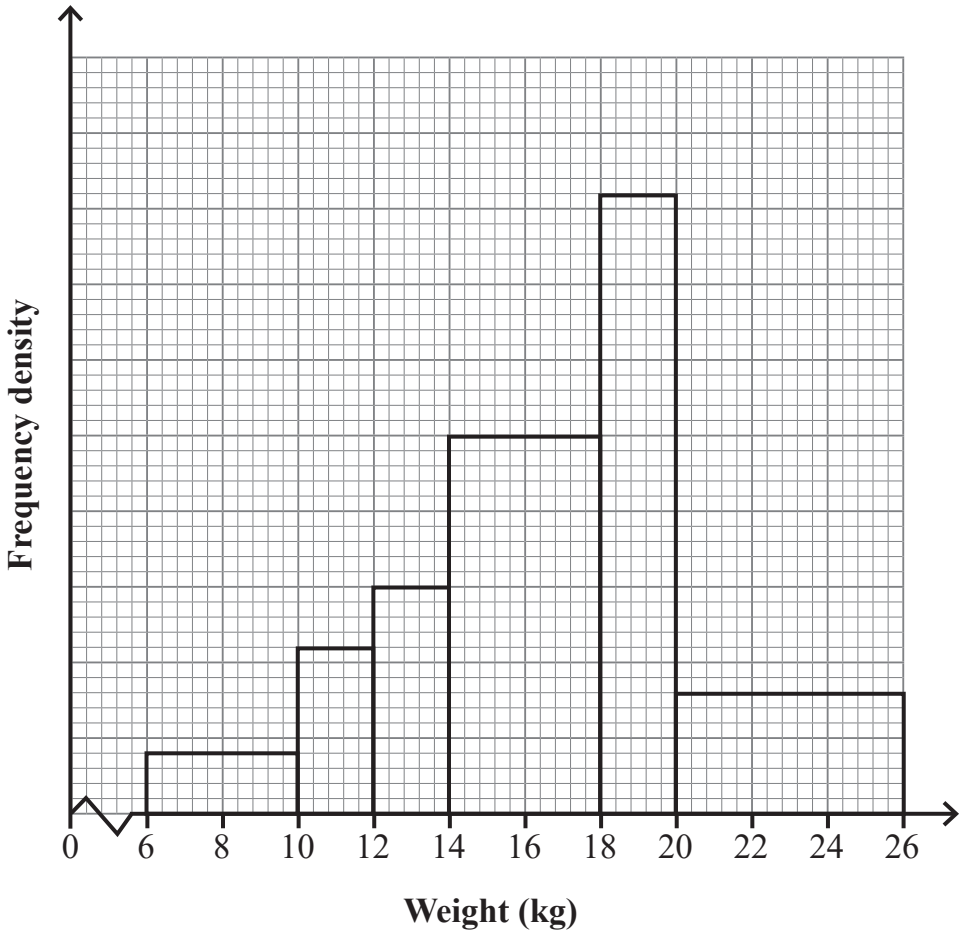
$x = \frac{76 \pm \sqrt{((-76)^2 - 4 \times 6 \times 18)}}{2 \times 6} = \frac{76 \pm \sqrt{5344}}{12}$

$x = 12.42522229...$ or $x = 0.2414443725...$

Answers $x =$ 12.43 or 0.24 [7]

[Turn over

24 The histogram shows the weights in kg of 894 dogs registered with a vet.



48 dogs weigh under 10 kg.

(a) Calculate an estimate of the number of dogs weighing between 14 kg and 21 kg.

6–10: 4 × FD = 48, FD = 12 = 4 squares, so 1 square = 3

14–18: 4 × (25 × 3) = 300

18–20: 2 × (41 × 3) = 246

20–21: 1 × (8 × 3) = 24

300 + 246 + 24 = 570

Answer 570 dogs [3]

(b) Calculate an estimate for the median weight of the dogs.

894 ÷ 2 = 447th value

below 14 kg: 48 + 2 × (11 × 3) + 2 × (15 × 3)

= 48 + 66 + 90 = 204

447 − 204 = 243 into the group 14–18 (300 dogs)

$\text{median} = 14 + \frac{243}{300} \times 4 = 17.24$

Answer 17.24 kg [3]

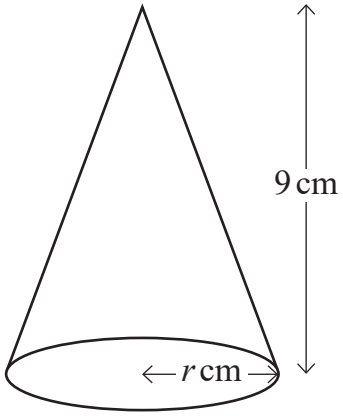
25 Simplify $\frac{x^3 - 3x^2}{x^2 - 9}$

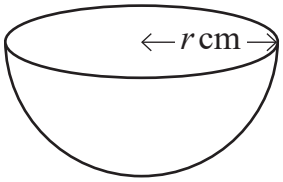
$\frac{x^3 - 3x^2}{x^2 - 9} = \frac{x^2(x - 3)}{(x + 3)(x - 3)} = \frac{x^2}{x + 3}$

Answer $\frac{x^2}{x + 3}$ [3]

[Turn over

26





diagrams not drawn accurately

A solid cone has a radius of r cm and a perpendicular height of 9 cm.

A solid hemisphere also has a radius of r cm.

The surface areas of both shapes are equal.

Calculate the value of the radius r.

$l^2 = 9^2 + r^2$, so $l = \sqrt{(81 + r^2)}$

S.A. of cone $= \pi r^2 + \pi r l = \pi r^2 + \pi r\sqrt{(81 + r^2)}$

S.A. of hemisphere $= \pi r^2 + \frac{1}{2}(4\pi r^2) = 3\pi r^2$

$3\pi r^2 = \pi r^2 + \pi r\sqrt{(81 + r^2)}$

$2\pi r^2 = \pi r\sqrt{(81 + r^2)}$

$2r = \sqrt{(81 + r^2)}$

$4r^2 = 81 + r^2$

$3r^2 = 81, \quad r^2 = 27$

$r = \sqrt{27} = 3\sqrt{3} = 5.196152423$

Answer $r =$ $3\sqrt{3}$ (5.196) [6]

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